3.8.5. The Brayton Cycle and Improvements

The Brayton cycle is a thermodynamic model for gas turbine engines. In a simple gas turbine engine (components shown schematically in Figure 3.52), air enters a compressor, which does work on the air to increase its pressure and temperature. Downstream of the compressor is a combustor. In the combustor, fuel is added to the air and the air/fuel mixture is ignited, increasing the working fluid temperature significantly. The combustion products move downstream through a turbine, which extracts power from the working fluid. Part of the power from the turbine is used to drive the compressor. In an energy generation application, the remainder of the power is used to drive a generator to create electricity. If the gas turbine engine is used as a jet engine, then only a small portion of the working fluid energy is extracted for excess power. Instead, the remainder of the flow energy is converted to kinetic energy via a nozzle at the engine outlet to provide thrust.

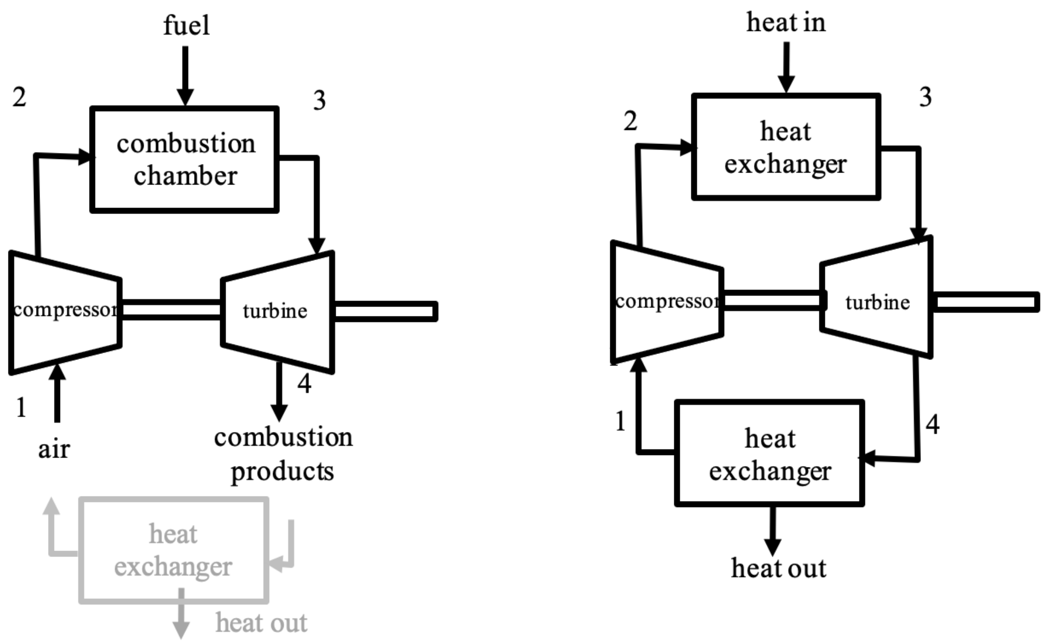


Figure 3.52. Illustrations of the components in open and closed gas turbine cycles.

The left side of Figure 3.52 is an open gas turbine engine cycle while the right side is a closed cycle. In a closed cycle, the working fluid (usually air) is recycled throughout the cycle. In an open cycle, however, fresh air enters the cycle and then is discharged to the atmosphere downstream of the turbine. The figure shows a grayed-out heat exchanger since discharging the exhaust to the atmosphere then pulling in fresh air from the atmosphere is effectively the same as running the air through a heat exchanger. Because the atmosphere is large, the high temperature air leaving the turbine eventually comes into thermal equilibrium with the surrounding atmosphere.

Like the internal combustion engine analyses described in the previous section, the basic study of a gas turbine engine cycle uses an air standard analysis. The assumptions made in this air standard analysis include:

(1) Air is modeled as an ideal gas.
(2) Combustion is modeled as a heat addition process and the working fluid remains as air. The combustion chemistry and changes to the working fluid are ignored.

In a cold air standard analysis, we further assume constant specific heats, i.e., a perfect gas assumption.

The processes in an ideal Brayton cycle analysis, used to model a gas turbine engine cycle, include (Figure 3.53):

- *Process 1 - 2*: isentropic compression of the working fluid through the compressor,
- *Process 2 - 3*: constant pressure heat addition to the working fluid through the heat exchanger (combustion),
- *Process 3 - 4*: isentropic expansion of the working fluid through the turbine,
- *Process 4 - 1*: constant pressure heat transfer from the working fluid as it flows through the heat exchanger.

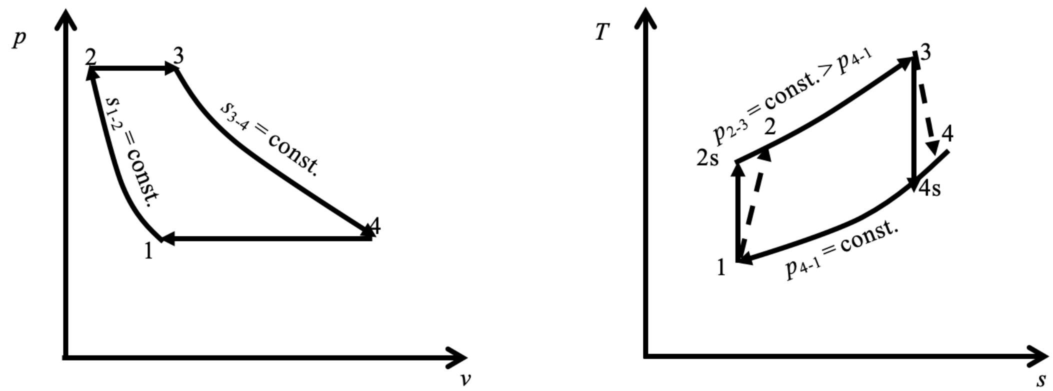


Figure 3.53. Illustrations of the processes in a Brayton cycle shown on p-v and T-s plots.

