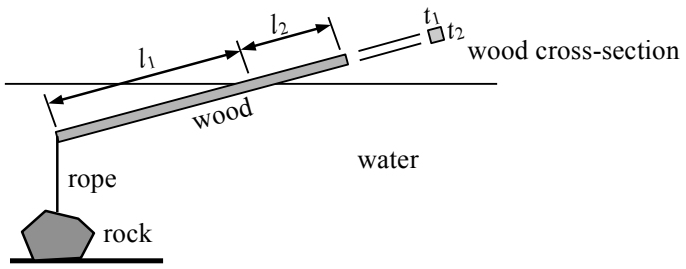
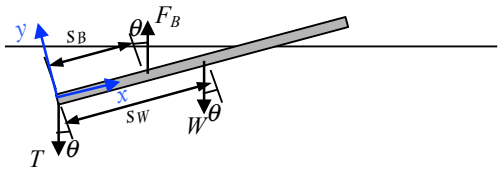
What is the specific weight of the wood in the figure below? What is the tension in the rope?



SOLUTION:
Draw a free body diagram of the stick.



First, sum forces in the vertical direction and set equal to zero (static system),

$$\sum F_y = 0 = -T\cos\theta + F_B\cos\theta - W\cos\theta \,, \tag{1}$$

where, assuming the cross-sectional dimensions of the stick are small compared to the stick length,

$$F_B = \rho_{\text{H2O}} g l_1 t_1 t_2 \,, \tag{2}$$

$$W = \rho_{\text{wood}} g\left(l_1 + l_2\right) t_1 t_2 \,. \tag{3}$$

Substitute and solve for T,

$$0 = -T\cos\theta + \rho_{\text{H2O}} g l_1 t_1 t_2 \cos\theta - \rho_{\text{wood}} g\left(l_1 + l_2\right) t_1 t_2 \cos\theta \,, \tag{4}$$

$$T = \rho_{\text{H2O}} g l_1 t_1 t_2 - \rho_{\text{wood}} g\left(l_1 + l_2\right) t_1 t_2 \,. \tag{5}$$

Note that at this point we don't know either the rope tension or the specific weight of the wood. We need another equation to uniquely solve for both. Balance moments about the origin in order to provide this other equation,

$$\sum M_O = 0 = s_B F_B \cos\theta - s_W W\cos\theta \,, \tag{6}$$

where,

$$s_B = \frac{1}{2} l_1 \,, \tag{7}$$

$$s_W = \frac{1}{2}\left(l_1 + l_2\right) \,. \tag{8}$$

Substitute and solve for the specific weight of the wood,

$$0 = \left(\frac{1}{2} l_1\right)\left(\rho_{\text{H2O}} g l_1 t_1 t_2\right)\cos\theta - \left[\frac{1}{2}\left(l_1 + l_2\right)\right]\left[\rho_{\text{wood}} g\left(l_1 + l_2\right) t_1 t_2\right]\cos\theta \,, \tag{9}$$

$$\rho_{\text{wood}} g\left(l_1 + l_2\right)^2 = \rho_{\text{H2O}} g l_1^2 \,, \tag{10}$$

$$\boxed{\rho_{\text{wood}} g = \rho_{\text{H2O}} g \frac{l_1^2}{\left(l_1 + l_2\right)^2} = \rho_{\text{H2O}} g \frac{1}{\left(1 + l_2/l_1\right)^2}} \,. \tag{11}$$

Substitute the previous result back into Eq. (5) and simplify,

$$T = \rho_{H2O} g l_1 t_1 t_2 - \rho_{H2O} g \frac{l_1^2}{\left(l_1 + l_2\right)^2}\left(l_1 + l_2\right) t_1 t_2 , \tag{12}$$

$$\boxed{T = \rho_{H2O} g l_1 t_1 t_2 \left(1 - \frac{1}{1 + l_2 / l_1}\right)} \tag{13}$$