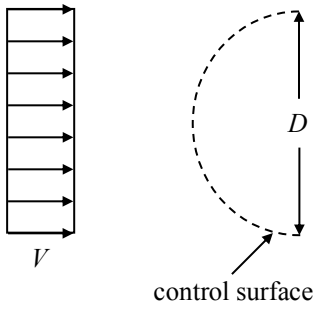
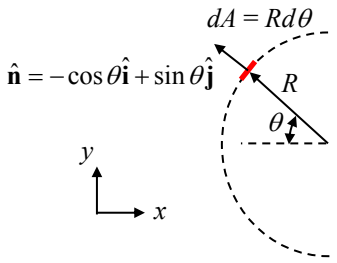
Calculate the mass flux through the control surface shown below. Assume a unit depth into the page.



SOLUTION:

The mass flux through the surface is given by:

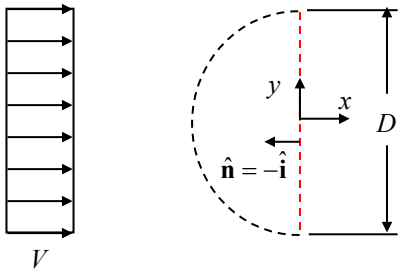



$$m = \int_{CS} \rho \mathbf{u}_{rel} \cdot d\mathbf{A} = \int_{\theta=-\pi/2}^{\theta=\pi/2} \rho \underbrace{V\hat{\mathbf{i}}}_{=\mathbf{u}_{rel}} \cdot \underbrace{\left(-\cos\theta\hat{\mathbf{i}} + \sin\theta\hat{\mathbf{j}}\right)}_{=\hat{\mathbf{n}}} \underbrace{\left(Rd\theta\right)}_{=dA}$$

$$= -\rho VR \int_{\theta=-\pi/2}^{\theta=\pi/2} \cos\theta d\theta = -\rho VR \sin\theta \Big|_{-\pi/2}^{\pi/2} = -2\rho VR$$

$$\boxed{\therefore m = -\rho VD}$$

We could have also figured out the mass flux by noticing that any mass passing through the curved control surface must also pass through a vertical control surface as shown below.




$$m = \int_{CS} \rho \mathbf{u}_{rel} \cdot d\mathbf{A} = \int_{y=-R}^{y=R} \rho \underbrace{V\hat{\mathbf{i}}}_{=\mathbf{u}_{rel}} \cdot \underbrace{\left(-\hat{\mathbf{i}}\right)}_{=\hat{\mathbf{n}}} \underbrace{\left(dy\right)}_{=dA} = -2\rho VR = -\rho VD$$ (The same answer as before!)