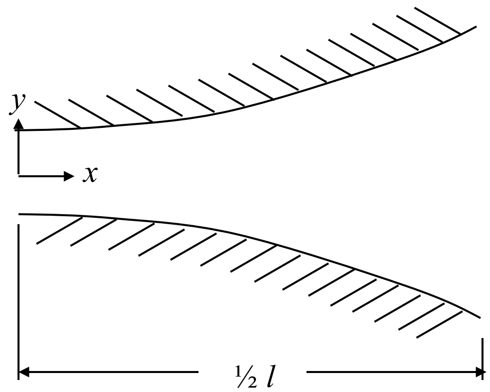
For the diffuser shown below, determine:

a. the acceleration of a fluid particle for any x and t, and
b. the value of c (other than c=0) for which the acceleration is zero for any x at t=2 s. Assume V_0=10 ft/s and l=5 ft.
c. Explain how the acceleration can be zero if the flow rate is increasing with time.



$$\mathbf{u} = V_0(1-\mathrm{e}^{-ct})(1-\frac{x}{l})\hat{\mathbf{i}}$$

where V_0, c, and l are constants

SOLUTION:

The acceleration of a fluid particle is the Lagrangian derivative of the velocity.

$$\mathbf{a} = \frac{D\mathbf{u}}{Dt} = \frac{\partial \mathbf{u}}{\partial t} + u\frac{\partial \mathbf{u}}{\partial x} \tag{1}$$

Substitute the given velocity field.

$$\mathbf{a} = \frac{\partial}{\partial t}\left[V_0\left(1-\mathrm{e}^{-ct}\right)\left(1-\frac{x}{l}\right)\hat{\mathbf{i}}\right] + \left[V_0\left(1-\mathrm{e}^{-ct}\right)\left(1-\frac{x}{l}\right)\right]\frac{\partial}{\partial x}\left[V_0\left(1-\mathrm{e}^{-ct}\right)\left(1-\frac{x}{l}\right)\hat{\mathbf{i}}\right]$$

$$= V_0 c\mathrm{e}^{-ct}\left(1-\frac{x}{l}\right)\hat{\mathbf{i}} + \left[V_0\left(1-\mathrm{e}^{-ct}\right)\left(1-\frac{x}{l}\right)\right]\left[-\frac{V_0}{l}\left(1-\mathrm{e}^{-ct}\right)\hat{\mathbf{i}}\right]$$

$$\boxed{\therefore \mathbf{a} = V_0\left(1-\frac{x}{l}\right)\left[c\mathrm{e}^{-ct} - \frac{V_0}{l}\left(1-\mathrm{e}^{-ct}\right)^2\right]\hat{\mathbf{i}}} \tag{2}$$

At $t = 2$ s:

$$\mathbf{a}(x, t=2) = \mathbf{0} = V_0\left(1-\frac{x}{l}\right)\left[c\mathrm{e}^{-2c} - \frac{V_0}{l}\left(1-\mathrm{e}^{-2c}\right)^2\right]\hat{\mathbf{i}}$$

$$c\mathrm{e}^{-2c} = \frac{V_0}{l}\left(1-\mathrm{e}^{-2c}\right)^2 \tag{3}$$

Solve numerically for c when $V_0 = 10$ ft/s and $l = 5$ ft.

$$\Rightarrow \boxed{c = 0.124\ \mathrm{s}^{-1}}$$

The acceleration of a fluid particle can be zero even though the flow rate is increasing because the local acceleration ($\partial u/\partial t$) exactly balances the convective deceleration ($u\partial u/\partial x$).