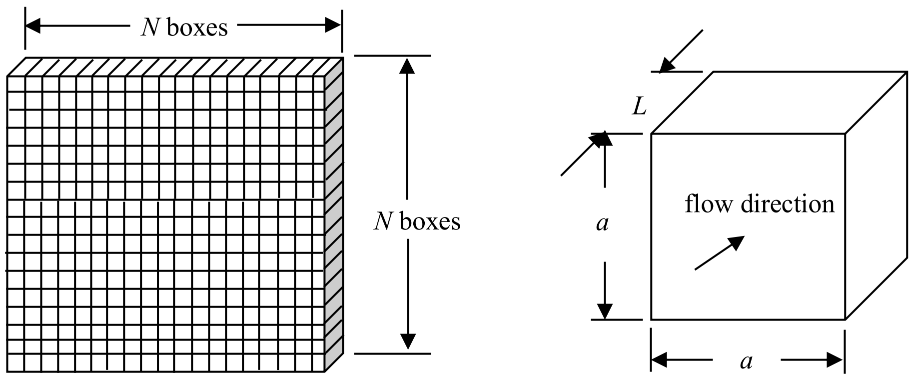
Flow straighteners are arrays of narrow ducts placed in wind tunnels to remove swirl and other in-plane secondary velocities. They can be idealized as square boxes constructed by vertical and horizontal plates as shown in the figure. The cross-section of the box is a by a and the box length is L. Assuming laminar flat plate flow and an array of N by N boxes, derive a formula for:

a. the total drag on the bundle of boxes.
b. the effective pressure drop across the bundle.



SOLUTION:

Determine the drag acting on one wall due to skin friction. From the Blasius solution, the drag coefficient for laminar, flat plate flow is:

$$c_D = \frac{1.328}{\mathrm{Re}_L^{1/2}} \tag{1}$$

where

$$c_D = \frac{D}{\frac{1}{2}\rho U^2 (La)} \tag{2}$$

$$\mathrm{Re}_L = \frac{UL}{\nu} \tag{3}$$

Note that in writing Eqn. (1) we've assumed that there is no pressure gradient in the cell's core flow. This is not exactly correct since there will, in fact, be a pressure gradient due to the growth of the boundary layer along the plate surface and hence an increase in the outer (*i.e.,* cell core) flow velocity (from conservation of mass). However, as a first estimate it is reasonable to assume a constant outer flow velocity and thus a Blasius boundary layer profile. A more precise analysis would account for the growth in the displacement thickness and the resulting increase in the outer velocity.

Using Eqns. (1) and (2), the skin friction drag acting on one wall of a cell is:

$$D_{\substack{\text{one}\\\text{wall}}} = \frac{1.328 \cdot \frac{1}{2}\rho U^2 (La)}{\mathrm{Re}_L^{1/2}} = \frac{0.664\rho U^2 (La)}{\mathrm{Re}_L^{1/2}} \tag{4}$$

The drag acting on a single cell, which consists of four walls, is:

$$D_{\text{cell}} = 4D_{\substack{\text{one}\\\text{wall}}} = \frac{2.656\rho U^2 (La)}{\mathrm{Re}_L^{1/2}} \tag{5}$$

The total drag acting on a grid of NxN cells is:

$$\boxed{D_{\substack{N\text{x}N\\\text{cells}}} = N^2 D_{\text{cell}} = \frac{2.656\rho U^2 (La) N^2}{\mathrm{Re}_L^{1/2}}} \tag{6}$$

where Re_L is given in Eqn. (3).

As stated previously, in deriving Eqn. (1) we've assumed that there is no pressure gradient within the cell which is not entirely correct but should instead be considered a first-cut estimate. Regardless, we can still determine an effective pressure drop across the flow straightener by dividing the drag force acting on the straightener by its area.

$$\boxed{\Delta p = \frac{-D_{N\text{x}N\ \text{cells}}}{N^2 a^2}} \quad \text{(The pressure decreases across the flow straightener.)} \tag{7}$$

A more accurate approach to solving this problem involves iteration. First, determine the velocity in the outer flow region using the displacement thickness, d_D.

$$\dot{m} = \text{constant} = \rho U a^2 = \rho U' \left(a - 2\delta_D\right)^2$$

$$U' = U\left(\frac{a}{a - 2\delta_D}\right)^2 \tag{8}$$

where, from the Blasius solution:

$$\frac{\delta_D}{L} = \frac{1.72}{\mathrm{Re}_L^{1/2}} \tag{9}$$

and

$$\mathrm{Re}_L = \frac{U'L}{\nu} \tag{10}$$

The pressure in the outer layer can be determined using Bernoulli's equation.

$$p + \tfrac{1}{2}\rho U^2 = p' + \tfrac{1}{2}\rho U'^2$$

$$\Delta p = p' - p = \tfrac{1}{2}\rho\left(U^2 - U'^2\right) \tag{11}$$

The iterative procedure is as follows:

1. Assume $U' = 0$.
2. Evaluate the displacement thickness using Eqns. (9) and (10).
3. Evaluate the new downstream velocity, U', using Eqn. (8).
4. Repeat steps 2 and 3 until a converged solution for U' occurs.
5. Use Eqn. (11) to determine Δp.