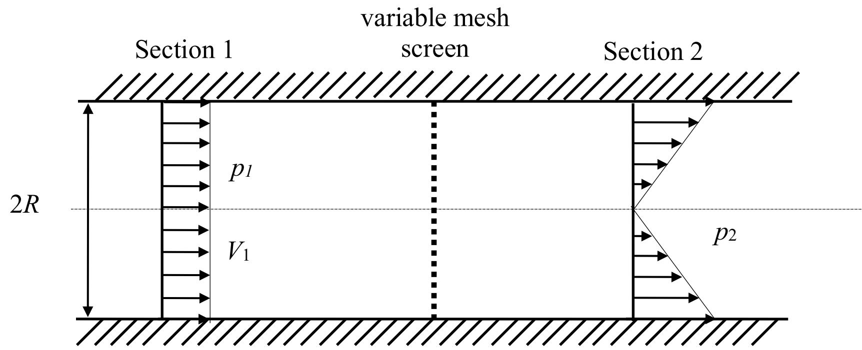
A variable mesh screen produces a linear and axi-symmetric velocity profile as shown in the figure. The static pressure upstream and downstream of the screen are p_1 and p_2 respectively (and are uniformly distributed). If the flow upstream of the mesh is uniformly distributed and equal to V_1, determine the force the mesh screen exerts on the fluid. Assume that the pipe wall does not exert any force on the fluid.



SOLUTION:

First, note that the linear velocity profile at the outlet may be written as,

$$V = V_{\max}\frac{r}{R}, \tag{1}$$

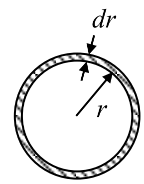
where $V_{\max}$ is the flow velocity at $r = R$. Now apply Conservation of Mass to the fixed control volume shown in the figure to find $V_{\max}$ in terms of the upstream properties,

$$\frac{d}{dt}\int_{\text{CV}} \rho dV + \int_{\text{CV}} \rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A} = 0, \tag{2}$$

where,

$$\frac{d}{dt}\int_{\text{CV}} \rho dV = 0 \quad \text{(steady state)}, \tag{3}$$

$$\int_{\text{CV}} \rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A} = \underbrace{-\rho V_1 \pi R^2}_{\text{left side}} + \underbrace{\int_{r=0}^{r=R} \rho \overbrace{\left(V_{\max}\frac{r}{R}\right)}^{=V} \overbrace{(2\pi r dr)}^{=dA}}_{\text{right side}} \tag{4}$$



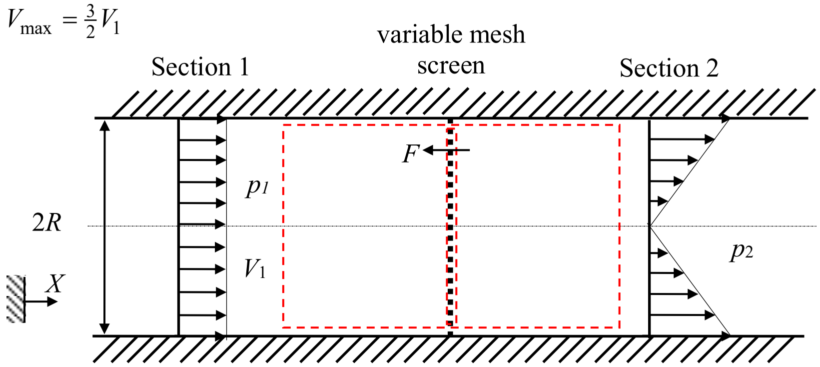
$dA = 2\pi r dr$

$$= -\rho V_1 \pi R^2 + \rho \tfrac{2}{3}\pi V_{\max} R^2$$

Substitute and simplify,

$$-\rho V_1 \pi R^2 + \rho \tfrac{2}{3}\pi V_{\max} R^2 = 0 \tag{5}$$

$$V_{\max} = \tfrac{3}{2} V_1 \tag{6}$$



The control volume weaves in and out of the mesh so that the mesh is not part of the control volume, and instead exerts a force, F, on the control volume.

Now apply the Linear Momentum Equation in the *X*-direction to the fixed control volume shown in the figure,

$$\frac{d}{dt}\int_{\text{CV}} u_X \rho dV + \int_{\text{CV}} u_X \left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = F_{B,X} + F_{S,X} \,, \tag{7}$$

where,

$$\frac{d}{dt}\int_{\text{CV}} u_X \rho dV = 0 \quad \text{(steady state)}, \tag{8}$$

$$\int_{CS} u_X(\rho \boldsymbol{u}_{rel} \cdot d\boldsymbol{A}) = \underbrace{V_1(-\rho V_1 \pi R^2)}_{\text{left side}} + \underbrace{\int_{r=0}^{r=R} \left(\frac{3}{2}V_1 \frac{r}{R}\right)\left[\rho \left(\frac{3}{2}V_1 \frac{r}{R}\right) \underbrace{(2\pi r dr)}_{=dA}\right]}_{\text{right side}}, \tag{9}$$

$$= -\rho V_1^2 \pi R^2 + \frac{9\pi}{2}\frac{\rho V_1^2}{R^2}\int_0^R r^3 dr, \tag{10}$$

$$= -\rho V_1^2 \pi R^2 + \frac{9}{8}\rho V_1^2 \pi R^2 = \frac{1}{8}\rho V_1^2 \pi R^2 \;, \tag{11}$$

$$F_{B,X} = 0 \,, \tag{12}$$

$$F_{S,X} = -F + p_1 \pi R^2 - p_2 \pi R^2 \,. \tag{13}$$

Substitute and simplify,

$$\frac{1}{8}\rho V_1^2 \pi R^2 = -F + p_1 \pi R^2 - p_2 \pi R^2, \tag{14}$$

$$\boxed{F = (p_1 - p_2)\pi R^2 - \frac{1}{8}\rho V_1^2 \pi R^2}, \tag{15}$$

This is the force the mesh applies to the control volume (i.e., the fluid). The fluid applies an equal and opposite force to the mesh.