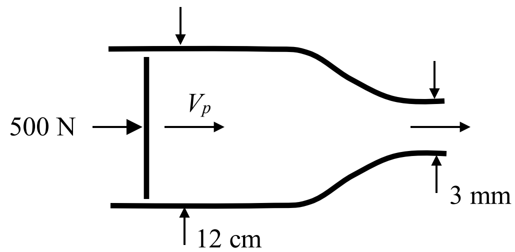
A force of 500 N pushes a piston of diameter 12 cm through an insulated cylinder containing air at 20 °C. The exit diameter is 3 mm and the atmospheric pressure is 1 atm. Estimate:

1. the exit velocity,
2. the velocity near the piston (V_p), and
3. mass flow rate out of the device.



SOLUTION:

The pressure at the piston face may be found from the piston force and piston diameter.

$$p_1 = \frac{F}{\frac{\pi}{4}d_1^2} + p_{\text{atm}} = \frac{(500\text{ N})}{\frac{\pi}{4}(0.12\text{ m})^2} + 101\text{ kPa} = 145\text{ kPa} \tag{1}$$

Assume the flow through the piston is isentropic. The velocity at the exit may be found by applying conservation of energy to the air inside the piston with 1 signifying the location adjacent to the piston face and 2 signifying the device's exit.

$$\left(h + \tfrac{1}{2}V^2\right)_2 - \left(h + \tfrac{1}{2}V^2\right)_1 = Q_{\substack{\text{into}\\\text{air}}} + W_{\substack{\text{on}\\\text{air}}} \tag{2}$$

Assuming perfect gas behavior, adiabatic conditions, and that $V_2 >> V_1$ (since the areas are so different):

$$c_P(T_2 - T_1) + \tfrac{1}{2}V_2^2 = 0 \tag{3}$$

$$V_2 = \sqrt{2c_P(T_1 - T_2)} \tag{4}$$

Also assume that the flow is isentropic so that:

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \tag{5}$$

Using the given data:

c_P = 1004 J/(kg·K)
T_1 = (20 + 273) K = 293 K
p_1 = $145*10^3$ Pa (from Eqn. (1))
p_2 = $101*10^3$ Pa (discharging into the atmosphere, assuming the exit Mach number is subsonic)
$\Rightarrow T_2$ = 264 K
$\boxed{\therefore V_2 = 241\text{ m/s}}$

Check that the exit Mach number is subsonic.

$$c_2 = \sqrt{\gamma R T_2} \Rightarrow c_2 = 326\text{ m/s} \tag{6}$$

Since $V_2 < c_2$, the exit flow is subsonic and the assumption that $p_2 = p_{\text{atm}}$ is a good one.

From conservation of mass applied to the same control volume:

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 \Rightarrow V_1 = V_2\left(\frac{\rho_2}{\rho_1}\right)\left(\frac{D_2}{D_1}\right)^2 = V_2\left(\frac{p_2}{p_1}\right)\left(\frac{T_1}{T_2}\right)\left(\frac{D_2}{D_1}\right)^2 \tag{7}$$

$\boxed{\therefore V_1 = 0.116\text{ m/s}}$ Clearly the assumption that $V_2 >> V_1$ was a good one.

The mass flow rate is:

$$m = \rho_2 V_2 A_2 = \frac{p_2}{RT_2}V_2 \tfrac{\pi}{4}D_2^2 \tag{8}$$

$\boxed{\therefore m = 2.27*10^{-3}\text{ kg/s}}$