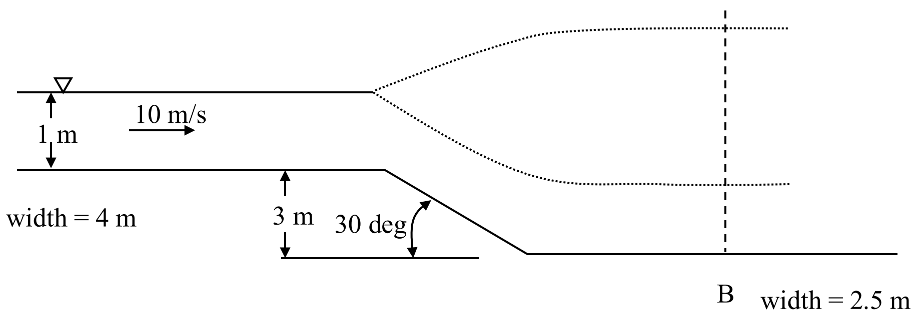
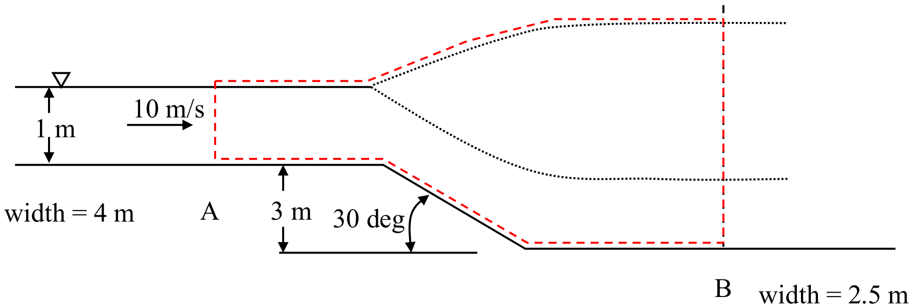
Water 1 m deep is flowing steadily at 10 m/s in a channel 4 m wide. The channel drops 3 m at 30 deg, and simultaneously narrows to 2.5 m as shown in the accompanying sketch.

Determine the two possible water depths at downstream station B. Neglect all losses.



SOLUTION:

Apply conservation of mass to the control volume shown in the figure below.



$$\frac{d}{dt}\int_{CV}\rho dV + \int_{CS}\rho \mathbf{u}_{rel}\cdot d\mathbf{A} = 0 \tag{1}$$

where

$$\frac{d}{dt}\int_{CV}\rho dV = 0 \quad \text{(steady flow)} \tag{2}$$

$$\int_{CS}\rho \mathbf{u}_{rel}\cdot d\mathbf{A} = -\left(\rho Vzw\right)_A + \left(\rho Vzw\right)_B \tag{3}$$

Substitute and simplify noting that the water density remains constant.

$$-\left(\rho Vzw\right)_A + \left(\rho Vzw\right)_B = 0 \;\Rightarrow\; \left(Vzw\right)_B = \left(Vzw\right)_A \;\Rightarrow\; V_B = V_A\left(\frac{z_A}{z_B}\right)\left(\frac{w_A}{w_B}\right) \tag{4}$$

Now apply Bernoulli's equation along the free surface of the stream from point A to point B.

$$\left(\frac{p}{\rho g} + \frac{V^2}{2g} + z\right)_B = \left(\frac{p}{\rho g} + \frac{V^2}{2g} + z\right)_A \tag{5}$$

where

$$p_A = p_B = p_{atm} \tag{6}$$

$$\frac{V_B^2}{2g} + z_B = \frac{V_A^2}{2g} + z_A \tag{7}$$

Substitute Eq. (4) and solve for z_B.

$$\frac{V_A^2\left(\frac{z_A}{z_B}\right)^2\left(\frac{w_A}{w_B}\right)^2}{2g} + z_B = \frac{V_A^2}{2g} + z_A \tag{8}$$

$$V_A^2\left(\frac{z_A}{z_B}\right)^2\left(\frac{w_A}{w_B}\right)^2 + 2gz_B = V_A^2 + 2gz_A \tag{9}$$

$$V_A^2 z_A^2\left(\frac{w_A}{w_B}\right)^2 + 2gz_B^3 = \left(V_A^2 + 2gz_A\right)z_B^2 \tag{10}$$

$$z_B^3 - \left(\frac{V_A^2}{2g} + z_A\right)z_B^2 + \frac{V_A^2}{2g}z_A^2\left(\frac{w_A}{w_B}\right)^2 = 0 \tag{11}$$

Using the given parameters:

V_A = 10 m/s
g = 9.81 m/s^2
z_A = 1 m
w_A = 4 m
w_B = 2.5 m

Eq. (11) may be written as,

$$z_B^3 - (6.097 \text{ m}) z_B^2 + (13.05 \text{ m}) = 0 \tag{12}$$

Solve Eq. (12) numerically to get,

$$z_B = 1.728 \text{ m}, 5.695 \text{ m}, -1.326 \text{ m} \tag{13}$$

Thus, the two possible depths at location B are: $\boxed{\text{1.7 m and 5.7 m}}$.