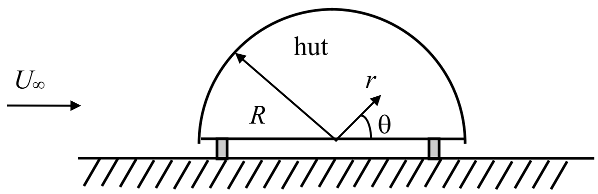
You are to design Quonset huts for a military base. The design wind speed is $U_\infty = 30$ m/s and the free-stream pressure and density are $p_\infty = 101$ kPa and $\rho_\infty = 1.2$ kg/m^3, respectively. The Quonset hut may be considered to be a closed (no leaks) semi-cylinder with a radius of $R = 5$ m which is mounted on tie-down blocks as shown in the figure. The flow is such that the velocity distribution over the top of the hut is approximated by:

$$u_r(r = R) = 0$$

$$u_\theta(r = R) = -2U_\infty \sin\theta$$

The air under the hut is at rest.



a. What is the pressure distribution over the top surface of the Quonset hut?

b. What is the net lift force acting on the Quonset hut due to the air? Don't forget to include the effect of the air under the hut.

c. What is the net drag force acting on the hut? (Hint: A calculation may not be necessary here but some justification is required.)

SOLUTION:

Apply Bernoulli's equation over a streamline adjacent to the upper surface of the hut to determine the pressure distribution. Neglect elevation effects since the fluid is a gas and the elevation differences are small.

$$\left(p+\tfrac{1}{2}\rho V^2\right)_\infty = \left(p+\tfrac{1}{2}\rho V^2\right)_{\text{surface}} \quad (1)$$

where

$p_\infty = 101 \text{ kPa}$

$V_\infty^2 = U_\infty^2 = \left(30 \text{ m/s}\right)^2 = 900 \text{ m}^2/\text{s}^2$

$p_{\text{surface}} = ?$

$V_{\text{surface}}^2 = u_\theta^2 = 4U_\infty^2 \sin^2\theta \quad (0 \le \theta \le \pi)$

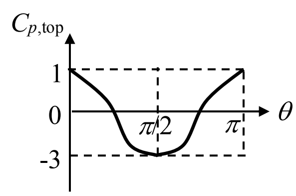
$\rho = 1.2 \text{ kg/m}^3$

Substitute and solve for the pressure on the hut's upper surface.

$$p_{\text{surface}} = p_\infty + \tfrac{1}{2}\rho\left(V_\infty^2 - V_{\text{surface}}^2\right)$$

$$p_{\text{surface}} = p_\infty + \tfrac{1}{2}\rho U_\infty^2\left(1-4\sin^2\theta\right)$$

$$\boxed{C_{p,\text{top}} = \frac{p_{\text{surface}} - p_\infty}{\tfrac{1}{2}\rho U_\infty^2} = 1-4\sin^2\theta} \quad (2)$$

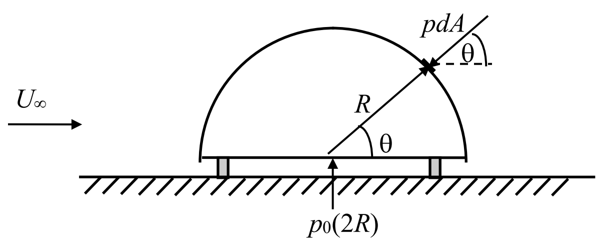


where C_p is known as a "pressure coefficient."

The pressure under the hut will be the stagnation pressure. It can also be found by applying Bernoulli's equation and noting that under the hut the velocity is zero.

$$C_{p,\text{bottom}} = \frac{p_0 - p_\infty}{\tfrac{1}{2}\rho U_\infty^2} = 1 \quad (3)$$

The net lift force is determined by integrating the vertical component of the pressure forces over the entire surface of the hut.



$$L = \underbrace{\int_{\theta=0}^{\theta=\pi}\left(-p\sin\theta\right)\underbrace{\left(Rd\theta\right)}_{=dA}}_{\text{top}} + \underbrace{p_0\left(2R\right)}_{\text{bottom}} \quad \text{(Note that positive lift is directed upwards.)} \quad (4)$$

where p_0 is the stagnation pressure.

$$L = -\int_{\theta=0}^{\theta=\pi}\left[p_\infty + \tfrac{1}{2}\rho U_\infty^2\left(1-4\sin^2\theta\right)\right]\sin\theta R d\theta + \left(p_\infty + \tfrac{1}{2}\rho U_\infty^2\right)\left(2R\right)$$

$$C_L = \frac{L}{\tfrac{1}{2}\rho U_\infty^2\left(2R\right)} = -\frac{1}{2}\int_{\theta=0}^{\theta=\pi}\left[\frac{p_\infty}{\tfrac{1}{2}\rho U_\infty^2} + \left(1-4\sin^2\theta\right)\right]\sin\theta d\theta + \left(\frac{p_\infty}{\tfrac{1}{2}\rho U_\infty^2}+1\right)$$

where C_L is a "lift coefficient."

$$C_L = -\frac{1}{2}\frac{p_\infty}{\frac{1}{2}\rho U_\infty^2}\underbrace{\int_{\theta=0}^{\theta=\pi}\sin\theta\,d\theta}_{=-\cos\theta|_0^\pi=1+1=2} - \frac{1}{2}\int_{\theta=0}^{\theta=\pi}\left(\sin\theta - 4\sin^3\theta\right)d\theta + \frac{p_\infty}{\frac{1}{2}\rho U_\infty^2} + 1$$

$$= -\frac{p_\infty}{\frac{1}{2}\rho U_\infty^2} - \frac{1}{2}\underbrace{\int_{\theta=0}^{\theta=\pi}\sin\theta\,d\theta}_{=-\cos\theta|_0^\pi=1+1=2} + 2\underbrace{\int_{\theta=0}^{\theta=\pi}\sin^3\theta\,d\theta}_{=-\frac{1}{3}\left(2+\sin^2\theta\right)\cos\theta|_0^\pi=\frac{2}{3}+\frac{2}{3}=\frac{4}{3}} + \frac{p_\infty}{\frac{1}{2}\rho U_\infty^2} + 1$$

$$\boxed{C_L = \frac{L}{\frac{1}{2}\rho U_\infty^2\left(2R\right)} = \frac{8}{3}} \tag{5}$$

The net drag force is determined by integrating the horizontal component of the pressure forces over the entire surface of the hut.

$$D = \int_{\theta=0}^{\theta=\pi}\left(-p\cos\theta\right)\left(Rd\theta\right) \tag{6}$$

$$D = -\int_{\theta=0}^{\theta=\pi}\left[p_\infty + \frac{1}{2}\rho U_\infty^2\left(1 - 4\sin^2\theta\right)\right]\cos\theta R\,d\theta$$

$$= -Rp_\infty\underbrace{\int_{\theta=0}^{\theta=\pi}\cos\theta\,d\theta}_{=\sin\theta|_0^\pi=0} - \frac{1}{2}\rho U_\infty^2 R\left[\underbrace{\int_{\theta=0}^{\theta=\pi}\cos\theta\,d\theta}_{=\sin\theta|_0^\pi=0} - 4\underbrace{\int_{\theta=0}^{\theta=\pi}\sin^2\theta\cos\theta\,d\theta}_{=\frac{1}{3}\sin^3\theta|_0^\pi=0}\right]$$

$$\boxed{\therefore D = 0} \tag{7}$$

We could have also anticipated that the drag would be zero since the velocity field is symmetric between the upstream and downstream sides of the hut.