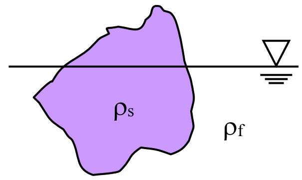
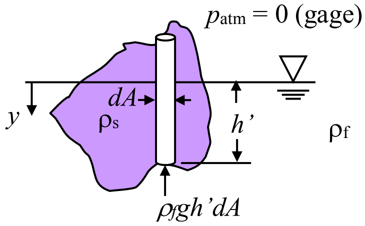
a. Determine the buoyant force and center of buoyancy for a partially submerged object as shown below.



b. Using your result from part (a), determine what percentage of an iceberg's volume remains below the water line. The specific gravity of ice is 0.890 and the specific gravity of sea water is 1.028.

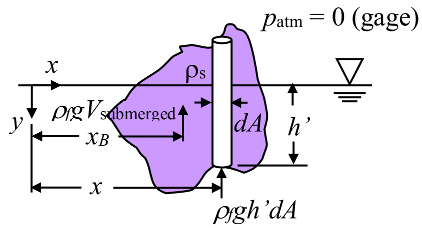
SOLUTION:

Consider a vertical force balance on a differential cylindrical element as shown below.



$$F_{\text{pressure,net}} = \int_{A_{\text{cross-section}}} -\rho_f g h' dA = -\rho_f g \int_{A_{\text{cross-section}}} h' dA$$

$$\boxed{F_{\text{buoyant}} = F_{\text{pressure,net}} = -\rho_f g V_{\text{submerged}}} \text{ (force is directed upward)} \quad (1)$$



To find the center of buoyancy, perform a moment balance.

$$\left(-\rho_f g V_{\text{submerged}}\right) x_B = \int_{A_{\text{cross-section}}} \left(-\rho_f g h' dA\right) x = -\rho_f g \int_{A_{\text{cross-section}}} h' dA x = -\rho_f g \int_{V_{\text{submerged}}} x dV$$

$$\boxed{x_B = \frac{1}{V_{\text{submerged}}} \int_{V_{\text{submerged}}} x dV}$$ The center of buoyancy is equal to the center of submerged volume. (2)

Perform a vertical force balance on the iceberg to determine what percentage of it remains below the water line.

$$W = F_{\text{buoyancy}}$$

$$\rho_S g V_{\text{total}} = \rho_f g V_{\text{submerged}}$$

$$\boxed{\frac{V_{\text{submerged}}}{V_{\text{total}}} = \frac{\rho_S}{\rho_f} = \frac{SG_S}{SG_f}} \quad (3)$$

For SG_S = 0.890 and SG_f = 1.028, $\boxed{V_{\text{submerged}}/V_{\text{total}} = 0.87 = 87\%}$.