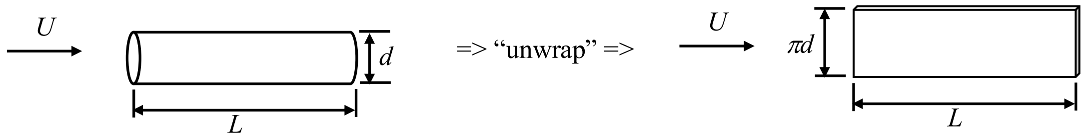
The U.S. Navy's *Ohio*-class guided-missile submarines have a length of 170.69 m (560 ft) and a beam, i.e., width, of 12.8 m (42 ft). Assume the submarine travels at 37.0 kph (= 20 kn) when fully submerged.

a. What percentage of the submarine's surface has a laminar boundary layer for these conditions?
b. Estimate the power required for the submarine to overcome skin friction drag for these conditions.

SOLUTION:

Model the submarine as a cylinder with a diameter of d = 12.8 m and a length of L = 170.69 m. “Unwrap” the cylinder and model the flow along its length of the cylinder as flow adjacent to a flat plate as shown in the figures below.



First calculate the distance from the leading edge at which the boundary layer transitions from laminar to turbulent flow,

$$\mathrm{Re}_{x_{\mathrm{crit}}} = \frac{U x_{\mathrm{crit}}}{\nu} = 500{,}000 \quad \Rightarrow \quad x_{\mathrm{crit}} = 500{,}000\left(\frac{\nu}{U}\right). \tag{1}$$

Using the given numbers,

U = 37.0 kph = 10.3 m/s,

ρ_{seawater} = 1025 kg/m^3,

μ_{seawater} = 1.08*10^{-3} Pa.s,

□□□ ν□□□□□□□□□□ μ□ρ □ 1.05*10^{-6} m^2/s, (2)

□□□ x_{crit} = 5.10*10^{-2} m = 5.1 cm!

Thus, the fraction of the length that’s laminar is,

x_{crit}/L = (5.10*10^{-2} m)/(170.69 m) = 0.030%. (3)

Clearly, the flow over the submarine can be assumed turbulent over the entire length without much error.

Assuming a turbulent boundary layer over the full length of the hull, the drag force is,

$$C_D \equiv \frac{D}{\tfrac{1}{2}\rho U^2 L(\pi d)} = \frac{0.0742}{\mathrm{Re}_L^{1/5}}, \tag{4}$$

$$D = \frac{0.0742}{\mathrm{Re}_L^{1/5}}\left(\tfrac{1}{2}\rho U^2 L H\right) \text{ where } \mathrm{Re}_L = \frac{UL}{\nu}. \tag{5}$$

Using the given numbers,

L = 170.69 m,

$\pi d = \pi$(12.8 m) = 40.2 m,

□□□□Re_L = 1.67*10^9,

=> D = 3.96*10^5 N (= 89,000 lb$_f$!)

The power to overcome this skin friction drag is,

$P = DU$,

=> P = 4.08*10^6 W (= 5470 hp!)