Small droplets of liquid are formed when a liquid jet breaks up in spray and fuel injection processes. The resulting droplet diameter, d, is thought to depend on liquid density, ρ, viscosity, μ, and surface tension, σ, as well as jet speed, V, and diameter, D. How many dimensionless ratios are required to characterize this process? Determine these ratios.

SOLUTION:

1. Write the dimensional functional relationship.

$$d = f_1(\rho, \mu, \sigma, V, D)$$

2. Determine the basic dimensions of each parameter.

$$[d] = L$$
$$[\rho] = M/L^3$$
$$[\mu] = M/LT$$
$$[\sigma] = F/L = M/T^2$$
$$[V] = L/T$$
$$[D] = L$$

3. Determine the number of Π terms required to describe the functional relationship.

   # of variables = 6 (d, ρ, μ, σ, V, D)
   # of reference dimensions = 3 (M, L, T)

   (# Π terms) = (# of variables) – (# of reference dimensions) = 6 – 3 = 3

4. Choose three repeating variables by which all other variables will be normalized (same # as the # of reference dimensions).

   ρ, V, D (Note that these repeating variables have independent dimensions.)

5. Make the remaining non-repeating variables dimensionless using the repeating variables.

$$\Pi_1 = d\rho^a V^b D^c$$
$$\Rightarrow \; M^0 L^0 T^0 = \left(L/1\right)\left(M/L^3\right)^a \left(L/T\right)^b \left(L/1\right)^c$$

M: $0 = a \quad \Rightarrow \; a = 0$
T: $0 = -b \quad \Rightarrow \; b = 0$
L: $0 = 1 - 3a + b + c \quad \Rightarrow \; c = -1$

$$\therefore \Pi_1 = \frac{d}{D}$$

$$\Pi_2 = \mu\rho^a V^b D^c$$
$$\Rightarrow \; M^0 L^0 T^0 = \left(\frac{M}{LT}\right)\left(M/L^3\right)^a \left(L/T\right)^b \left(L/1\right)^c$$

M: $0 = 1 + a \quad \Rightarrow \; a = -1$
T: $0 = -1 - b \quad \Rightarrow \; b = -1$
L: $0 = -1 - 3a + b + c \quad \Rightarrow \; c = -1$

$$\therefore \Pi_2 = \frac{\mu}{\rho V D} \text{ or } \Pi_2 = \frac{\rho V D}{\mu} \text{ (a Reynolds number!)}$$

$$\Pi_3 = \sigma\rho^a V^b D^c$$

$$\Rightarrow \; M^0 L^0 T^0 = \left(\frac{M}{T^2}\right)\left(M/L^3\right)^a \left(L/T\right)^b \left(L/1\right)^c$$

M: $0 = 1 + a \quad \Rightarrow \; a = -1$

T: $0 = -2 - b \quad \Rightarrow \; b = -2$

L: $0 = -3a + b + c \quad \Rightarrow \; c = -1$

$$\therefore \Pi_3 = \frac{\sigma}{\rho V^2 D} \text{ or } \Pi_3 = \frac{\rho V^2 D}{\sigma} \text{ (a Weber number!)}$$

6. Verify that each Π term is, in fact, dimensionless.

$$[\Pi_1] = \left[\frac{d}{D}\right] = L/1 \; 1/L = 1 \;\text{ OK!}$$

$$[\Pi_2] = \left[\frac{\rho V D}{\mu}\right] = M/L^3 \; L/T \; L/1 \; LT/M = 1 \;\text{ OK!}$$

$$[\Pi_3] = \left[\frac{\rho V^2 D}{\sigma}\right] = M/L^3 \; L^2/T^2 \; L/1 \; T^2/M = 1 \;\text{ OK!}$$

7. Re-write the original relationship in dimensionless terms.

$$\boxed{\frac{d}{D} = f_2\left(\underbrace{\frac{\rho V D}{\mu}}_{\text{Reynolds \#}}, \underbrace{\frac{\rho V^2 D}{\sigma}}_{\text{Weber \#}}\right)} \qquad (1)$$