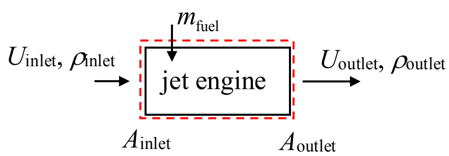
A new jet engine is being tested in a wind tunnel. Air, with properties characteristic of U.S. Standard Atmosphere at 6000 m enters the engine at a velocity of 275 m/s through a circular intake port of radius 0.5 m. Fuel enters the engine at a mass flow rate of 2.5 kg/s. If the gas leaves the engine with an average velocity of 300 m/s through an exit port of radius 0.4 m, calculate the density of the exhaust gas.

SOLUTION:

Apply conservation of mass to a control volume surrounding the jet engine as shown in the figure below.



$$\frac{d}{dt}\int_{CV} \rho dV + \int_{CS} \rho \mathbf{u}_{rel} \cdot d\mathbf{A} = 0 \tag{1}$$

where

$$\frac{d}{dt}\int_{CV} \rho dV = 0 \quad \text{(steady flow)} \tag{2}$$

$$\int_{CS} \rho \mathbf{u}_{rel} \cdot d\mathbf{A} = -\rho_{inlet} U_{inlet} A_{inlet} - m_{fuel} + \rho_{outlet} U_{outlet} A_{outlet} \tag{3}$$

Substitute and solve for the outlet density.

$$-\rho_{inlet} U_{inlet} A_{inlet} - m_{fuel} + \rho_{outlet} U_{outlet} A_{outlet} = 0 \tag{4}$$

$$\boxed{\rho_{outlet} = \rho_{inlet} \frac{U_{inlet}}{U_{outlet}} \frac{A_{inlet}}{A_{outlet}} + \frac{m_{fuel}}{U_{outlet} A_{outlet}}} \tag{5}$$

Using the given data:

ρ_{inlet} = 0.660 kg/m^3 (from U.S. Standard Atmosphere at an altitude of 6000 m)

U_{inlet} = 275 m/s

U_{outlet} = 300 m/s

A_{inlet} = π(0.5 m)^2 = 0.785 m^2

A_{outlet} = π(0.4 m)^2 = 0.503 m^2

m_{fuel} = 2.5 kg/s

$\boxed{\therefore \rho_{outlet} = 0.961 \text{ kg/m}^3}$