A more detailed analysis of the Brayton cycle involves applying the First Law to control volumes surrounding the various components. For example, for control volumes surrounding the turbine and the compressor,

$$\frac{\dot{W}_{\text{by turbine}}}{\dot{m}} = h_3 - h_4 \quad \text{and} \quad \frac{\dot{W}_{\text{on compressor}}}{\dot{m}} = h_2 - h_1. \tag{3.240}$$

Applying the First Law to control volumes surrounding the heat exchangers,

$$\frac{\dot{Q}_{\text{added}}}{\dot{m}} = h_3 - h_2 \quad \text{and} \quad \frac{\dot{Q}_{\text{removed}}}{\dot{m}} = h_4 - h_1. \tag{3.241}$$

The thermal efficiency of this power cycle is,

$$\eta_{\text{Brayton}} = \frac{\dot{W}_{\text{net, out}}}{\dot{Q}_{\text{added}}} = \frac{\dot{W}_{\text{by turb}}/\dot{m} - \dot{W}_{\text{on comp}}/\dot{m}}{\dot{Q}_{\text{added}}/\dot{m}}, \tag{3.242}$$

$$= \frac{(h_3 - h_4) - (h_2 - h_1)}{(h_3 - h_2)} = \frac{(h_3 - h_2) - (h_4 - h_1)}{(h_3 - h_2)}, \tag{3.243}$$

$$= 1 - \frac{(h_4 - h_1)}{(h_3 - h_2)} = 1 - \frac{\dot{Q}_{\text{removed}}}{\dot{Q}_{\text{added}}}. \tag{3.244}$$

The back work ratio (bwr) for the cycle is,

$$\text{bwr} = \frac{\dot{W}_{\text{on compressor}}}{\dot{W}_{\text{by turbine}}} = \frac{h_2 - h_1}{h_3 - h_2}. \tag{3.245}$$

The bwr for a typical gas turbine is 40 – 80%. The typical bwr for a vapor power plant (Rankine cycle) is 1 – 3%. The difference is due to the fact that the specific volume for a gas is large, but is small for a liquid. Recall that for a steady, internally reversible, process with one inlet and one outlet and negligible changes in kinetic and potential energies: $\dot{W}_{\text{by}}/\dot{m} = \int_1^2 v\,dp$.

Notes:

(1) Across the compressor and turbine, the typical pressure ratios ($p_2/p_1 = p_3/p_4$) are 5 – 20 and typical engine thermal efficiencies are 35 – 60%.

(2) A larger pressure ratio across the compressor (p_2/p_1) gives a larger efficiency since a larger pressure at State 2 corresponds to a larger average temperature through the combustor where heat addition occurs (recall that $\eta_{\text{rev}} = 1 - T_C/T_H$). Alternately, increasing the temperature leading into the turbine (T_3) also leads to a larger efficiency; however, the temperature at the turbine inlet is typically limited by metallurgical considerations.

(3) Consider the case when T_3 is fixed (for example, due to metallurgical factors), but p_2/p_1 is varied, as shown in Figure 3.54. Cycle A has a larger thermal efficiency than Cycle B since the pressure ratio for Cycle A is larger.

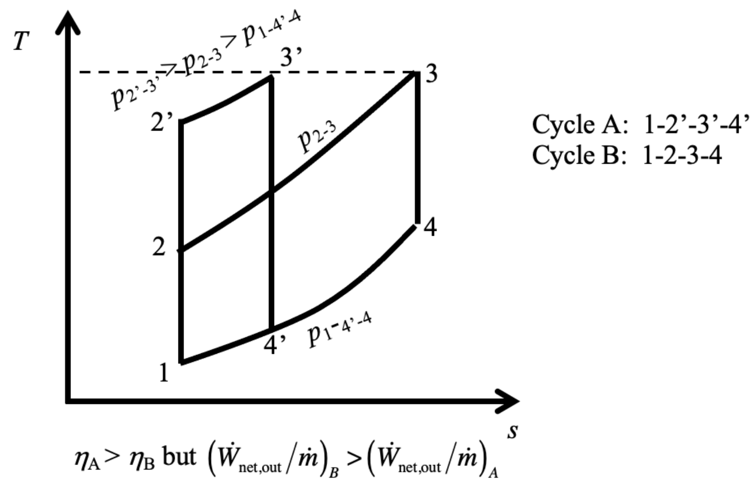


Figure 3.54. Two Brayton cycles, but different pressure ratios. Both cycles have the same temperature leaving the combustor ($T_3 = T_3'$), but Cycle A (1-2'-3'-4') has a larger compressor pressure ratio than Cycle B (1-2-3-4), i.e., $p_2'/p_1 > p_2/p_1$.

