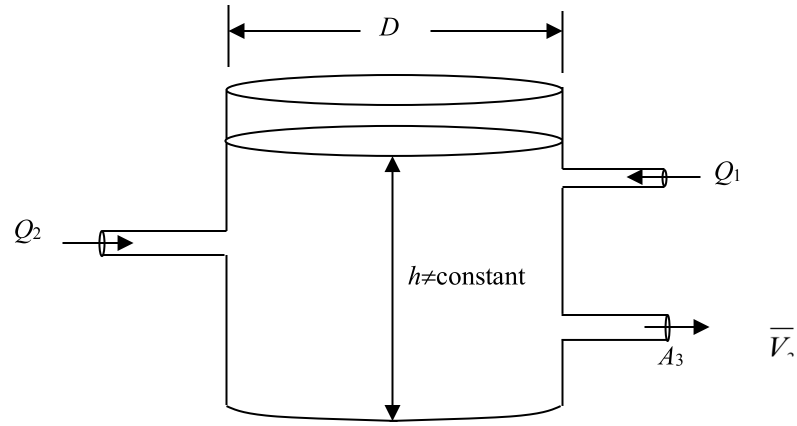
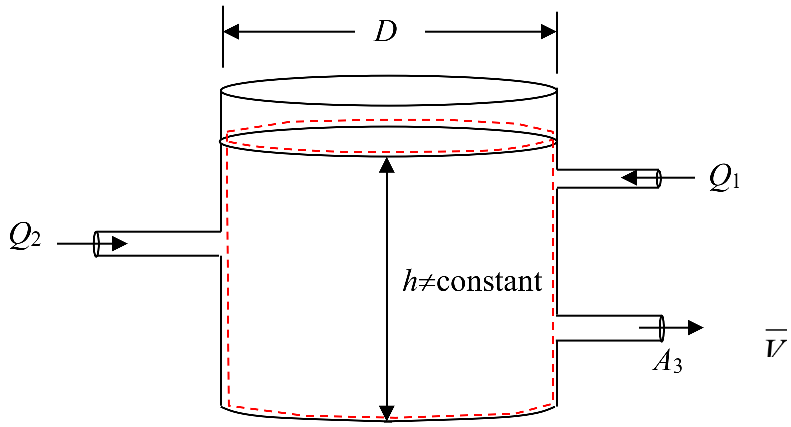
Water enters a cylindrical tank with diameter, D, through two pipes at volumetric flow rates of Q_1 and Q_2 and leaves through a pipe with area, A_3, with an average velocity, $\overline{V}_3$. The level in the tank, h, does <u>not</u> remain constant. Determine the time rate of change of the level in the tank.



SOLUTION:

Apply conservation of mass to a control volume that deforms to follow the free surface of the liquid.



$$\frac{d}{dt}\int_{CV} \rho dV + \int_{CS} \rho \mathbf{u}_{rel} \cdot d\mathbf{A} = 0 \tag{1}$$

where

$$\frac{d}{dt}\int_{CV} \rho dV = \frac{d}{dt}\underbrace{\left(\rho h \frac{\pi D^2}{4}\right)}_{=M_{CV}} = \rho \frac{dh}{dt}\frac{\pi D^2}{4}$$

$$\int_{CS} \rho \mathbf{u}_{rel} \cdot d\mathbf{A} = -\rho Q_2 - \rho Q_1 + \rho \bar{V}_3 A_3$$

Substitute and re-arrange.

$$\rho \frac{dh}{dt}\frac{\pi D^2}{4} - \rho Q_2 - \rho Q_1 + \rho \bar{V}_3 A_3 = 0$$

$$\boxed{\frac{dh}{dt} = \frac{Q_2 + Q_1 - \bar{V}_3 A_3}{\pi D^2 / 4}} \tag{2}$$

We could have also chosen a fixed control volume through which the free surface moves. Using this time of control volume, conservation of mass is given by:

$$\frac{d}{dt}\int_{\mathrm{CV}} \rho dV + \int_{\mathrm{CS}} \rho \mathbf{u}_{\mathrm{rel}} \cdot d\mathbf{A} = 0 \qquad (3)$$

where

$$\frac{d}{dt}\int_{\mathrm{CV}} \rho dV = 0 \quad \text{(the mass of fluid in the fixed control volume remains constant)}$$

$$\int_{\mathrm{CS}} \rho \mathbf{u}_{\mathrm{rel}} \cdot d\mathbf{A} = -\rho Q_2 - \rho Q_1 + \rho \bar{V}_3 A_3 + \underbrace{\rho \overbrace{\frac{dh}{dt}}^{=V_{\mathrm{top}}} \frac{\pi D^2}{4}}_{m_{\mathrm{top}}}$$

Substitute and re-arrange.

$$\frac{dh}{dt} = \frac{\rho Q_2 + \rho Q_1 - \bar{V}_3 A_3}{\pi D^2 / 4}$$

$$\boxed{\frac{dh}{dt} = \frac{Q_2 + Q_1 - \bar{V}_3 A_3}{\pi D^2 / 4}} \quad \text{(This is the same answer as before!)} \qquad (4)$$