The x-velocity component of a steady, 2D, incompressible flow is given by:

$$u_x = y - x$$

Determine the most general velocity component in the y-direction for this flow.

SOLUTION:

Consider the continuity equation:

$$\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}=0 \tag{1}$$

$$\frac{\partial u_y}{\partial y}=-\frac{\partial u_x}{\partial x}=-\frac{\partial}{\partial x}\left(y-x\right)=1$$

Integrate u_y with respect to y.

$$\boxed{u_y=y+f\left(x\right)} \tag{2}$$

where $f(x)$ is an unknown function of x.

Double check:

$$\frac{\partial u_x}{\partial x}=\frac{\partial}{\partial x}\left(y-x\right)=-1 \tag{3}$$

$$\frac{\partial u_y}{\partial y}=\frac{\partial}{\partial y}\left[y+f\left(x\right)\right]=1 \tag{4}$$

$$\Rightarrow\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}=-1+1=0 \quad \text{OK!} \tag{5}$$