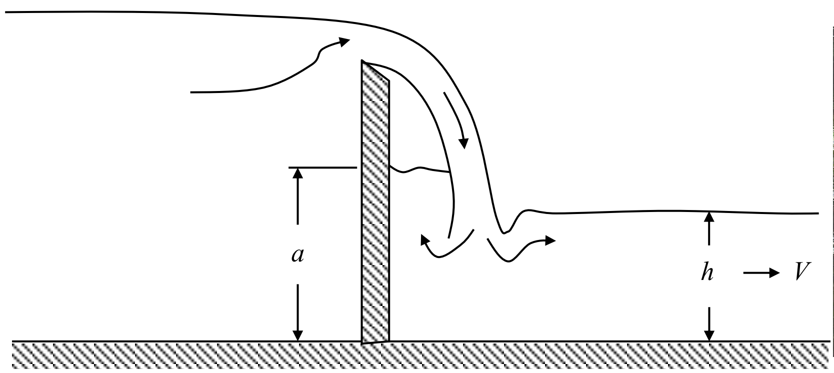
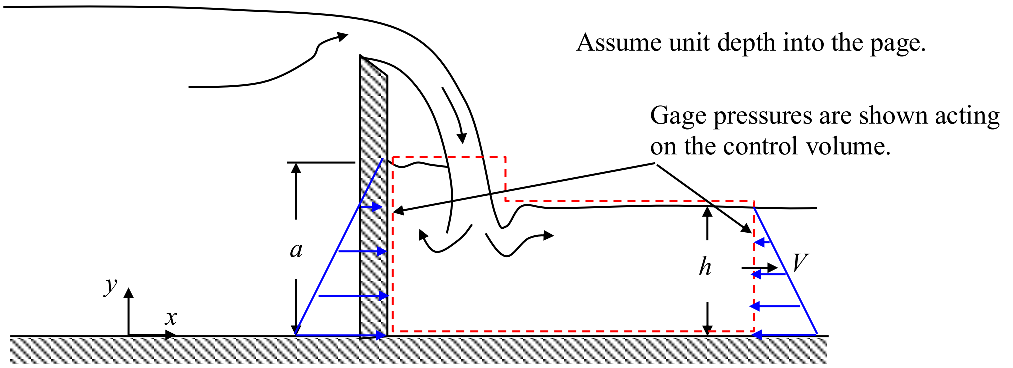
A weir discharges into a channel of constant breadth as shown in the figure. It is observed that a region of still water backs up behind the jet to a height a. The velocity and height of the flow in the channel are given as V and h, respectively, and the density of the water is ρ. You may assume that friction and the horizontal momentum of the fluid falling over the weir are negligible.



What is the height a in terms of the other parameters?

SOLUTION:

Apply the linear momentum equation in the *x*-direction to the control volume shown below. Use the fixed frame of reference shown in the figure.



$$\frac{d}{dt}\int_{CV} u_x \rho dV + \int_{CS} u_x \left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = F_{B,x} + F_{S,x}$$

where

$$\frac{d}{dt}\int_{CV} u_x \rho dV = 0 \quad \text{(steady flow)}$$

$$\int_{CS} u_x \left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = \rho V^2 h \quad \text{(assume incoming flow has negligible horizontal velocity)}$$

$$F_{B,x} = 0$$

$$F_{S,x} = \tfrac{1}{2}\rho g a^2 - \tfrac{1}{2}\rho g h^2 \quad \text{(net horizontal pressure forces)}$$

Substitute and simplify.

$$\rho V^2 h = \tfrac{1}{2}\rho g a^2 - \tfrac{1}{2}\rho g h^2 \tag{1}$$

$$a^2 = h^2 + \frac{2V^2 h}{g}$$

$$a = h\sqrt{1 + \frac{2V^2}{gh}}$$

$$\boxed{\therefore \frac{a}{h} = \sqrt{1 + 2\text{Fr}^2}} \tag{2}$$

where Fr = $V/(gh)^{1/2}$ is a dimensionless parameter known as the Froude number.