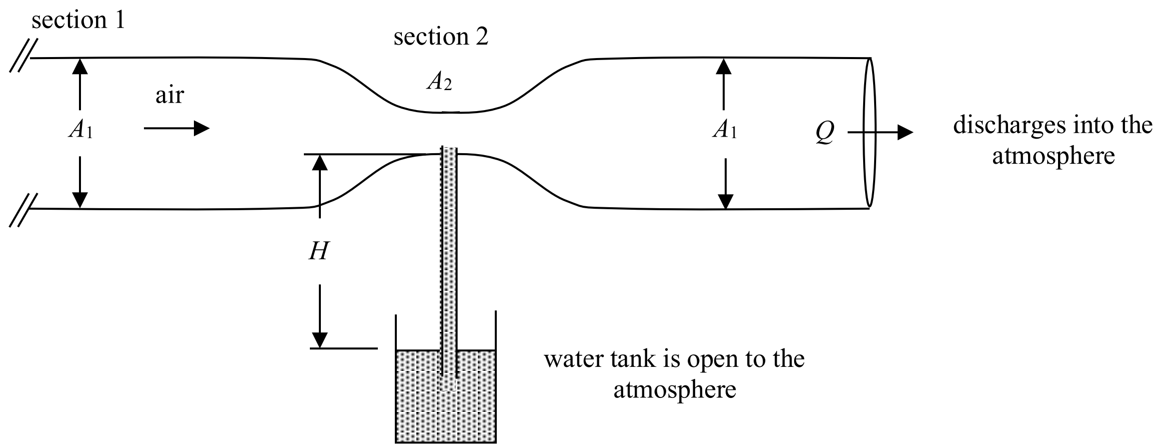
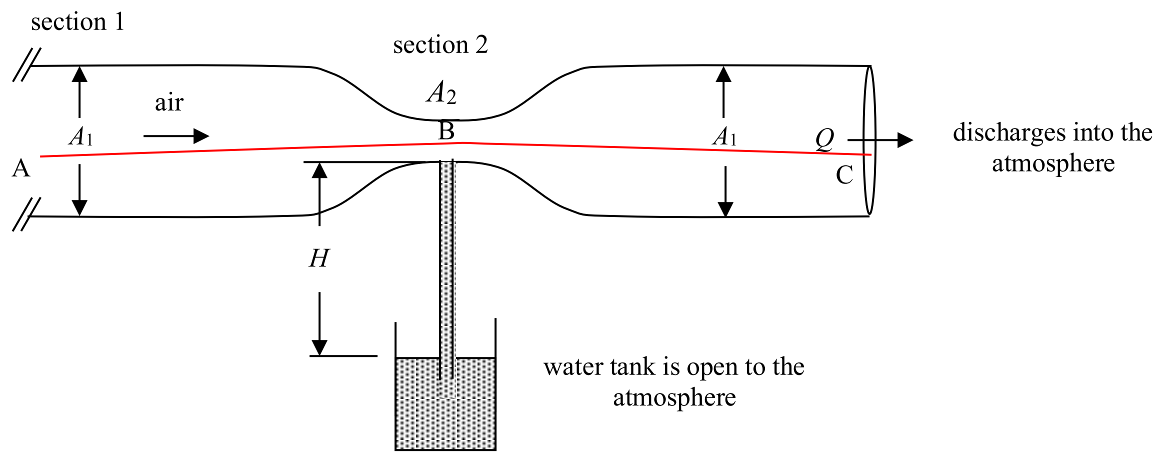
Air flows through the Venturi tube that discharges to the atmosphere as shown in the figure. If the flow rate is large enough, the pressure in the constriction will be low enough to draw the water up into the tube. Determine the flow rate, Q, needed to just draw the water into the tube. What is the pressure at section 1? Assume the air flow is frictionless.



SOLUTION:



Apply Bernoulli's equation along a streamline from point B to point C in order to determine the pressure at point B. Neglect changes in elevation.

$$\left(\frac{p}{\rho g}+\frac{V^2}{2g}\right)_B=\left(\frac{p}{\rho g}+\frac{V^2}{2g}\right)_C \tag{1}$$

where

$$p_C = p_{\text{atm}}$$

$$V_B=\frac{Q}{A_2}$$

$$V_C=\frac{Q}{A_1}$$

Substitute into Eqn. (1) and solve for p_B:

$$\frac{p_B}{\rho g}=\frac{p_{\text{atm}}}{\rho g}+\frac{Q^2}{2g}\left(\frac{1}{A_1^2}-\frac{1}{A_2^2}\right) \tag{2}$$

Note that the ρ in the previous equation is the density of air.The pressure at point B can also be determined from manometry.

$$p_B = p_{\text{atm}} - \rho_{\text{H}_2\text{0}} gH \tag{3}$$

Substitute Eqn. (3) into Eqn. (2) and solve for Q:

$$\boxed{Q=\sqrt{2gH\left(\frac{\rho_{\text{H}_2\text{0}}}{\rho}\right)\left(\frac{1}{A_2^2}-\frac{1}{A_1^2}\right)^{-1}}} \tag{4}$$

The pressure at point A can be determined by applying Bernoulli's Equation along a streamline from point A to point C. Note that since the areas at point A and point C are the same (= A_1), the pressure must also be the same. Hence:

$$\boxed{p_A = p_C = p_{\text{atm}}} \tag{5}$$

Note that these pressures can only be equal in an inviscid flow. If viscous effects were included, then $p_A > p_C$ in order to drive the flow from left to right.