The area enclosed by Cycle B is larger than the area for Cycle A; hence, Cycle B has a larger work per unit mass flow rate. In order for Cycle A to produce the same work, a larger mass flow rate would be required, potentially requiring a larger set of components, which might be unacceptable for use on an aircraft where weight is a significant design factor. Hence, in aircraft applications, aircraft engine designers typically design for maximum work per unit mass flow rate, i.e., $\left(\dot{W}_{\text{net,out}}/\dot{m}\right)_{\text{max,fixed}T_3}$.

(4) A larger value for the specific heat ratio k results in a larger thermal efficiency. The specific heat ratio is governed by the type of fuel used.

(5) Recall that from the First Law, the Second Law, and the Tds equations combined together, for a steady state flow with a single inlet-outlet and negligible changes in kinetic and potential energies:

$$\frac{\dot{W}_{\text{out}}}{\dot{m}} = w_{\text{out}} = -\int_1^2 v\,dp \quad \text{and} \quad \frac{\dot{W}_{\text{in}}}{\dot{m}} = w_{\text{in}} = \int_1^2 v\,dp \tag{3.246}$$

Thus, the specific work extracted by the turbine can be increased if the specific volume of the working fluid can be increased, and the specific work into the compressor can be decreased if the specific volume can be decreased. These are the ideas behind the concept of "reheating" and "intercooling". Intercooling between successive compressor stages is used to decrease the specific volume of the working fluid. From the Ideal Gas Law,

$$v = \frac{RT}{p} \tag{3.247}$$

For $p =$ constant in an intercooling heat exchanger (refer to Figure 3.55), if T decreases, then v also decreases and, from Eq. (3.246), w_{in} decreases. Thus, intercooling between compressor stages can reduce the specific work required to drive the compressor.

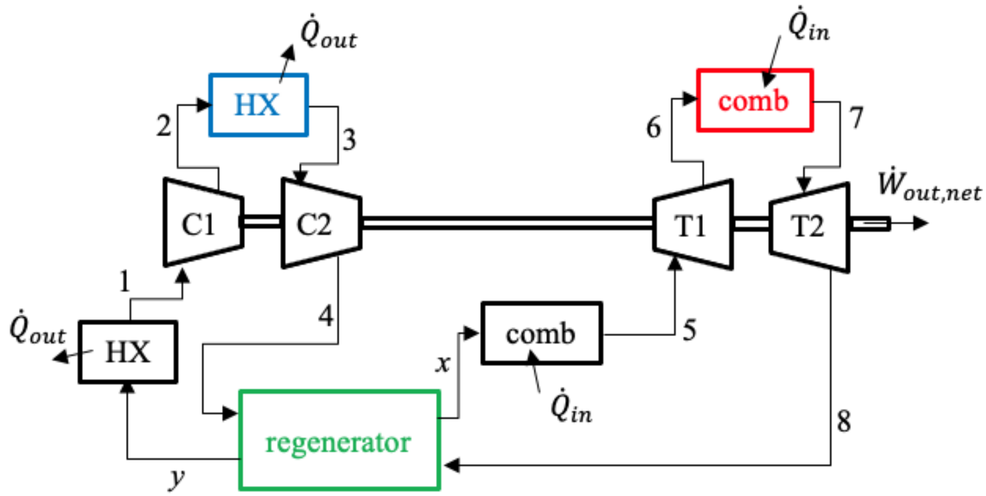


FIGURE 3.55. A gas turbine engine cycle with intercooling between compressor stages, reheating between the turbine stages, and a regenerator.

Reheating between successive turbine stages is used to increase the specific volume of the working fluid (Figure 3.55). Again, from the Ideal Gas Law, if the pressure remains constant in the reheating heat exchanger and the temperature of the working fluid increases, then the specific volume will increase. From Eq. (3.246), w_{out} will increase. Similar to intercooling, reheating between turbine stages can increase the specific work obtained from the turbine.

Another method for improving a Brayton Cycle is to use regenerative heating of the working fluid. Regeneration is when the working fluid is preheated in a heat exchanger using the hot combustion gas in order to reduce the amount of heat (and fuel) needed in the combustor. From the First Law applied to the combustor (Figure 3.55),

$$\dot{Q}_{\text{in}} = \dot{m}(h_5 - h_x), \tag{3.248}$$

where $h_x > h_4$ due to heat transfer with the hot combustion gases. Thus, $\dot{Q}_{\text{in}}$ in the combustor decreases with the use of the regenerator and, as a result, the cycle thermal efficiency increases since $\dot{Q}_{\text{in}}$ decreases.

To determine the effectiveness of using the exhaust working fluid to preheat the working fluid leading into the combustor, we can define a regenerator effectiveness,

$$\eta_{\text{ref}} := \frac{h_x - h_4}{h_8 - h_4}. \tag{3.249}$$

The effectiveness is defined in this manner since the largest temperature that State x can reach is the temperature at State 8 (Figure 3.56). Thus, we compare the actual specific heat transfer into the working fluid between States 4 and x to the ideal specific heat transfer, assuming State x has the same temperature as State 8 (and, thus, the same specific enthalpy since specific enthalpy is a function only of temperature for an ideal gas).

