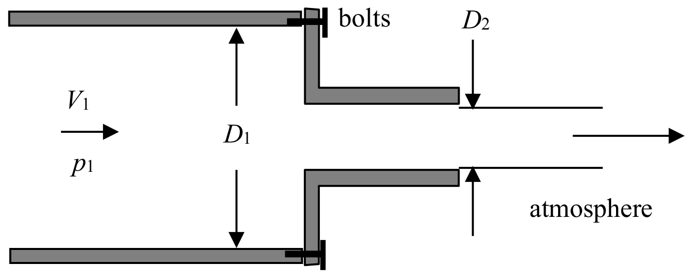
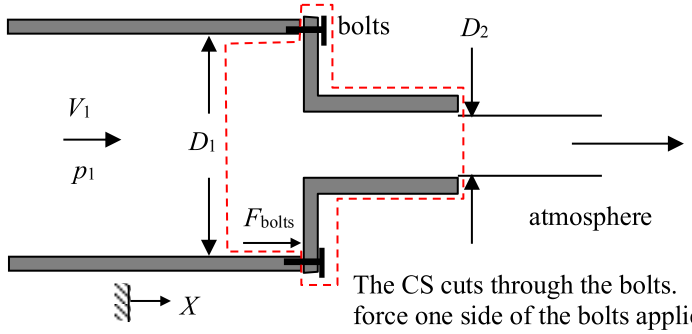
A fluid enters a horizontal, circular cross-sectioned, sudden contraction nozzle. At section 1, which has diameter D_1, the velocity is uniformly distributed and equal to V_1. The gage pressure at 1 is p_1. The fluid exits into the atmosphere at section 2, with diameter D_2. Determine the force in the bolts required to hold the contraction in place. Neglect gravitational effects and assume that the fluid is inviscid.



SOLUTION:

Apply conservation of linear momentum in the X-direction to the fixed control volume shown below.



The CS cuts through the bolts. So that F_{bolts} is the force one side of the bolts applies to the other side.

$$\frac{d}{dt}\int_{\text{CV}} u_X \rho dV + \int_{\text{Cs}} u_X \left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = F_{B,X} + F_{S,X} \qquad (1)$$

where

$$\frac{d}{dt}\int_{\text{CV}} u_X \rho dV = 0 \quad \text{(steady flow)}$$

$$\int_{\text{Cs}} u_X \left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = \rho \underbrace{V_1}_{=u_X} \left(\underbrace{V_1 \hat{\mathbf{i}}}_{=\mathbf{u}_{\text{rel}}} \cdot \underbrace{-\frac{\pi D_1^2}{4}\hat{\mathbf{i}}}_{=\mathbf{A}}\right) + \rho \underbrace{V_2}_{=u_X} \left(\underbrace{V_2 \hat{\mathbf{i}}}_{=\mathbf{u}_{\text{rel}}} \cdot \underbrace{\frac{\pi D_2^2}{4}\hat{\mathbf{i}}}_{=\mathbf{A}}\right) = -\rho V_1^2 \frac{\pi D_1^2}{4} + \rho V_2^2 \frac{\pi D_2^2}{4}$$

(Note that V_2 is unknown for now.)

$$F_{B,X} = 0$$

$$F_{S,X} = p_{1,\text{gage}} \frac{\pi D_1^2}{4} + F_{\text{bolts}}$$

(Note that $p_{2,\text{gage}} = 0$ since $p_{2,\text{abs}} = p_{\text{atm}}$. We could have also worked the problem using absolute pressures everywhere. The pressure force on the left hand side would be $p_{1,\text{abs}}\pi D_1^2/4$ and the pressure force on the right hand side would be $p_{\text{atm}}\pi D_1^2/4$ (note that the diameter is D_1 and not D_2).)

Substitute and re-arrange.

$$-\rho V_1^2 \frac{\pi D_1^2}{4} + \rho V_2^2 \frac{\pi D_2^2}{4} = p_{1,\text{gage}} \frac{\pi D_1^2}{4} + F_{\text{bolts}}$$

$$F_{\text{bolts}} = -\rho V_1^2 \frac{\pi D_1^2}{4} + \rho V_2^2 \frac{\pi D_2^2}{4} - p_{1,\text{gage}} \frac{\pi D_1^2}{4} \qquad (2)$$

To determine V_2, apply conservation of mass to the same control volume.

$$\frac{d}{dt}\int_{\text{CV}} \rho dV + \int_{\text{Cs}} \rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A} = 0 \tag{3}$$

where

$$\frac{d}{dt}\int_{\text{CV}} \rho dV = 0$$

$$\int_{\text{Cs}} \rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A} = -\rho V_1 \frac{\pi D_1^2}{4} + \rho V_2 \frac{\pi D_2^2}{4}$$

Substitute and simplify.

$$-\rho V_1 \frac{\pi D_1^2}{4} + \rho V_2 \frac{\pi D_2^2}{4} = 0$$

$$V_2 = V_1 \left(\frac{D_1}{D_2}\right)^2 \tag{4}$$

Substitute Eqn. (4) into Eqn. (2) and simplify.

$$F_{\text{bolts}} = -\rho V_1^2 \frac{\pi D_1^2}{4} + \rho V_1^2 \left(\frac{D_1}{D_2}\right)^4 \frac{\pi D_2^2}{4} - p_{1,\text{gage}} \frac{\pi D_1^2}{4}$$

$$\boxed{F_{\text{bolts}} = \rho V_1^2 \frac{\pi D_1^2}{4}\left[\left(\frac{D_1}{D_2}\right)^2 - 1\right] - p_{1,\text{gage}} \frac{\pi D_1^2}{4}} \tag{5}$$

Note that F_{bolts} was assumed to be positive when acting in the $+X$ direction (causing compression in the bolts). If $F_{\text{bolts}} < 0$ then the bolts will be in tension.