3.8. Common Thermodynamic Cycles

Important: The reader should read the Conservation of Mass, First Law, and Second Law sections of Chapter 4 prior to reading this section since this section involves applying these relations to control volumes, which have mass crossing the control volume boundaries.

In this section we review several thermodynamic cycles encountered in practice, including:

- the Carnot cycle (not common in practice, but real cycles are often compared to this ideal cycle),
- the Rankine cycle and improvements to the Rankine cycle,
- the Otto, Diesel, and dual cycles, and
- the Brayton cycle and improvements to the Brayton cycle.

3.8.1. Carnot Cycle

The Carnot cycle is one particular type of internally reversible cycle and serves as a point of comparison for other real and internally reversible cycles. Carnot cycles include power, refrigeration, and heat pump cycles. This section, however, focuses specifically on Carnot power cycles. The Carnot refrigeration and heat pump cycles are similar to the power cycle, but operate in the opposite direction.

The Carnot power cycle consists of the following four internally reversible processes:

(1) *Process 1 – 2*: Isothermal expansion at T_H.
(2) *Process 2 – 3*: Adiabatic expansion ($Q_{23} = 0$).
(3) *Process 3 – 4*: Isothermal compression at T_C.
(4) *Process 4 – 1*: Adiabatic compression ($Q_{41} = 0$).

As mentioned previously, a Carnot refrigeration or heat pump cycle would operate in reverse, i.e., Process 1 - 2 would be an isothermal compression, Process 2 - 3 would be an adiabatic compression, etc.

Notes:

(1) Figure 3.39 illustrates the four Carnot power cycle processes for a piston-cylinder arrangement. The corresponding processes are shown on the p-v and T-s diagrams for a substance remaining entirely in the vapor phase throughout the cycle. Note that Carnot cycles can also involve working fluids that change phase.
(2) One can also have a Carnot cycle involving a system of components, as shown in Figure 3.40. The same four processes described at the beginning of this section occur in this system. In the particular case shown in the p-v and T-s plots, the working fluid is a saturated liquid vapor mixture.
(3) Since each of the processes in a Carnot cycle is internally reversible, the entire Carnot cycle is also internally reversible. Furthermore, in a Carnot cycle, the absolute temperatures at which the energy is added to or removed from the working fluid via heat transfer are T_H and T_C. Hence, the efficiency and coefficients of performance of a Carnot cycle are given by,

$$\eta_{\text{rev}} = 1 - \frac{T_C}{T_H}, \tag{3.189}$$

$$COP_{\text{ref,rev}} = \frac{T_C}{T_H - T_C}, \tag{3.190}$$

$$COP_{\text{HP,rev}} = \frac{T_H}{T_H - T_C}. \tag{3.191}$$

(4) In order to be internally reversible, the heat addition and removal processes with the thermal reservoirs must not occur over a finite temperature gradient, i.e., the temperature of the system must equal the temperature of the thermal reservoir during the heat addition and heat removal processes. In a real system, there must be some finite temperature difference to drive the heat transfer process, which is one reason a real cycle would not be reversible. Similarly, in order to be internally reversible, the working fluid must have no viscosity, there should be no friction in the system, and all of the processes must be in quasi-equilibrium.
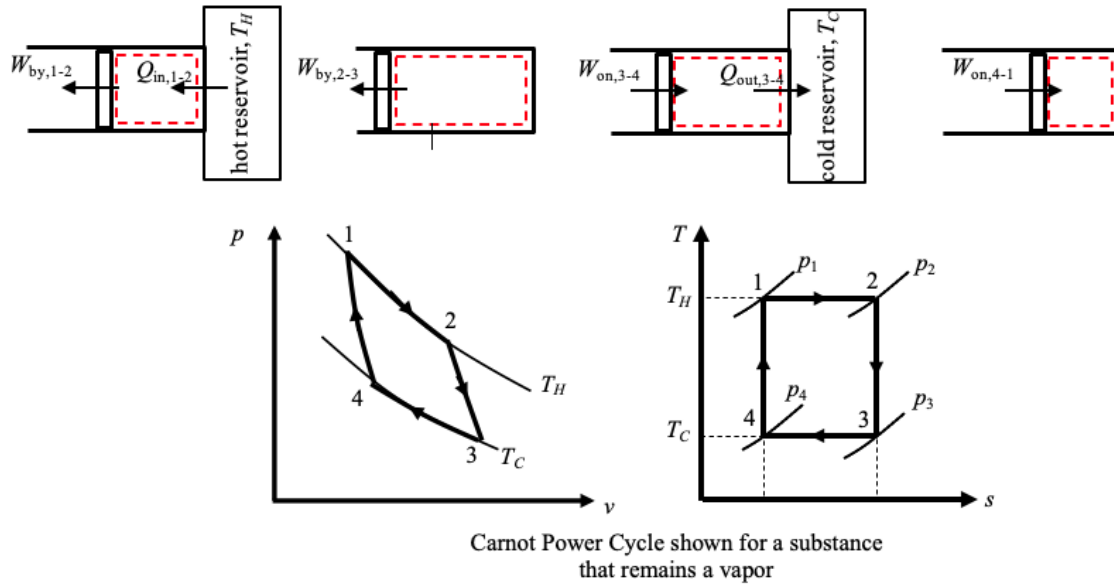
Carnot Power Cycle shown for a substance
that remains a vapor

