a. A fluid velocity field is given by:

$$\mathbf{u} = 2t\hat{\mathbf{e}}_x$$

Will a fluid particle accelerate in this flow? Why?

b. Now consider the following flow:

$$\mathbf{u} = x\hat{\mathbf{e}}_x$$

Will a fluid particle accelerate in this flow? Why?

SOLUTION:

Part (a):
The acceleration is given by:

$$\mathbf{a} = \frac{D\mathbf{u}}{Dt} = \underbrace{\frac{\partial \mathbf{u}}{\partial t}}_{=2\hat{\mathbf{e}}_x} + \underbrace{u_x}_{2t}\underbrace{\frac{\partial \mathbf{u}}{\partial x}}_{=\mathbf{0}} + \underbrace{u_y \frac{\partial \mathbf{u}}{\partial y}}_{=\mathbf{0}} + \underbrace{u_z \frac{\partial \mathbf{u}}{\partial z}}_{=\mathbf{0}}$$

Hence, for the given flow:

$\boxed{\mathbf{a} = 2\hat{\mathbf{e}}_x}$ Yes, fluid particles will accelerate due to the local (or Eulerian) derivative.

Part (b):
The acceleration is given by:

$$\mathbf{a} = \frac{D\mathbf{u}}{Dt} = \underbrace{\frac{\partial \mathbf{u}}{\partial t}}_{=\mathbf{0}} + \underbrace{u_x}_{x}\underbrace{\frac{\partial \mathbf{u}}{\partial x}}_{=\hat{\mathbf{e}}_x} + \underbrace{u_y \frac{\partial \mathbf{u}}{\partial y}}_{=\mathbf{0}} + \underbrace{u_z \frac{\partial \mathbf{u}}{\partial z}}_{=\mathbf{0}}$$

Hence, for the given flow:

$\boxed{\mathbf{a} = x\hat{\mathbf{e}}_x}$ Yes, fluid particles will accelerate due to the convective derivative.