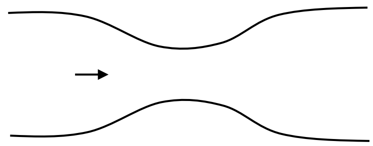
A rocket engine is designed to operate at a pressure ratio (inlet reservoir pressure/back pressure) of 37. Find:

a. the ratio of the exit area to the throat area which is necessary for the supersonic exhaust to be correctly expanded,
b. the Mach number of the exit flow under correctly expanded conditions,
c. the lowest pressure ratio (p_0/p_b) at which the same nozzle would be choked, and
d. the pressure ratio (p_0/p_b) at which there would be a normal shock wave at the exit.

Assume the specific heat ratio of the gas is 1.4.

SOLUTION:



$p_0/p_e = 37$

The area ratio may be found from the isentropic sonic area ratio and the isentropic pressure ratio.

$$\frac{p_e}{p_0} = \left(1 + \frac{\gamma - 1}{2}\mathrm{Ma}_e^2\right)^{\frac{\gamma}{1-\gamma}} \Rightarrow \boxed{\mathrm{Ma}_e = 3.0} \text{ (since at design conditions, the flow is isentropic)} \quad (1)$$

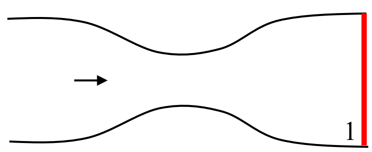
$$\frac{A_e}{A^*} = \frac{1}{\mathrm{Ma}_e}\left(\frac{1 + \frac{\gamma - 1}{2}\mathrm{Ma}_e^2}{1 + \frac{\gamma - 1}{2}}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \Rightarrow \boxed{A_e/A_t = 4.3} \text{ (since } A_t = A^*\text{)} \quad (2)$$

The lowest pressure ratio for which the nozzle will be choked may be found Eqns. (2) and (1), but using the subsonic Mach number.

$$\frac{A_e}{A^*} = \frac{A_e}{A_t} = \frac{1}{\mathrm{Ma}_e}\left(\frac{1 + \frac{\gamma - 1}{2}\mathrm{Ma}_e^2}{1 + \frac{\gamma - 1}{2}}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \Rightarrow \mathrm{Ma}_e = 0.14 \quad (3)$$

$$\frac{p_e}{p_0} = \left(1 + \frac{\gamma - 1}{2}\mathrm{Ma}_e^2\right)^{\frac{\gamma}{1-\gamma}} \Rightarrow \boxed{p_0/p_b = 1.01} \text{ Note that } p_e = p_b \text{ when the flow just becomes choked.} \quad (4)$$

Now consider a case where a shock wave occurs at the exit of the device.



From Eqn. (1), $\mathrm{Ma}_{e1} = 3.0$ and $p_{01}/p_{e1} = 37$. From the normal shock relations,

$$\mathrm{Ma}_2^2 = \frac{(\gamma - 1)\mathrm{Ma}_1^2 + 2}{2\gamma \mathrm{Ma}_1^2 - (\gamma - 1)} \Rightarrow \mathrm{Ma}_2 = 0.475 \quad (5)$$

$$\frac{p_{02}}{p_{01}} = \left[\frac{(\gamma + 1)\mathrm{Ma}_1^2}{2 + (\gamma - 1)\mathrm{Ma}_1^2}\right]^{\frac{\gamma}{\gamma-1}} \left[\frac{\gamma + 1}{2\gamma \mathrm{Ma}_1^2 - (\gamma - 1)}\right]^{\frac{1}{\gamma-1}} \Rightarrow p_{02}/p_{01} = 0.327 \quad (6)$$

and the isentropic relations:

$$\frac{p_2}{p_{02}} = \left(1 + \frac{\gamma - 1}{2}\mathrm{Ma}_2^2\right)^{\frac{\gamma}{1-\gamma}} \Rightarrow p_2/p_{02} = 0.857 \quad (7)$$

Since the flow downstream of the shock is subsonic, $p_2 = p_b$. Thus,

$$\frac{p_{01}}{p_b} = \left(\frac{p_{01}}{p_{02}}\right)\left(\frac{p_{02}}{p_2}\right) \Rightarrow \boxed{p_{01}/p_b = 3.6} \quad (8)$$