A velocity field for an incompressible flow is given by

$$\mathbf{u} = (-2xz)\hat{\mathbf{i}} + (2xy + z^2)\hat{\mathbf{j}} + (z^2 - 2xz - 2yz)\hat{\mathbf{k}}$$

Is this flow physically possible?

SOLUTION:

Does the given velocity field satisfy the continuity equation?

$$\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}+\frac{\partial u_z}{\partial z}=0 \tag{1}$$

Using the given velocity field:

$$\frac{\partial u_x}{\partial x}=\frac{\partial}{\partial x}\left(-2xz\right)=-2z$$

$$\frac{\partial u_y}{\partial y}=\frac{\partial}{\partial y}\left(2xy+z^2\right)=2x$$

$$\frac{\partial u_z}{\partial z}=\frac{\partial}{\partial z}\left(z^2-2xz-2yz\right)=2z-2x-2y$$

Substitute into Eqn. (1).

$$\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}+\frac{\partial u_z}{\partial z}=-2z+2x+2z-2x-2y=-2y\neq 0$$

Hence, the given flow field is <u>not</u> physically possible since it does not satisfy the continuity equation.