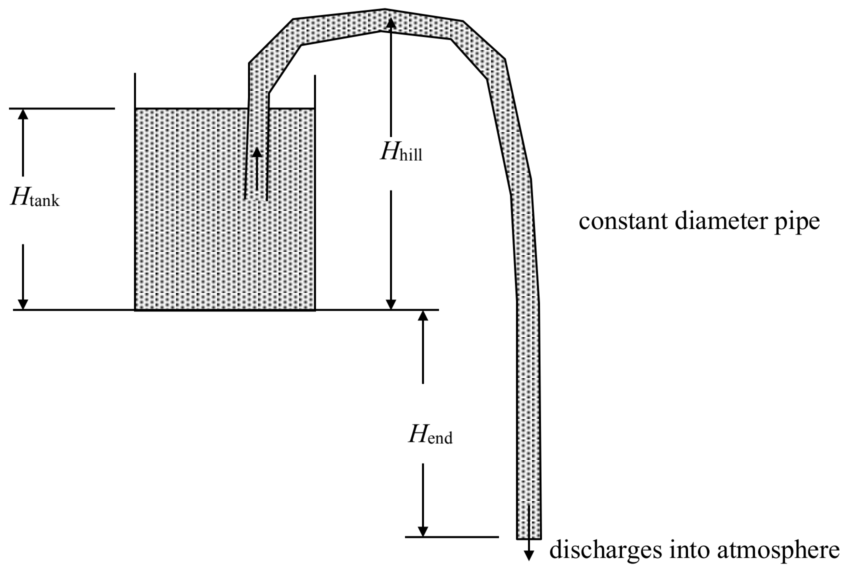
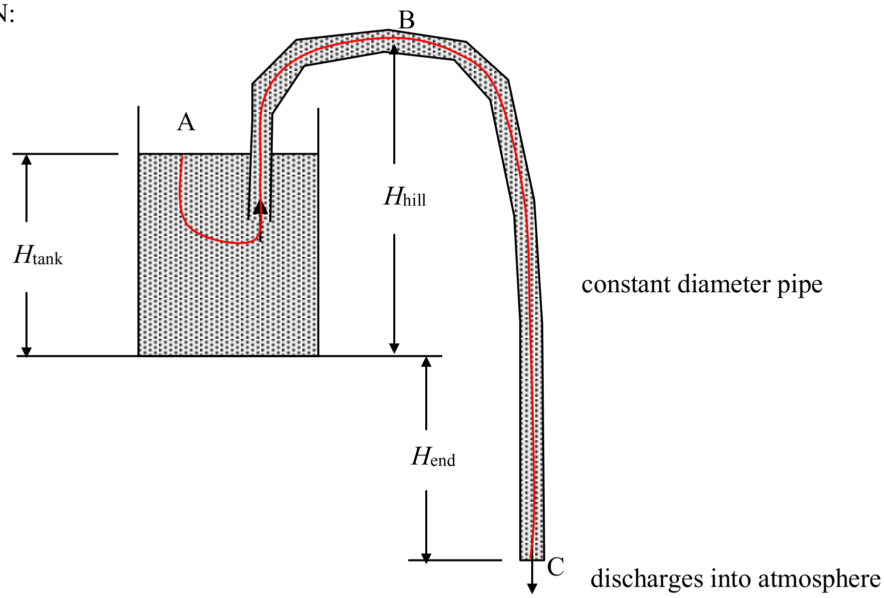
Water is siphoned from a large tank through a constant diameter hose as shown in the figure. Determine the maximum height of the hill, H_{hill}, over which the water can be siphoned without cavitation occurring. Assume that the vapor pressure of the water is p_v, the height of the water free surface in the tank is H_{tank}, and the vertical distance from the end of the hose to the base of the tank is H_{end}.



SOLUTION:



Apply Bernoulli's equation along a streamline from the tank free surface (point A) to the end of the tube (point C).

$$\left(\frac{p}{\rho g}+\frac{V^2}{2g}+z\right)_A=\left(\frac{p}{\rho g}+\frac{V^2}{2g}+z\right)_C \tag{1}$$

where

$p_A = p_C = p_{\text{atm}}$

$V_A \approx 0$ (free surface of a large tank)

$z_A - z_C = H_{\text{tank}} + H_{\text{end}}$

Solving Eqn. (1) for V_C gives:

$$V_C = \sqrt{2g\left(H_{\text{tank}} + H_{\text{end}}\right)} \tag{2}$$

Now apply Bernoulli's equation along a streamline from the tank free surface (point A) to the top of the tube (point B). Note that the velocity everywhere within the tube will be equal to V_C (from conservation of mass).

$$\left(\frac{p}{\rho g}+\frac{V^2}{2g}+z\right)_A=\left(\frac{p}{\rho g}+\frac{V^2}{2g}+z\right)_B \tag{3}$$

where

$p_A = p_{\text{atm}}$

$p_B = p_{\text{v}}$

(From Eqn. (3) we see that the pressure at point B will decrease as H_{hill} increases so we should use the smallest allowable pressure at point B to determine the maximum H_{hill}.)

$V_A \approx 0$ (free surface of a large tank)

$V_B = V_C = \sqrt{2g\left(H_{\text{tank}} + H_{\text{end}}\right)}$ (from conservation of mass)

$z_A - z_B = H_{\text{tank}} - H_{\text{hill}}$

Substituting into Eqn. (3) and solving for H_{hill} gives:

$$\frac{p_{\text{atm}}}{\rho g} + H_{\text{tank}} - H_{\text{hill}} = \frac{p_v}{\rho g} + H_{\text{tank}} + H_{\text{end}}$$

$$\boxed{H_{\text{hill}} = \frac{p_{\text{atm}} - p_v}{\rho g} - H_{\text{end}}} \qquad (4)$$