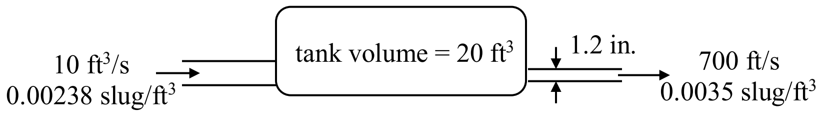
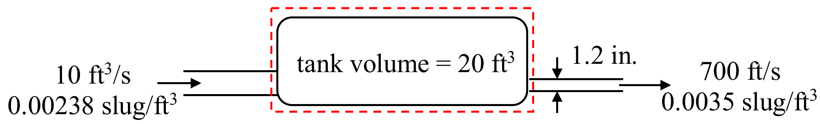
Air at standard conditions enters the compressor shown in the figure below at a rate of 10 ft^3/s. The air leaves the tank through a 1.2 in. diameter pipe with a density of 0.0035 $slug/ft^3$ and a uniform speed of 700 ft/s. Determine the average time rate of change of air density within the tank.



SOLUTION:

Apply conservation of mass to a control volume surrounding the tank as shown in the figure below.



$$\frac{d}{dt}\int_{\mathrm{CV}} \rho dV + \int_{\mathrm{CS}} \rho \mathbf{u}_{\mathrm{rel}} \cdot d\mathbf{A} = 0 \tag{1}$$

where

$$\frac{d}{dt}\int_{\mathrm{CV}} \rho dV = \frac{d}{dt}\left(\rho_t V_t\right) = V_t \frac{d\rho_t}{dt} \tag{2}$$

$$\int_{\mathrm{CS}} \rho \mathbf{u}_{\mathrm{rel}} \cdot d\mathbf{A} = -\rho_{\mathrm{i}} Q_{\mathrm{i}} + \rho_{\mathrm{o}} U_{\mathrm{o}} A_{\mathrm{o}} \tag{3}$$

Substitute and solve for the rate at which the air density within the tank is changing.

$$V_t \frac{d\rho_t}{dt} - \rho_{\mathrm{i}} Q_{\mathrm{i}} + \rho_{\mathrm{o}} U_{\mathrm{o}} A_{\mathrm{o}} = 0 \tag{4}$$

$$\boxed{\frac{d\rho_t}{dt} = \frac{\rho_{\mathrm{i}} Q_{\mathrm{i}} - \rho_{\mathrm{o}} U_{\mathrm{o}} A_{\mathrm{o}}}{V_t}} \tag{5}$$

Using the given data:

ρ_{i} = $2.38*10^{-3}$ slug/ft^3
Q_{i} = 10 ft^3/s
ρ_{o} = $3.5*10^{-3}$ slug/ft^3
U_{o} = 700 ft/s
A_{o} = $\pi/4*(1.2 \text{ in.}\cdot 1 \text{ ft}/12 \text{ in.})^2 = 7.85*10^{-3}$ ft^2
V_t = 20 ft^3

$$\boxed{\therefore \frac{d\rho_t}{dt} = 2.28*10^{-4} \ \ \mathrm{slug/ft^3/s}}$$