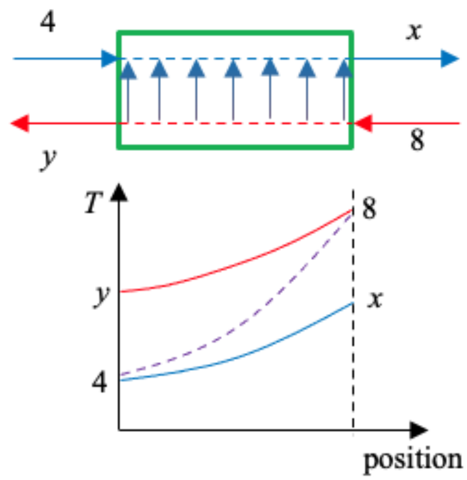
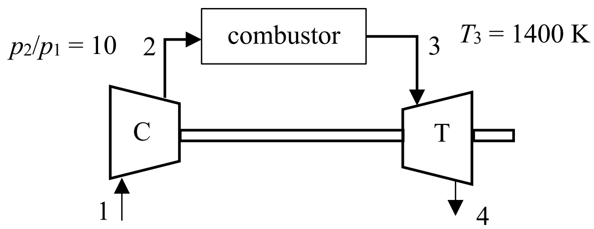


Figure 3.56. Top: A schematic of a counterflow regenerator. Bottom: A plot of the working fluid temperatures as a function of position in the regenerator. State 4 to state x is the flow leading into the combustor. State 8 to state y is the flow leading into the heat removal heat exchanger. The cooler working fluid traveling from State 4 to State x heats up due to heat transfer from the warmer fluid traveling from State 8 to State y. The dashed line in the plot shows the ideal temperature profile for the fluid traveling from State 4 to State x.

Air enters the compressor of an ideal cold air-standard Brayton cycle at 100 kPa (abs) and 300 K, with a mass flow rate of 6 kg/s. The compressor pressure ratio is 10 and the turbine inlet temperature is 1400 K. For a specific heat ratio of 1.4, calculate:
1. the thermal efficiency of the cycle,
2. the back work ratio, and
3. the net power developed.

SOLUTION:



Calculate the thermal efficiency for the Brayton cycle,

$$\eta = 1 - \left(\frac{p_2}{p_1}\right)^{\frac{1-k}{k}}, \tag{1}$$

where,

$k = 1.4$,

$p_2/p_1 = 10$,

⇨ $\boxed{\eta = 0.482}$.

The back work ratio (bwr) is,

$$bwr \equiv \frac{\dot{W}_{into\ comp}}{\dot{W}_{by\ turb}}, \tag{2}$$

where,

$$\dot{W}_{into\ comp} = \dot{m}(h_2 - h_1) = \dot{m}c_p(T_2 - T_1), \tag{3}$$

$$\dot{W}_{by\ turb} = \dot{m}(h_3 - h_4) = \dot{m}c_p(T_3 - T_4), \tag{4}$$

so that Eq. (2) becomes,

$$bwr = \frac{T_2 - T_1}{T_3 - T_4}. \tag{5}$$

Note that Eqs. (3) and (4) were derived by applying the 1st Law to CVs that surround the compressor and turbine, respectively, and assuming steady flow, one inlet and one outlet, adiabatic conditions, and neglecting changes in kinetic and potential energies. In addition, the air is assumed to be a perfect gas (constant specific heats).

The temperature ratios T_2/T_1 and T_4/T_3 may be found by noting that the flow through the compressor and turbine are assumed to be adiabatic an reversible => isentropic. Since the air is also assumed to be a perfect gas, the temperature and pressure ratios are related by,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}. \tag{6}$$

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{k-1}{k}}. \tag{7}$$

For the given values,

$p_2/p_1 = 10$, $k = 1.4$ => $T_2/T_1 = 1.9307$,

$p_4/p_3 = 1/10$, $k = 1.4$ => $T_4/T_3 = 0.51795$,

(Since the pressure remains constant in the combustor, $p_3 = p_2$. In addition, the pressure at 4 will be the same as the pressure at 1, i.e., $p_4 = p_1$, since both are either open to the atmosphere or are connected via another heat exchanger.)

Given that $T_1 = 300$ K and $T_3 = 1400$ K,

⇨ $T_2 = 579.2$ K,

⇨ $T_4 = 725.1$ K.

Substituting these temperature values into Eq. (5) gives,

$\boxed{bwr = 0.414}$.

The net power developed is,

$$\dot{W}_{by,net} = \dot{W}_{by\ turb} - \dot{W}_{into\ comp} = \dot{W}_{by\ turb}\left(1 - \frac{\dot{W}_{into\ comp}}{\dot{W}_{by\ turb}}\right) = \dot{W}_{by\ turb}(1 - bwr), \tag{8}$$

$$\dot{W}_{by,net} = \dot{m}c_p(T_3 - T_4)(1 - bwr), \tag{9}$$

where Eq. (4) has been used to derive Eq. (9). Using the given values,

$c_p = 1.005$ kJ/(kg.K), (value for air at 300 K, since it's a cold air-standard analysis)

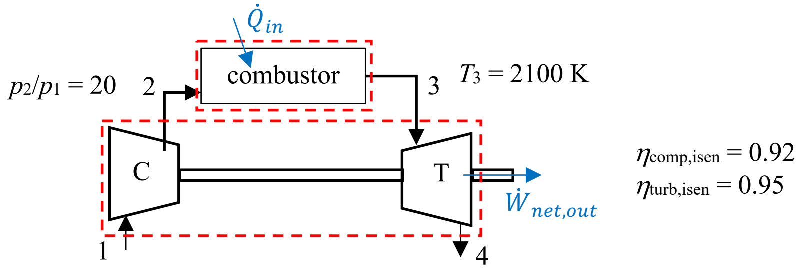
$\dot{m} = 6$ kg/s,

⇨ $\boxed{\dot{W}_{by,net} = 2390\text{ kW}}$.

