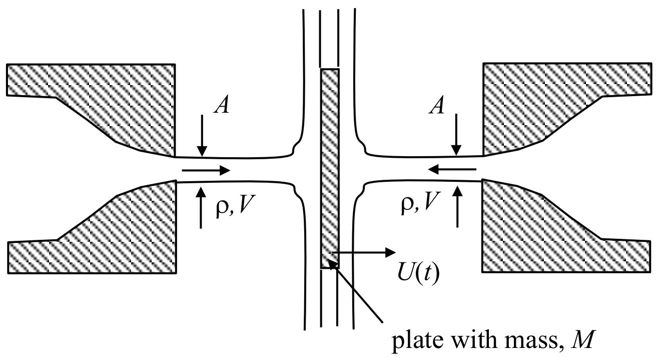
A flat plate of mass, M, is located between two equal and opposite jets of liquid as shown in the figure. At time t=0, the plate is set into motion. Its initial speed is U_0 to the right; subsequently its speed is a function of time, $U(t)$. The motion is without friction and parallel to the jet axes. The mass of liquid that adheres to the plate is negligible compared to M.

Obtain algebraic expressions (as functions of time for t>0) for:

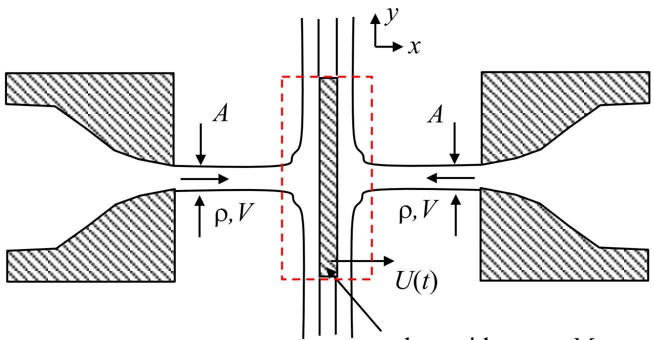
a. the velocity of the plate and
b. the acceleration of the plate.
c. What is the maximum displacement of the plate from its original position?

Express all of your answers in terms of (a subset of) U_0, V, A, ρ, M, and t.



SOLUTION:

Apply the linear momentum equation in the x-direction to a control volume that surrounds the plate as shown in the figure below. Use a frame of reference (FOR) that is fixed to the control volume (non-inertial).



$$\frac{d}{dt}\int_{CV} u_x \rho dV + \int_{CS} u_x\left(\rho \mathbf{u}_{rel}\cdot d\mathbf{A}\right) = F_{B,x} + F_{S,x} - \int_{CV} a_{x/X}\rho dV \quad (1)$$

where

$$\frac{d}{dt}\int_{CV} u_x \rho dV \approx 0 \quad \text{(The CV's } x\text{-linear momentum is approximately zero in the given FOR.)} \quad (2)$$

$$\begin{aligned}\int_{CS} u_x\left(\rho \mathbf{u}_{rel}\cdot d\mathbf{A}\right) &= \underbrace{\left[(V-U)\right]\left[-\rho(V-U)A\right]}_{\text{left side}} + \underbrace{\left[-(V+U)\right]\left[-\rho(V+U)A\right]}_{\text{right side}} \\ &= -\rho(V-U)^2 A + \rho(V+U)^2 A \\ &= \rho\left(-V^2 + 2UV - U^2 + V^2 + 2UV + U^2\right)A \\ &= 4\rho UVA \end{aligned} \quad (3)$$

$$F_{B,x} = F_{S,x} = 0 \quad \text{(No body or surface forces in the } x\text{-direction. The pressure everywhere is } p_{atm}.\text{)} \quad (4)$$

$$\int_{CV} a_{x/X}\rho dV \approx M\frac{dU}{dt} \quad \text{(Assume the plate mass is much larger than the water mass in the CV.)} \quad (5)$$

Substitute and simplify.

$$4\rho UVA = -M\frac{dU}{dt} \quad (6)$$

$$\therefore \frac{dU}{dt} = -\frac{4\rho UVA}{M} \quad (7)$$

$$\int_{U=U_0}^{U=U}\frac{dU}{U} = -\frac{4\rho VA}{M}\int_{t=0}^{t=t} dt \quad (8)$$

$$\ln\left(\frac{U}{U_0}\right) = -\frac{4\rho VAt}{M} \quad (9)$$

$$\boxed{\therefore \frac{U}{U_0} = \exp\left(-\frac{4\rho VAt}{M}\right)} \quad (10)$$

The acceleration is found by differentiating the velocity.

$$\boxed{a = \frac{dU}{dt} = -\frac{4\rho U_0 VA}{M}\exp\left(-\frac{4\rho VAt}{M}\right)} \tag{11}$$

The displacement of the plate is found by integrating the velocity in time.

$$U = \frac{dx}{dt} = U_0 \exp\left(-\frac{4\rho VAt}{M}\right) \tag{12}$$

$$\int_{x=0}^{x=x} dx = U_0 \int_{t=0}^{t=t} \exp\left(-\frac{4\rho VAt}{M}\right) dt \tag{13}$$

$$\therefore x = \frac{MU_0}{4\rho VA}\left[1 - \exp\left(-\frac{4\rho VAt}{M}\right)\right] \tag{14}$$

The maximum displacement occurs as $t \to \infty$.

$$\boxed{\therefore x_{\max} = \frac{MU_0}{4\rho VA}} \tag{15}$$