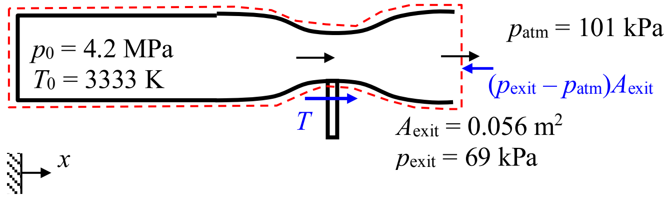
A small, solid fuel rocket motor is tested on a horizontal thrust stand at atmospheric conditions. The chamber (essentially a large tank) absolute pressure and temperature are maintained at 4.2 MPa (abs) and 3333 K, respectively. The rocket's converging-diverging nozzle is designed to expand the exhaust gas isentropically to an absolute pressure of 69 kPa. The nozzle exit area is 0.056 m^2. The gas may be treated as a perfect gas with a specific heat ratio of 1.2 and an ideal gas constant of 300 J/(kg·K). Determine, for design conditions:

a. the mass flow rate of propellant gas, and
b. the thrust force exerted on the test stand.

SOLUTION:



Determine the mass flow rate using the conditions at the exit. The Mach number at the exit may be found from the isentropic stagnation pressure ratio:

$$\frac{p_{exit}}{p_0} = \left(1 + \frac{k-1}{2}\mathrm{Ma}_{exit}^2\right)^{\frac{k}{1-k}} \tag{1}$$

Using p_{exit} = 69e3 Pa, p_0 = 4.2e6 Pa, and k = 1.2:

$$\therefore \mathrm{Ma}_{exit} = 3.136 \tag{2}$$

The exit temperature may be found using the stagnation temperature ratio:

$$\frac{T_{exit}}{T_0} = \left(1 + \frac{k-1}{2}\mathrm{Ma}_{exit}^2\right)^{-1} \tag{3}$$

Using T_0 = 3333 K, Ma_{exit} = 3.136, and k = 1.2:

$$\therefore T_{exit} = 1680 \text{ K} \tag{4}$$

The exit density may be found using the ideal gas law:

$$p_{exit} = \rho_{exit} R T_{exit} \tag{5}$$

Using p_{exit} = 69e3 Pa, T_{exit} = 1680 K, and R = 300 J/(kg·K):

$$\therefore \rho_{exit} = 0.1369 \text{ kg/m}^3 \tag{6}$$

The exit velocity may be found using the speed of sound at the exit and the Mach number definition:

$$c_{exit} = \sqrt{kRT_{exit}} \tag{7}$$

$$V_{exit} = c_{exit}\mathrm{Ma}_{exit} \tag{8}$$

Using k = 1.2, R = 300 J/(kg·K), T_{exit} = 1680 K, and Ma_{exit} = 3.136:

$$\therefore c_{exit} = 777.8 \text{ m/s} \tag{9}$$

$$\therefore V_{exit} = 2439 \text{ m/s} \tag{10}$$

The mass flow rate through the nozzle is:

$$\dot{m} = \rho_{exit} V_{exit} A_{exit} \tag{11}$$

Using ρ_{exit} = 0.1369 kg/m³, V_{exit} = 2439 m/s, and A_{exit} = 0.056 m²:

$$\boxed{\therefore \dot{m} = 18.69 \text{ kg/s}} \tag{12}$$

The thrust force, T, acting on the stand may be determined using the linear momentum equation in the x-direction for the control volume shown in the figure.

$$\frac{d}{dt}\int_{\mathrm{CV}} u_x \rho dV + \int_{\mathrm{CS}} u_x \left(\rho \mathbf{u}_{\mathrm{rel}} \cdot d\mathbf{A}\right) = F_{B,x} + F_{S,x} \tag{13}$$

where

$$\frac{d}{dt}\int_{\mathrm{CV}} u_x \rho dV = 0 \quad \text{(steady flow)} \tag{14}$$

$$\int_{\mathrm{CS}} u_x \left(\rho \mathbf{u}_{\mathrm{rel}} \cdot d\mathbf{A}\right) = \dot{m} V_{\mathrm{exit}} \tag{15}$$

$$F_{B,x} = 0 \tag{16}$$

$$F_{S,x} = T - \left(p_{\mathrm{exit}} - p_{\mathrm{atm}}\right) A_{\mathrm{exit}} \tag{17}$$

Substitute and simplify.

$$\dot{m} V_{\mathrm{exit}} = T - \left(p_{\mathrm{exit}} - p_{\mathrm{atm}}\right) A_{\mathrm{exit}} \tag{18}$$

$$\therefore T = \dot{m} V_{\mathrm{exit}} + \left(p_{\mathrm{exit}} - p_{\mathrm{atm}}\right) A_{\mathrm{exit}} \tag{19}$$

Using $\dot{m}$ = 18.69 kg/s, V_{exit} = 2439 m/s, p_{exit} = 69e3 Pa, p_{atm} = 101e3 Pa, and A_{exit} = 0.056 m^2:

$$\boxed{\therefore T = 4.380\text{e}4 \text{ N}} \tag{20}$$