Air enters the compressor of an air-standard Brayton cycle with a volumetric flow rate of 60 m^3/s at 0.8 bar (abs) and 280 K. The compressor pressure ratio is 20 and the maximum cycle temperature is 2100 K. The compressor and turbine isentropic efficiencies are 92% and 95%, respectively. Determine:

a. the net power developed from the cycle,
b. the rate of heat addition in the combustor, and
c. the thermal efficiency of the cycle.

SOLUTION:



Apply the 1st Law to a control volume surrounding the compressor and turbine,

$$0 = \dot{m}(h_1 - h_2 + h_3 - h_4) - \dot{W}_{out,net}, \tag{1}$$

(assuming steady state, negligible KE and PE, and adiabatic conditions)

$$\dot{W}_{out,net} = \dot{m}(h_1 - h_2 + h_3 - h_4). \tag{2}$$

The mass flow rate can be found from the conditions at State 1 and using the ideal gas law,

$$\dot{m} = \rho_1 \dot{V}_1 = \left(\frac{p_1}{RT_1}\right)\dot{V}_1, \tag{3}$$

⇨ $\dot{m} = 59.731$ kg/s

Now determine the specific enthalpies at each of the states.

State 1:

$T_1 = 280$ K => $h_1 = 280.1$ kJ/kg, $p_r(T_1) = 1.0889$ (from the Ideal Gas Table)

State 3:

$T_3 = 2100$ K => $h_3 = 2377$ kJ/kg, $p_r(T_3) = 2559$ (from the IGT)

State 2:

$$\eta_{comp,isen} = \frac{h_{2s} - h_1}{h_2 - h_1} \;=>\; h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_{comp,isen}}. \tag{4}$$

State 2s: $p_2/p_1 = p_{2s}/p_1 = 20$,

$$\frac{p_{2s}}{p_1} = \frac{p_r(T_{2s})}{p_r(T_1)} \;=>\; p_r(T_{2s}) = p_r(T_1)\left(\frac{p_{2s}}{p_1}\right) \text{ (isentropic compression of an ideal gas),} \tag{5}$$

=> $p_r(T_{2s}) = 21.778$ => $T_{2s} = 649.33$ K, $h_{2s} = 659.29$ kJ/kg (from IGT)

Using Eq. (4) and the given and computed values,

$h_2 = 692.26$ kJ/kg.

State 4:

$$\eta_{turb,isen} = \frac{h_4 - h_3}{h_{4s} - h_3} \Rightarrow h_4 = h_3 + \eta_{turb,isen}(h_{4s} - h_3). \quad (6)$$

State 4*s*: $p_3/p_4 = p_3/p_{4s} = p_2/p_1 = 20$,

$$\frac{p_3}{p_{4s}} = \frac{p_r(T_3)}{p_r(T_{4s})} \Rightarrow p_r(T_{4s}) = p_r(T_3)\left(\frac{p_{4s}}{p_3}\right) \text{ (isentropic expansion of an ideal gas)}, \quad (7)$$

=> $p_r(T_{4s}) = 127.95$ => $T_{4s} = 1029.19$ K, $h_{4s} = 1079.57$ kJ/kg (from IGT)

Using Eq. (6) and the given and computed values,

$h_4 = 1144.44$ kJ/kg.

Using the state data, mass flow rate, and Eq. (2),

$\boxed{\dot{W}_{out,net} = 49.0 \text{ MJ}}$.

The rate of heat addition into the combustor is found by applying the 1st Law to a control volume surrounding the combustor,

$$0 = \dot{m}(h_2 - h_3) + \dot{Q}_{in}, \quad (8)$$

(assuming steady state, negligible KE and PE, and a passive device)

$$\dot{Q}_{in} = \dot{m}(h_3 - h_2). \quad (9)$$

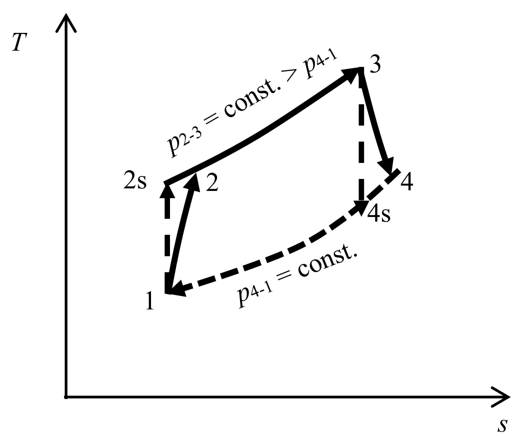
Using the previously calculated quantities,

$\boxed{\dot{Q}_{in} = 101 \text{ MW}}$.

