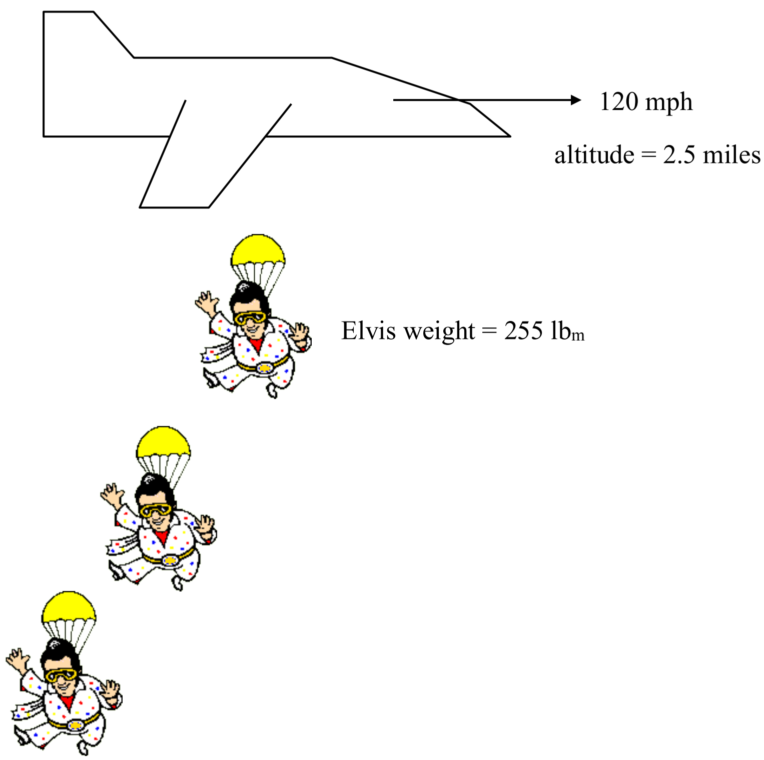
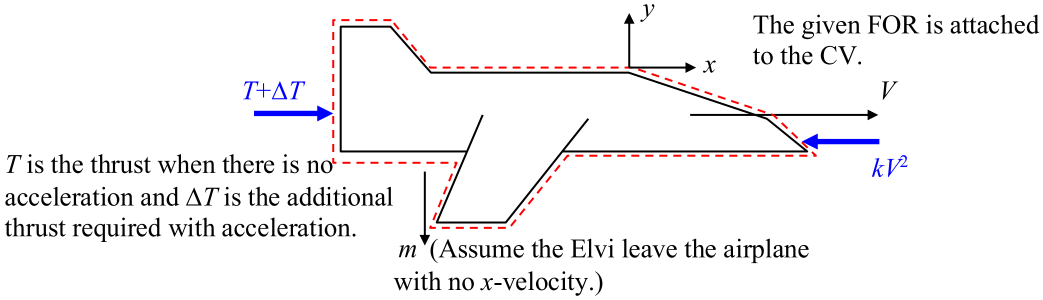
Flying Elvi, 20 of them in all, jump out of an airplane at a rate of one Elvis, weighing 255 lb_m, every 5 seconds. If the airplane is flying horizontally at a velocity of 120 mph and tries to accelerate at a rate of 1 ft/s^2, determine the change in the thrust that must be supplied by the airplane propellers as a function of time until all of the Elvi have left the ~~building~~ plane.

Assume that there is a drag force acting on the plane that can be modeled by $F_D = -kV^2$ where $k = 0.2$ $lb_f/(ft^2/s^2)$ and V is the velocity of the plane relative to the air. The air density at an altitude of 2.5 miles is approximately 0.61 times the air density at sea level. You may assume that the mass rate at which fuel is burned is very small in comparison to the mass rate at which Elvi leave the plane. The plane weight at altitude not counting Elvi is 30000 lb_m.



SOLUTION:

Apply the linear momentum equation in the x-direction to a control volume surrounding the airplane. Use a frame of reference fixed to the airplane (non-inertial since the airplane is accelerating).



$$\frac{d}{dt}\int_{CV} u_x \rho dV + \int_{CS} u_x \left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = F_{B,x} + F_{S,x} - \int_{CV} a_{x/X} \rho dV$$

where

$$\frac{d}{dt}\int_{CV} u_x \rho dV = 0 \quad \text{(in this frame of reference, everything within the CV has zero velocity)}$$

$$\int_{CS} u_x \left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = 0 \quad \text{(assuming the Elvi jump out of the plane with no } x \text{ velocity)}$$

$$F_{B,x} = 0$$

$$F_{S,x} = \left(T + \Delta T\right) - kV^2$$

$$\int_{CV} a_{x/X} \rho dV = \frac{dV}{dt} M$$

Substitute and solve for ΔT.

$$\Delta T = M\frac{dV}{dt} + kV^2 - T$$

Note that before accelerating:

$$T = kV^2$$

so that the additional thrust required when accelerating is:

$$\Delta T = M\frac{dV}{dt} \tag{1}$$

Now apply conservation of mass to the same control volume.

$$\frac{d}{dt}\int_{CV} \rho dV + \int_{CS} \rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A} = 0$$

where

$$\frac{d}{dt}\int_{CV} \rho dV = \frac{dM}{dt}$$

$$\int_{CS} \rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A} = m$$

Substitute and solve the resulting differential equation for M.

$$\frac{dM}{dt} = -m$$

$$M = M_0 - mt \quad \text{(assuming the mass flow rate is a constant)} \tag{2}$$

Substitute Eqn. (2) into Eqn. (1).

$$\boxed{\Delta T = \left(M_0 - mt\right)\frac{dV}{dt}} \qquad (3)$$

Using the given data:

$M_0 \quad = \quad 30000\ \text{lb}_\text{m} + 20*255\ \text{lb}_\text{m} = 35100\ \text{lb}_\text{m}$

$m \quad = \quad 255\ \text{lb}_\text{m} / 5\ \text{s} = 51\ \text{lb}_\text{m}/\text{s}$

$dV/dt \quad = \quad 1\ \text{ft/s}^2$

$\boxed{\therefore \Delta T \quad = \quad 1090\ \text{lb}_\text{f} - (1.6\ \text{lb}_\text{f}/\text{s})\ t}$ $\quad (0 \le t \le 100\ \text{s since there are 20 Elvi}) \qquad (4)$

Note that $1\ \text{lb}_\text{f} = 32.2\ \text{lb}_\text{m}*\text{ft/s}^2$.