Figure 3.39. A sketch of the four processes of a Carnot power cycle for a piston-cylinder system. The corresponding processes are sketched on p-v and T-s plots for a working fluid that remains entirely in the vapor phase throughout the cycle.

(5) It's possible for a system to have the same set of processes as the Carnot cycle (isothermal expansion, adiabatic expansion, isothermal compression, adiabatic compression), but not be reversible. For example, the heat transfer may occur over a finite temperature difference or the working fluid may be viscous. In that case, the efficiency of the cycle will be less than the Carnot efficiency.
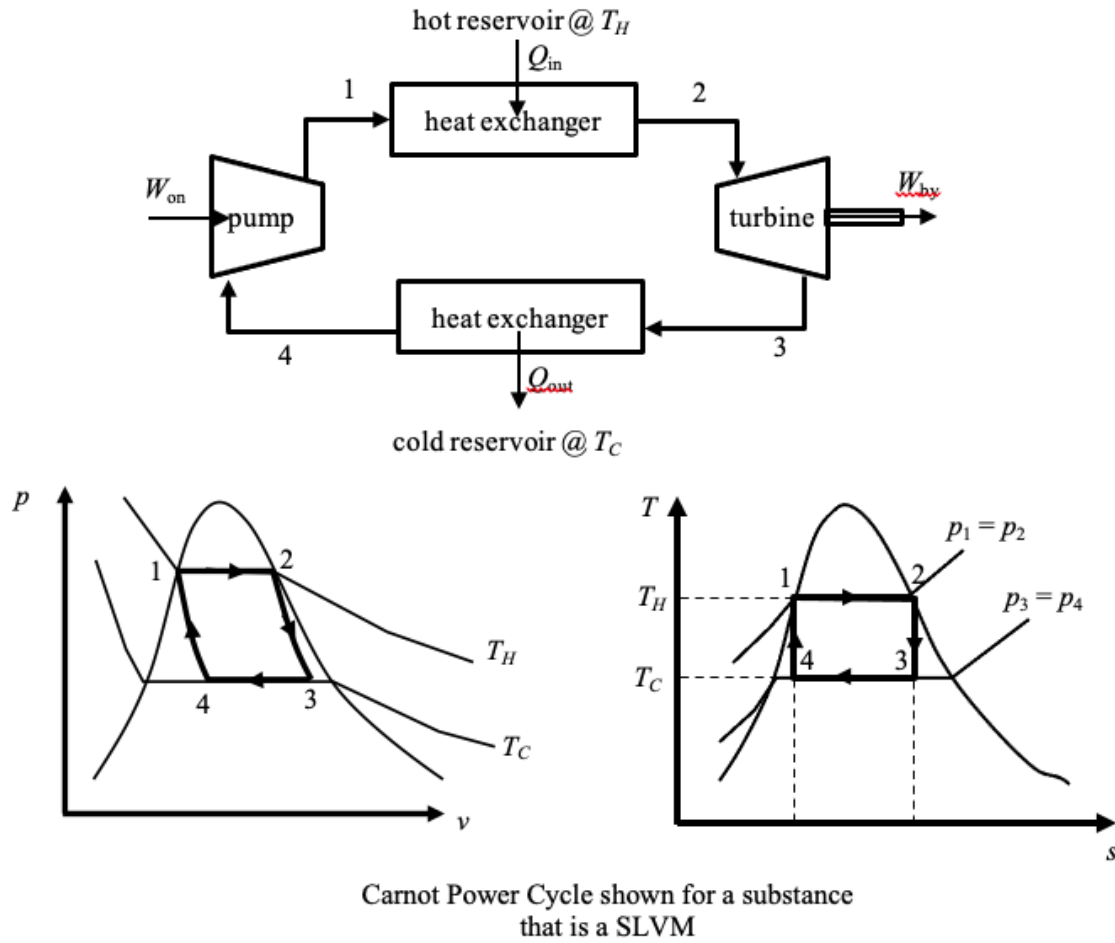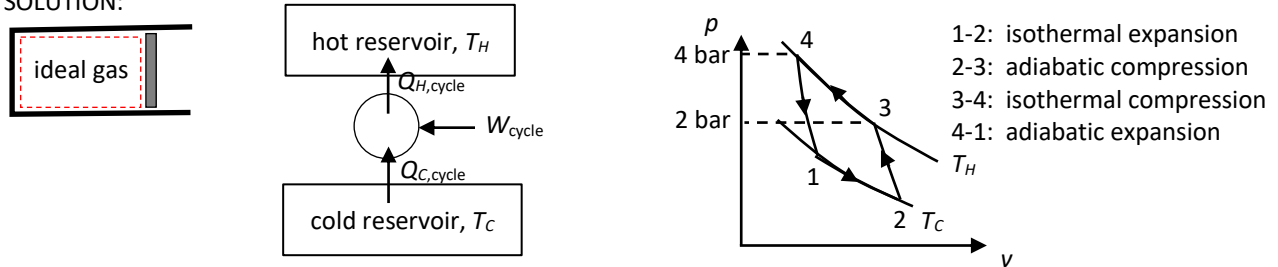
FIGURE 3.40. A sketch of a Carnot power cycle and p-v, and T-s plots for a simple vapor power cycle in which the working fluid remains as a saturated liquid-vapor mixture.

