A fluid velocity field is given by,

$$\mathbf{u} = (cy^2)\hat{\mathbf{i}} + (cx^2)\hat{\mathbf{j}},$$

where c is a constant. Determine

a. the components of the acceleration and
b. the points in the flow field where the acceleration is zero.

SOLUTION:

The acceleration of a fluid element is given by,

$$\mathbf{a} = \frac{D\mathbf{u}}{Dt} = \frac{\partial \mathbf{u}}{\partial t} + u_x \frac{\partial \mathbf{u}}{\partial x} + u_y \frac{\partial \mathbf{u}}{\partial y} \tag{1}$$

where,

$$\frac{\partial \mathbf{u}}{\partial t} = \mathbf{0} \quad \text{(steady flow)}$$

$$u_x \frac{\partial \mathbf{u}}{\partial x} = \left(cy^2\right)\left(2cx\hat{\mathbf{j}}\right) = 2c^2xy^2\hat{\mathbf{j}}$$

$$u_y \frac{\partial \mathbf{u}}{\partial y} = \left(cx^2\right)\left(2cy\hat{\mathbf{i}}\right) = 2c^2x^2y\hat{\mathbf{i}}$$

$$\boxed{\therefore \mathbf{a} = 2c^2x^2y\hat{\mathbf{i}} + 2c^2xy^2\hat{\mathbf{j}}} \tag{2}$$

Set the acceleration equal to zero,

$$\mathbf{a} = \mathbf{0} = 2c^2x^2y\hat{\mathbf{i}} + 2c^2xy^2\hat{\mathbf{j}}$$

$\boxed{\therefore \text{either } x = 0 \text{ or } y = 0}$ (This is locus of points where the total acceleration is zero.) (3)