The cycle's thermal efficiency is,

$$\eta = \frac{\dot{W}_{out,net}}{\dot{Q}_{in}},$$

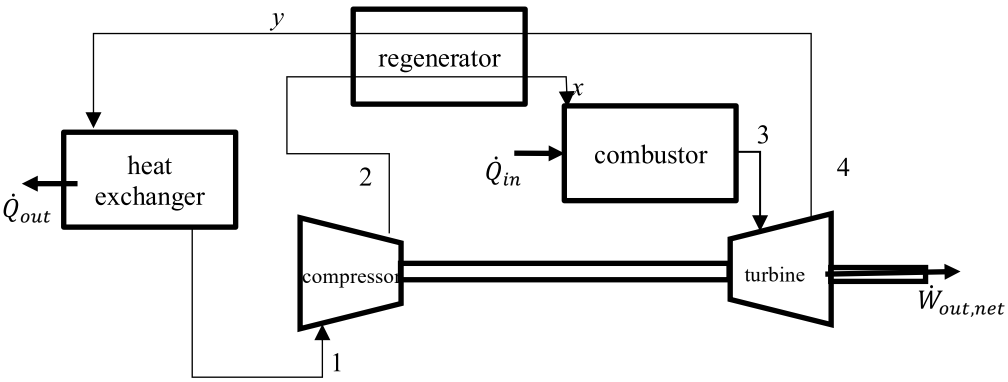
=> $\boxed{\eta = 0.487 = 48.7\%}$.



Air enters the compressor of a regenerative air-standard Brayton cycle with a volumetric flow rate of 60 m^3/s at 0.8 bar (abs) and 280 K. The compressor pressure ratio is 20 and the maximum cycle temperature is 2100 K. The compressor and turbine have isentropic efficiencies of 92% and 95%, respectively. For a regenerator effectiveness of 85%, determine:

a. the net power developed,
b. the rate of heat addition in the combustor,
c. the thermal efficiency of the cycle.

SOLUTION:



To determine the net power developed, apply the 1st Law to a CV surrounding the compressor and turbine,

$\dot{W}_{out,net} = \dot{m}(h_1 + h_3 - h_2 - h_4)$ (assuming SSSF, adiabatic, and negligible KE and PE). (1)

The rate of heat transfer in the combustor is found by applying the 1st Law to a CV surrounding the combustor,

$\dot{Q}_{in} = \dot{m}(h_3 - h_x)$ (assuming SSSF, passive device, and negligible KE and PE). (2)

Now find the properties at the various states.

State 1:

$\dot{V} = 60$ m^3/s, $p_1 = 0.8$ bar (abs) = 80 kPa (abs), $T_1 = 280$ K

=> $h_1 = 280.1$ kJ/kg and $p_r(T_1) = 1.0889$ (from the Ideal Gas Table for air)

Also, from the ideal gas law,

=> $\rho_1 = \frac{p_1}{RT_1} = 0.9955$ kg/m^3 => $\dot{m} = \rho\dot{V} = 59.731$ kg/s (3)

State 3:

$T_3 = 2100$ K,

=> $h_3 = 2377$ kJ/kg and $p_r(T_3) = 2559$ (from the Ideal Gas Table for air)

State 2:

$p_2/p_1 = 20 = p_{2s}/p_1$ and $\eta_{\text{comp,isen}} = 0.92$ (given),

$\eta_{comp,isen} = \frac{w_{in,isen}}{w_{in}} = \frac{h_{2s}-h_1}{h_2-h_1}$ => $h_2 = h_1 + \frac{h_{2s}-h_1}{\eta_{comp,isen}}$. (4)

For an ideal gas undergoing an isentropic process,

$\frac{p_{2s}}{p_1} = \frac{p_r(T_{2s})}{p_r(T_1)}$ => $p_r(T_{2s}) = p_r(T_1)\left(\frac{p_{2s}}{p_1}\right)$, (5)

=> $p_r(T_{2s}) = 21.778$ => $T_{2s} = 649.33$ K, $h_{2s} = 659.29$ kJ/kg (IGT),

=> $h_2 = 692.26$ kJ/kg.

State 4:

$p_3/p_4 = 20 = p_3/p_{4s}$ ($= p_2/p_1$) and $\eta_{\text{turb,isen}} = 0.95$ (given),

$$\eta_{turb,isen} = \frac{w_{out}}{w_{out,isen}} = \frac{h_3 - h_4}{h_3 - h_{4s}} \Rightarrow h_4 = h_3 - \eta_{turb,isen}(h_3 - h_{4s}). \quad (6)$$

For an ideal gas undergoing an isentropic process,

$$\frac{p_{4s}}{p_3} = \frac{p_r(T_{4s})}{p_r(T_3)} \Rightarrow p_r(T_{4s}) = p_r(T_3)\left(\frac{p_{4s}}{p_3}\right), \quad (7)$$

$\Rightarrow p_r(T_{4s}) = 127.95 \Rightarrow T_{4s} = 1029.19$ K, $h_{4s} = 1079.57$ kJ/kg (IGT),

$\Rightarrow h_4 = 1144.44$ kJ/kg.

State x:

From the definition of the regenerator effectiveness,

$$\eta_{reg} = \frac{h_x - h_2}{h_4 - h_2} \Rightarrow h_x = h_2 + \eta_{reg}(h_4 - h_2), \quad (8)$$

Using the previously calculated specific enthalpy values and the given $\eta_{reg} = 0.85$,

$\Rightarrow h_x = 1076.61$ kJ/kg.