An ideal gas within a piston-cylinder assembly undergoes a Carnot refrigeration cycle. The isothermal compression occurs at 325 K from 2 bar (abs) to 4 bar (abs). The isothermal expansion occurs at 250 K. Determine:
a.   the coefficient of performance,
b.   the heat transfer to the gas during the isothermal compression, in kJ per kmol of gas,
b.   the heat transfer to the gas during the isothermal expansion, in kJ per kmol of gas, and
c.   the magnitude of the net work input over the cycle, in kJ per kmol of gas.

SOLUTION:

hot reservoir, T_H

$Q_{H,cycle}$

W_{cycle}

$Q_{C,cycle}$

cold reservoir, T_C

1-2: isothermal expansion
2-3: adiabatic compression
3-4: isothermal compression
4-1: adiabatic expansion

Since a Carnot cycle is reversible, the coefficient of performance is,

$$\mathrm{COP}_{\substack{\mathrm{ref,}\\ \mathrm{rev}}} = \frac{T_C}{T_H - T_C} \,, \tag{1}$$

with T_H = 325 K and T_C = 250 K => $\boxed{\mathrm{COP_{ref,rev} = 3.33}}$.

The heat transfer to the gas during isothermal compression (process 3-4) can be found using the 1st Law for the process. From the 1st Law applied to the system identified in the figure, specifically for the process from 3 to 4,

$$\Delta E_{\mathrm{sys},34} = Q_{\mathrm{into\ sys},34} + W_{\mathrm{on\ sys},34} \Rightarrow Q_{\mathrm{into\ sys},34} = W_{\mathrm{by\ sys},34} \,, \tag{2}$$

where $\Delta E_{\mathrm{sys},34}$ = 0 since we're assuming no change in kinetic or potential energy during the process, and since the process is isothermal and we're working with an ideal gas in which $u = u(T)$, $\Delta U_{34} = 0$. The work done by the system during this compression is,

$$W_{\mathrm{by\ sys},34} = \int_{V_3}^{V_4} p\, dV = \int_{V_3}^{V_4} \frac{mRT_H}{V} dV = mRT_H \int_{V_3}^{V_4} \frac{dV}{V} = mRT_H \ln\left(\frac{V_4}{V_3}\right) = mRT_H \ln\left(\frac{p_3}{p_4}\right), \tag{3}$$

where the ideal gas law has been used a couple of times ($pV = mRT$) couple with the fact that the process is isothermal with $T = T_H$ and m = constant. Since we're asked to find the heat in kJ per kmol, re-write Eq. (3) on a per mole basis (recall $m = n/M$ and $R = MR_u$, where M is molecular weight),

$$W_{\mathrm{by\ sys},34} = (n/M)(M\bar{R}_u)T_H \ln\left(\frac{p_3}{p_4}\right) \Rightarrow \frac{W_{\mathrm{by\ sys},34}}{n} = \bar{R}_u T_H \ln\left(\frac{p_3}{p_4}\right). \tag{4}$$

Combining Eqs. (4) and (2) gives,

$$\frac{Q_{\mathrm{into\ sys},34}}{n} = \bar{R}_u T_H \ln\left(\frac{p_3}{p_4}\right). \tag{5}$$

