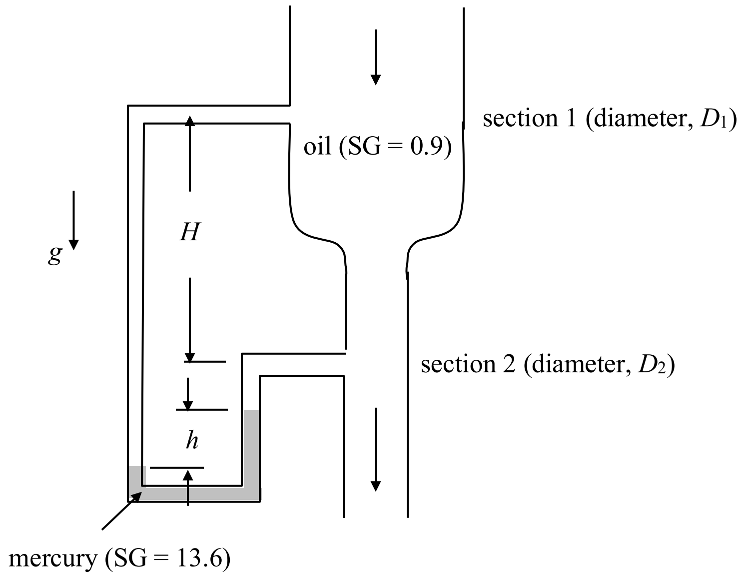
Oil flows through a contraction with circular cross-section as shown in the figure below. A manometer, using mercury as the gage fluid, is used to measure the pressure difference between sections 1 and 2 of the pipe. Assuming frictionless flow, determine:

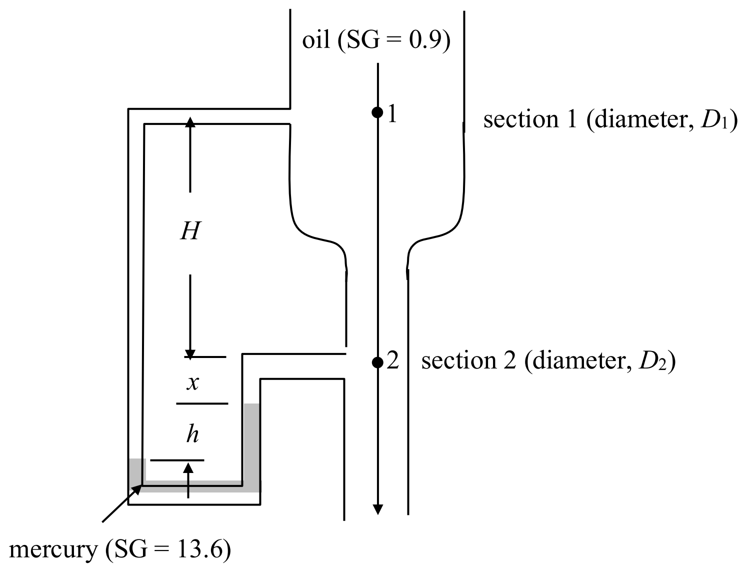
a. the pressure difference, p_1-p_2, between sections 1 and 2, and
b. the mass flow rate through the pipe.



D_1 = 300 mm
D_2 = 100 mm
H = 600 mm
h = 100 mm

SOLUTION:

First determine the pressure difference using the manometer.



$$p_2 = p_1 + \rho_{\text{oil}} g\left(H + x + h\right) - \rho_{\text{Hg}} gh - \rho_{\text{oil}} gx$$
$$p_2 = p_1 + SG_{\text{oil}} \rho_{\text{H20}} g\left(H + h\right) - SG_{\text{Hg}} \rho_{\text{H20}} gh$$
$$\boxed{p_1 - p_2 = \rho_{\text{H20}} g\left[SG_{\text{Hg}} h - SG_{\text{oil}}\left(H + h\right)\right]} \quad (1)$$

Use the given parameters.

ρ_{H20}	=	1000 kg/m^3
g	=	9.81 m/s^2
SG_{Hg}	=	13.6
h	=	100e-3 m
SG_{oil}	=	0.9
H	=	600e-3 m
⇒	$p_1 - p_2$ =	7.2 kPa

Now apply Bernoulli's equation along a streamline from 1 to 2 to determine the mass flow rate.

$$\left(\frac{p}{\rho_{\text{oil}} g} + \frac{V^2}{2g} + z\right)_2 = \left(\frac{p}{\rho_{\text{oil}} g} + \frac{V^2}{2g} + z\right)_1$$

where

$$p_2 - p_1 = 7200 \text{ N/m}^2 \quad \text{(found previously)}$$
$$V_2 = \frac{Q}{\frac{\pi D_2^2}{4}} = \frac{4Q}{\pi D_2^2} \qquad V_1 = \frac{Q}{\frac{\pi D_1^2}{4}} = \frac{4Q}{\pi D_1^2}$$
$$z_1 - z_2 = H$$

Substitute and simplify.

$$\frac{p_2 - p_1}{\rho_{\text{oil}} g} - H = \frac{8Q^2}{\pi^2 g}\left(\frac{1}{D_1^4} - \frac{1}{D_2^4}\right)$$
$$\boxed{\dot{m}_{\text{oil}} = \rho_{\text{oil}} Q = \rho_{\text{oil}} \sqrt{\frac{\pi^2 g}{8}\left(\frac{D_1^4 D_2^4}{D_2^4 - D_1^4}\right)\left(\frac{p_2 - p_1}{\rho_{\text{oil}} g} - H\right)}} \quad (2)$$

Use the given parameters.

ρ_{H20}	=	1000 kg/m^3
SG_{oil}	=	0.9
g	=	9.81 m/s^2
H	=	600e-3 m
D_1	=	300e-3 m
D_2	=	100e-3 m
$p_1 - p_2$	=	7200 N/m^2
$\Rightarrow$		$\boxed{\dot{m} = 37.5 \text{ kg/s}}$