Using these state data, Eq. (1) gives,

$\boxed{\dot{W}_{out,net} = 49.0 \text{ MW}}$,

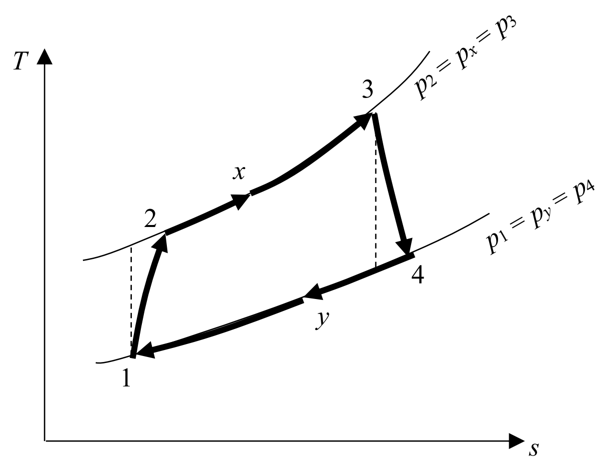
and Eq. (2) gives,

$\boxed{\dot{Q}_{in} = 77.7 \text{ MW}}$.

The thermal efficiency for the cycle is,

$$\boxed{\eta_{cycle} = \frac{\dot{W}_{out,net}}{\dot{Q}_{in}} = 0.631 = 63.1\%.} \quad (9)$$

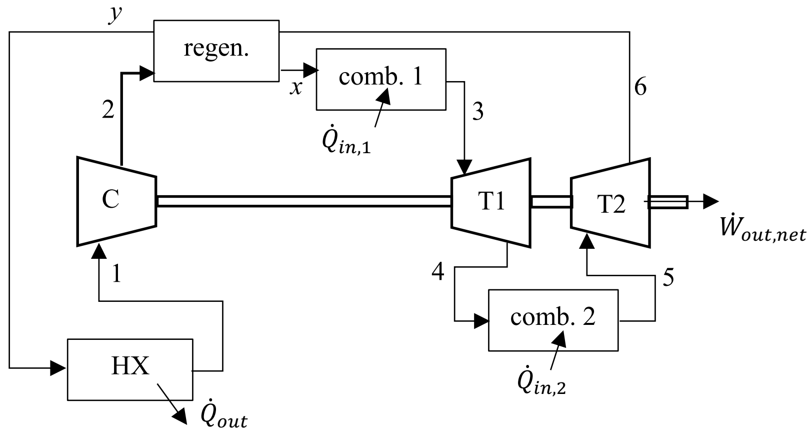
Note that as η_{reg} increases, then h_x approaches h_4 and $\dot{Q}_{in}$ decreases. As a result, the thermal efficiency for the cycle would increase. In the limit of $\eta_{reg} = 100\%$, $\dot{Q}_{in,min} = 73.6$ MW and $\eta_{cycle,max} = 66.6\%$. In contrast, without the regenerator ($\eta_{reg} = 0$), then $\dot{Q}_{in,max} = 100.6$ MW and $\eta_{cycle,min} = 48.7\%$. Thus, we observe that including a regenerator can substantially improve the cycle's thermal efficiency.



Air enters the compressor of a cold air-standard Brayton cycle with regeneration and reheat at 100 kPa (abs), 300 K, with a mass flow rate of 6 kg/s. The compressor pressure ratio is 10 and the inlet temperature for each turbine stage is 1400 K. The pressure ratios across each turbine stage are equal. The turbine stages and compressor each have isentropic efficiencies of 80% and the regenerator effectiveness is 80%. For a specific heat ratio of 1.4, calculate:

a. the thermal efficiency of the cycle,
b. the back work ratio, and
c. the net power developed by the cycle.

SOLUTION:



Since we're assuming a cold air-standard analysis, state the properties of the air in the analysis:

$R = 0.287$ kJ/(kg.K) and $c_{p@300\text{ K}} = 1.005$ kJ/(kg.K).

The net power from the cycle is found by applying the 1st Law to a CV surrounding the compressor and turbines (assuming SSSF, adiabatic operation, negligible KE and PE),

$$\dot{W}_{out,net} = \dot{m}(h_1 - h_2 + h_3 - h_4 + h_5 - h_6), \tag{1}$$

and, since we're performing a cold air-standard analysis, meaning the air is a perfect gas,

$$\dot{W}_{out,net} = \dot{m}c_p(T_1 - T_2 + T_3 - T_4 + T_5 - T_6). \tag{2}$$

The power into the compressor is found by applying the 1st Law to a CV surrounding just the compressor (assuming SSSF, adiabatic operation, negligible KE and PE),

$$\dot{W}_{in} = \dot{m}(h_2 - h_1), \tag{3}$$

$$\dot{W}_{in} = \dot{m}c_p(T_2 - T_1) \quad \text{(assuming perfect gas behavior).} \tag{4}$$

The back work ratio (bwr) is,

$$bwr = \frac{\dot{W}_{in}}{\dot{W}_{out}} = \frac{\dot{W}_{in}}{\dot{W}_{out,net} + \dot{W}_{in}}. \tag{5}$$

The rate at which heat is added into the two combustors is found by applying the 1st Law to CVs surrounding each combustor (assuming SSSF, passive devices, negligible KE and PE),

$$\dot{Q}_{in} = \dot{Q}_{in,1} + \dot{Q}_{in,2} = \dot{m}(h_3 - h_x) + \dot{m}(h_5 - h_4), \tag{6}$$

$$\dot{Q}_{in} = \dot{m}c_p(T_3 - T_x + T_5 - T_4) \quad \text{(assuming perfect gas behavior).} \tag{7}$$

The thermal efficiency of the cycle is,

$$\eta = \frac{\dot{W}_{out,net}}{\dot{Q}_{in}}. \tag{8}$$

Now find the temperatures at the various states.