Using the given data,
    R_u = 8.314 kJ/(kmol.K)
    T_H = 325 K
    p_3 = 2 bar (abs)
    p_4 = 4 bar (abs)
    => $\boxed{Q_{\mathrm{into\ sys},34}/n = \text{-1870 kJ/kmol}}$ (heat is leaving the system during this compression process)

To determine the heat transfer during isothermal expansion, make use of the definition of the absolute temperature scale,

$$\left.\frac{Q_C}{Q_H}\right|_{\substack{\text{rev,}\\\text{cycle}}} = \frac{T_C}{T_H} \Rightarrow Q_C = Q_H \frac{T_C}{T_H} \Rightarrow Q_{12} = -Q_{34} \frac{T_C}{T_H} \ , \tag{6}$$

where $Q_C = Q_{into,12}$ and $Q_H = -Q_{into,34}$. Using the given data,

=> $\boxed{Q_{into,12}/n = 1440 \text{ kJ/kmol.}}$

The magnitude of the net work input over the cycle may be found using the definition of the coefficient of performance for a refrigeration cycle,

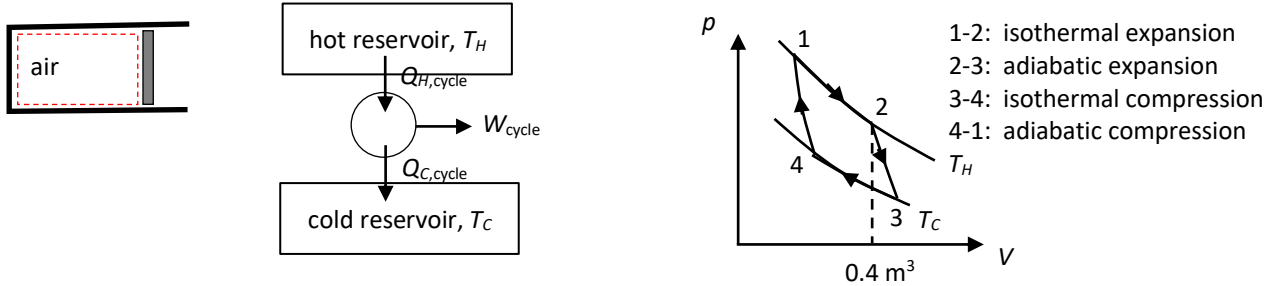$$COP_{ref} \equiv \frac{Q_C}{W_{on\ sys,net}} \Rightarrow W_{on\ sys,net} = \frac{Q_C}{COP_{ref}} \ . \tag{7}$$

Using the given data,

=> $\boxed{W_{on\ sys,net}/n = 433 \text{ kJ/kmol}}$.

Two kilograms of air within a piston-cylinder assembly execute a Carnot power cycle with maximum and minimum temperatures of 750 K and 300 K, respectively. The heat transfer to the air during the isothermal expansion is 60 kJ. At the end of the isothermal expansion the volume is 0.4 m³. Assuming the ideal gas model for the air, determine:
a.   the thermal efficiency,
b.   the pressure and volume at the beginning of the isothermal expansion, in kPa (abs) and m³, respectively,
c.   the work and heat transfer for each of the four processes, in kJ, and
d.   sketch the cycle on a p-V diagram.

SOLUTION:

1-2: isothermal expansion
2-3: adiabatic expansion
3-4: isothermal compression
4-1: adiabatic compression

Since the cycle is a Carnot power cycle, which is reversible, the efficiency is given by,

$$\eta_{rev} = 1 - \frac{T_C}{T_H}.$$ (1)

Using the given temperatures (T_C = 300 K and T_H = 750 K) => $\boxed{\eta_{rev} = 0.6}$.

The pressure and volume at the beginning of the isothermal expansion (state 1) may be found combining the 1st Law applied to the system shown in the figure, with the given heat transfer, and the definition of the power cycle efficiency. Applying the 1st Law to the system for the process from 1 to 2,

$$\Delta E_{12} = Q_{into,12} + W_{on,12} \Rightarrow W_{by,12} = Q_{into,12},$$ (2)

where $W_{by,12}$ = -$W_{on,12}$. Furthermore, ΔE_{12} = 0 since the change in kinetic and potential energy in going from state 1 to state 2 is zero, and since the process is isothermal and the system is an ideal gas ($u = u(T)$) => ΔU_{12} = 0. The heat added to the system in going from 1 to 2 is given (= 60 kJ) and thus the work done by the system is also known. The work done by the system may also be found using,

