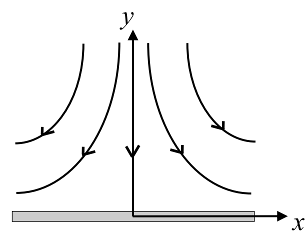
The velocity field near a planar stagnation point (see the figure below) is given as,

$\mathbf{u} = U_0\left(\frac{x}{L}\right)\hat{\mathbf{e}}_x - U_0\left(\frac{y}{L}\right)\hat{\mathbf{e}}_y$ where U_0 and L are positive constants

Determine the acceleration of a fluid particle along the line $x = 0$.



SOLUTION:

The acceleration of a fluid particle is,

$$\mathbf{a} = \frac{D\mathbf{u}}{Dt} = \frac{\partial \mathbf{u}}{\partial t} + u_x \frac{\partial \mathbf{u}}{\partial x} + u_y \frac{\partial \mathbf{u}}{\partial y}, \quad (1)$$

where,

$$\frac{\partial \mathbf{u}}{\partial t} = \mathbf{0} \quad \text{(steady flow)}$$

$$u_x \frac{\partial \mathbf{u}}{\partial x} = \left[U_0\left(\frac{x}{L}\right)\right]\left[U_0\left(\frac{1}{L}\right)\hat{\mathbf{e}}_x\right] = U_0^2\left(\frac{x}{L^2}\right)\hat{\mathbf{e}}_x$$

$$u_y \frac{\partial \mathbf{u}}{\partial y} = \left[-U_0\left(\frac{y}{L}\right)\right]\left[-U_0\left(\frac{1}{L}\right)\hat{\mathbf{e}}_y\right] = U_0^2\left(\frac{y}{L^2}\right)\hat{\mathbf{e}}_y$$

$$\therefore \mathbf{a} = U_0^2\left(\frac{x}{L^2}\right)\hat{\mathbf{e}}_x + U_0^2\left(\frac{y}{L^2}\right)\hat{\mathbf{e}}_y \quad (2)$$

Along the line $x = 0$,

$$\boxed{\therefore \mathbf{a}(0, y) = U_0^2\left(\frac{y}{L^2}\right)\hat{\mathbf{e}}_y} \quad (3)$$