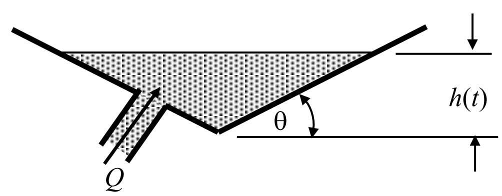
The (symmetric) V-shaped container shown in the figure has width, b, into the page and is filled from the inlet pipe at volume flow rate, Q. Derive expressions for:

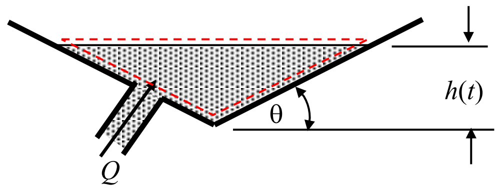
a. the rate of change of the surface height, dh/dt

b. the time required for the surface to rise from h_1 to h_2.



SOLUTION:

Apply conservation of mass to the deformable control volume shown in the figure below.



$$\frac{d}{dt}\int_{CV}\rho dV + \int_{CS}\rho \mathbf{u}_{\text{rel}}\cdot d\mathbf{A} = 0$$

where

$$\frac{d}{dt}\int_{CV}\rho dV = \frac{d}{dt}\left(\rho 2\cdot\tfrac{1}{2}h\frac{h}{\tan\theta}\cdot b\right) = \frac{\rho b}{\tan\theta}\frac{d}{dt}\left(h^2\right) = \frac{2\rho bh}{\tan\theta}\frac{dh}{dt}$$

$$\int_{CS}\rho \mathbf{u}_{\text{rel}}\cdot d\mathbf{A} = -\rho Q$$

Substitute and simplify.

$$\frac{2\rho bh}{\tan\theta}\frac{dh}{dt} - \rho Q = 0$$

$$\boxed{\frac{dh}{dt} = \frac{\tan\theta}{2hb}Q} \qquad (1)$$

Solve the differential equation to determine the time required for a specified change in the liquid level.

$$\frac{dh}{dt} = \frac{\tan\theta}{2hb}Q$$

$$\int_{h=h_1}^{h=h_2} hdh = \frac{\tan\theta}{2b}Q\int_{t=t_1}^{t=t_2} dt$$

$$\tfrac{1}{2}\left(h_2^2 - h_1^2\right) = \frac{\tan\theta}{2b}Q\left(t_2 - t_1\right)$$

$$\boxed{t_2 - t_1 = \frac{b\left(h_2^2 - h_1^2\right)}{Q\tan\theta}} \qquad (2)$$