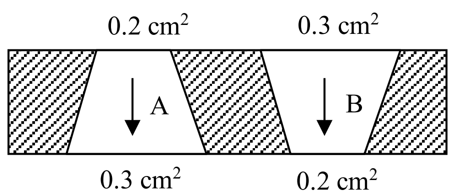
The orientation of a hole can make a difference. Consider holes A and B in the figure below which are identical but reversed. For the given air properties on either side, compute the mass flow rate through each hole and explain why they are different.

p_0 = 150 kPa (abs)
T_0 = 20 °C



p_{back} = 100 kPa (abs)

SOLUTION:

First consider flow through hole B which can be considered a converging nozzle. First check to see if the flow is choked.

$$\frac{p_B}{p_0} = \frac{100\text{ kPa}}{150\text{ kPa}} = 0.6667 > \frac{p^*}{p_0} = 0.5283 \quad \Rightarrow \text{ The flow is not choked. (Note that } \gamma_{\text{air}} = 1.4.) \qquad (1)$$

The mass flow rate can be found from the conditions at the hole exit.

$$\dot{m} = \rho_E V_E A_E \qquad (2)$$

where

$$\rho_E = \rho_0 \left(1 + \frac{\gamma - 1}{2}\text{Ma}_E^2\right)^{\frac{1}{1-\gamma}} \qquad (3)$$

$$\rho_0 = \frac{p_0}{RT_0} \qquad (4)$$

$$\frac{p_E}{p_0} = \frac{p_B}{p_0} = \left(1 + \frac{\gamma - 1}{2}\text{Ma}_E^2\right)^{\frac{\gamma}{1-\gamma}} \qquad (5)$$

$$V_E = \text{Ma}_E \sqrt{\gamma R T_E} \qquad (6)$$

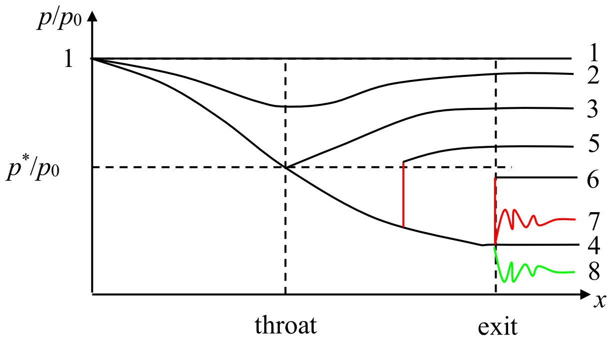
$$T_E = T_0 \left(1 + \frac{\gamma - 1}{2}\text{Ma}_E^2\right)^{-1} \qquad (7)$$

Using the given data:

γ = 1.4
R = 287 J/(kg·K)
p_0 = 150e3 Pa
T_0 = 20 °C = 293 K
p_E = 100e3 Pa (Note that since the exit flow is subsonic, $p_E = p_B$.)
A_E = 0.2 cm^2 = 2.0e-5 m^2
Ma_E = 0.784
ρ_0 = 1.784 kg/m^3
ρ_E = 1.335 kg/m^3
T_E = 261 K
V_E = 254 m/s

$\therefore\ \dot{m}_B$ = 6.78e-3 kg/s

Now consider hole A which can be modeled as a converging-diverging nozzle. Check to see what p_B/p_0 ratio will result in choked flow (case 3 in the figure below).



$$\frac{A_{E,\text{crit}}}{A^*} = \frac{A_E}{A_T} = \frac{1}{\text{Ma}_{E,\text{crit}}}\left(\frac{1+\frac{\gamma-1}{2}\text{Ma}_{E,\text{crit}}^2}{1+\frac{\gamma-1}{2}}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \Rightarrow \text{Ma}_{E,\text{crit}} = 0.43 \quad (8)$$

using $A_E = 0.3\ \text{cm}^2$ and $A_T = 0.2\ \text{cm}^2$.

$$\frac{p_{E,\text{crit}}}{p_0} = \left(1+\frac{\gamma-1}{2}\text{Ma}_{E,\text{crit}}^2\right)^{\frac{\gamma}{1-\gamma}} \Rightarrow \frac{p_{E,\text{crit}}}{p_0} = 0.8805 \quad (9)$$

For the given situation, $p_B/p_0 = 0.6667$ (refer to Eq. (1)) $< p_{E,\text{crit}}/p_0 = 0.8805$ so the flow for hole A must be choked! The mass flow rate through the hole can be found using the (sonic) conditions at the throat.

$$\dot{m} = \rho_T V_T A_T \quad (10)$$

where

$$\rho_T = \rho^* = \rho_0\left(1+\frac{\gamma-1}{2}\right)^{\frac{1}{1-\gamma}} = 0.6339 \ \ (\text{using } \gamma_{\text{air}} = 1.4) \quad (11)$$

$$\rho_0 = \frac{p_0}{RT_0} \quad (12)$$

$$\frac{p_E}{p_0} = \frac{p_B}{p_0} = \left(1+\frac{\gamma-1}{2}\text{Ma}_E^2\right)^{\frac{\gamma}{1-\gamma}} \quad (13)$$

$$V_T = c^* = \sqrt{\gamma R T^*} \quad (14)$$

$$T^* = T_0\left(1+\frac{\gamma-1}{2}\right)^{-1} = 0.8333 \quad (15)$$

Using the given data:

A_T = 0.2 cm^2 = 2.0e-5 m^2
ρ^* = 1.131 kg/m^3
T^* = 244.2 K
V_T = 313.2 m/s
$\therefore \dot{m}_A$ = 7.08e-3 kg/s

The different mass flow rates through holes A and B are because the flow through hole A is choked (the hole acts as a converging-diverging nozzle) while through hole B the flow is not choked (the hole acts as a converging nozzle).