$$W_{by,12} = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} \frac{mRT_H}{V}\,dV = mRT_H \int_{V_1}^{V_2} \frac{dV}{V} = mRT_H \ln\left(\frac{V_2}{V_1}\right),$$ (3)

where the ideal gas law has been used ($pV = mRT$) along with the fact that the process from 1 to 2 is isothermal with $T = T_H$. The mass is constant too. Since the volume at the end of the isothermal expansion process is given (V_2 = 0.4 m³), we can solve for V_1,

$$W_{by,12} = mRT_H \ln\left(\frac{V_2}{V_1}\right) \Rightarrow V_1 = V_2 \exp\left(-\frac{W_{by,12}}{mRT_H}\right),$$ (4)

with,
   V_2 = 0.4 m³,
   $W_{by,12}$ = 60 kJ,
   m = 2 kg,
   R_{air} = 0.287 kJ/(kg.K),
   T_H = 750 K,
   => $\boxed{V_1 = 0.348 \text{ m}^3}$.

The pressure at state 1 may be found using the ideal gas law,

$$p_1 = \frac{mRT_H}{V_1} \quad (T_1 = T_H),$$ (5)

=> $\boxed{p_1 = 1.24 \text{ MPa (abs)}}$.

The work and heat for process 1-2 have already been calculated. The heat for processes 2-3 and 4-1 are zero, i.e., $\boxed{Q_{23} = Q_{41} = 0}$, since these processes are adiabatic.

The work done on the system during process 2-3 may be found using the 1st Law,

$$\Delta E_{23} = Q_{into,23} + W_{on,23} \Rightarrow W_{on,23} = \Delta U_{23} = m(u_3 - u_2),$$ (6)

where $u_2 = u_2(T_2 = T_H)$ and $u_3 = u_3(T_3 = T_C)$ since the air is being treated as an ideal gas. These specific internal energies may be looked up in a table, e.g., Table A-22 from Moran et al., 8th ed.,

$u_2 = 551.99$ kJ/kg and $u_3 = 214.07$ kJ/kg => $\boxed{W_{on,23} = \Delta U_{23} = 676 \text{ kJ}}$. (7)

If a perfect gas model is used, then,

$\Delta U_{23} = mc_v(T_3 - T_2)$ => $\Delta U_{23} = 673$ kJ, (8)

where $c_v = 0.748$ kJ/(kg.K) (Table A-20, Moran et al., 8th ed.). This result is less than 1% different than the ideal gas result.

The work done on the system during process 4-1 will be identical in magnitude, but opposite in sign to the work done on the system during process 2-3 since both processes are adiabatic and operate between the same temperatures. Hence, $\boxed{W_{on,41} = -676 \text{ kJ}}$.

For process 3-4, the heat transferred into the system may be found using the definition of the absolute temperature scale (since the process is assumed reversible),

$$\left.\frac{Q_C}{Q_H}\right|_{\substack{rev, \\ cycle}} = \frac{T_C}{T_H} \Rightarrow Q_C = Q_H\left(\frac{T_C}{T_H}\right) \Rightarrow Q_{out\,of,34} = Q_{into,12}\left(\frac{T_C}{T_H}\right),$$ (9)

Using the given data, $\boxed{Q_{out\,of,34} = 24 \text{ kJ}}$.

Since the process from 3 to 4 is isothermal, we're dealing with an ideal gas, and there is no change in kinetic or potential energy, the 1st Law gives,

$$\Delta E_{34} = Q_{into,34} + W_{on,34} \Rightarrow W_{on,34} = Q_{out\,of,34},$$ (10)

=> $\boxed{W_{on,34} = 24 \text{ kJ}}$.

As a check, we can substitute the heat and work values into the definition for the power cycle efficiency,

$$\eta \equiv \frac{W_{by\,sys,net}}{Q_{into\,sys,12}},$$ (11)

where $W_{by\,sys,net} = W_{by,12} + W_{by,23} + W_{by,34} + W_{by,41} = (60 \text{ kJ}) + (676 \text{ kJ}) + (-24 \text{ kJ}) + (-676 \text{ kJ}) = 36$ kJ,

=> $\eta = 0.6$, which is exactly the value expected from part (a).

Also, from the 1st Law applied to the entire cycle (recall $\Delta E_{cycle} = 0$), we must have,

$Q_{into,net} = W_{by,net}$,

where $Q_{into,net} = Q_{into,12} + Q_{into,23} + Q_{into,34} + Q_{into,41} = (60 \text{ kJ}) + (0) + (-24 \text{ kJ}) + (0) = 36$ kJ, which is precisely $W_{by,net}$ found previously.