State 1:
$\dot{m} = 6$ kg/s, $p_1 = 100$ kPa (abs), $T_1 = 300$ K (given)

State 2s: (assuming the process is isentropic and involves a perfect gas)

$$\frac{T_{2s}}{T_1} = \left(\frac{p_{2s}}{p_1}\right)^{\frac{k-1}{k}} \Rightarrow T_{2s} = 579.21 \text{ K using } p_{2s}/p_1 = p_2/p_1 = 10. \tag{9}$$

State 2:

$$\eta_{comp,isen} = \frac{w_{in,isen}}{w_{in}} = \frac{h_{2s}-h_1}{h_2-h_1} = \frac{c_p(T_{2s}-T_1)}{c_p(T_2-T_1)} \Rightarrow T_2 = T_1 + \frac{T_{2s}-T_1}{\eta_{comp,isen}}, \tag{10}$$

=> $T_2 = 649.01$ K.

State 3:
$T_3 = 1400$ K (given)

State 4s:

$$\frac{T_{4s}}{T_3} = \left(\frac{p_{4s}}{p_3}\right)^{\frac{k-1}{k}} \Rightarrow T_{4s} = 1007.56 \text{ K using } p_3/p_{4s} = 3.162. \tag{11}$$

Note that since we're told that the pressure drops across both turbine stages are equal,

$$\frac{p_2}{p_1} = \frac{p_3}{p_6} = \left(\frac{p_3}{p_4}\right)\underbrace{\left(\frac{p_4}{p_5}\right)}_{=1}\underbrace{\left(\frac{p_5}{p_6}\right)}_{=p_3/p_4} = \left(\frac{p_3}{p_4}\right)^2 \Rightarrow \frac{p_3}{p_4} = \sqrt{\frac{p_2}{p_1}} \Rightarrow \frac{p_3}{p_4} = \frac{p_5}{p_6} = 3.162. \tag{12}$$

State 4:

$$\eta_{turb,isen} = \frac{w_{out}}{w_{out,isen}} = \frac{h_3-h_4}{h_3-h_{4s}} = \frac{c_p(T_3-T_4)}{c_p(T_3-T_{4s})} \Rightarrow T_4 = T_3 - \eta_{turb,isen}(T_3 - T_{4s}), \tag{13}$$

=> $T_4 = 1086.05$ K.

State 5:
$T_5 = 1400$ K (given)

State 6s:

$$\frac{T_{6s}}{T_5} = \left(\frac{p_{6s}}{p_5}\right)^{\frac{k-1}{k}} \Rightarrow T_{6s} = 1007.56 \text{ K using } p_{6s}/p_5 = 3.162 \text{ (Eq. (12))} \tag{14}$$

State 6:

$$\eta_{turb,isen} = \frac{w_{out}}{w_{out,isen}} = \frac{h_5-h_6}{h_5-h_{6s}} = \frac{c_p(T_5-T_6)}{c_p(T_5-T_{6s})} \Rightarrow T_6 = T_5 - \eta_{turb,isen}(T_5 - T_{6s}), \tag{15}$$

=> $T_6 = 1086.05$ K.

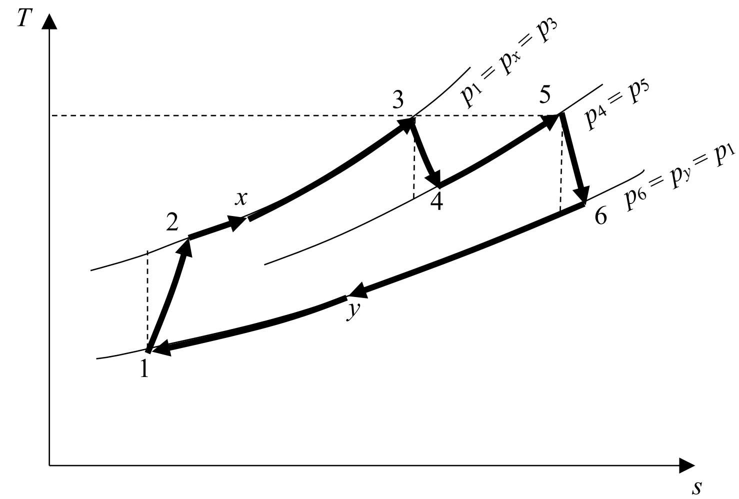
State x:

$$\eta_{reg} = \frac{h_x-h_2}{h_6-h_2} = \frac{c_p(T_x-T_2)}{c_p(T_6-T_2)} \Rightarrow T_x = T_2 + \eta_{reg}(T_6 - T_2). \tag{16}$$

=> $T_x = 998.64$ K using $\eta_{reg} = 0.80$ (given) and the previously calculated values.

Using the calculated temperatures and Eqs. (2), (4), (5), (7), and (8),

$\boxed{\dot{W}_{out,net} = 1680 \text{ kW}}$,
$\boxed{\dot{W}_{in} = 2100 \text{ kW}}$,
$\boxed{\dot{Q}_{in} = 4300 \text{ kW}}$,
$\boxed{bwr = 0.556 = 55.6\%}$,
$\boxed{\eta = 0.390 = 39.0\%}$.


T
s
1
2
x
3
4
5
6
y
$p_1 = p_x = p_3$
$p_4 = p_5$
$p_6 = p_y = p_1$