3.8.4. The Otto, Diesel, and Dual Cycles

The Otto, Diesel, and dual cycles are idealizations of the cycles observed in internal combustion, piston-cylinder engines. Before analyzing these cycles, it is useful to first describe the components and processes involved in a four-stroke internal combustion (IC) engine, which is the most common type of engine.

Figure 3.46 illustrates the components in a typical IC engine piston-cylinder arrangement. The piston moves vertically within the cylinder as the crankshaft turns. When the piston is at its lowest point it's considered to be at bottom dead center, bdc and when the piston is at its highest point it's at the top dead center, tdc. The volumes within the cylinder at these two points are, respectively, V_{bdc} and V_{tdc}. The piston stroke is the vertical distance between the bdc and tdc. At the top of the cylinder are valves, which open and close during the piston movement. Opening an intake valve allows for fresh air (and possibly fuel) to enter the cylinder while opening an outlet valve allows the piston to push out exhaust gases after combustion has occurred.

The figure also shows a spark plug at the top center of the cylinder. Spark plugs are used to ignite the air/fuel mixture in spark ignition IC engines. In a compression ignition IC engine, the spark plug is removed and instead fuel is injected at this location. The high temperature generated during compression of the air/fuel mixture in a compression ignition engine is enough to initiate combustion.

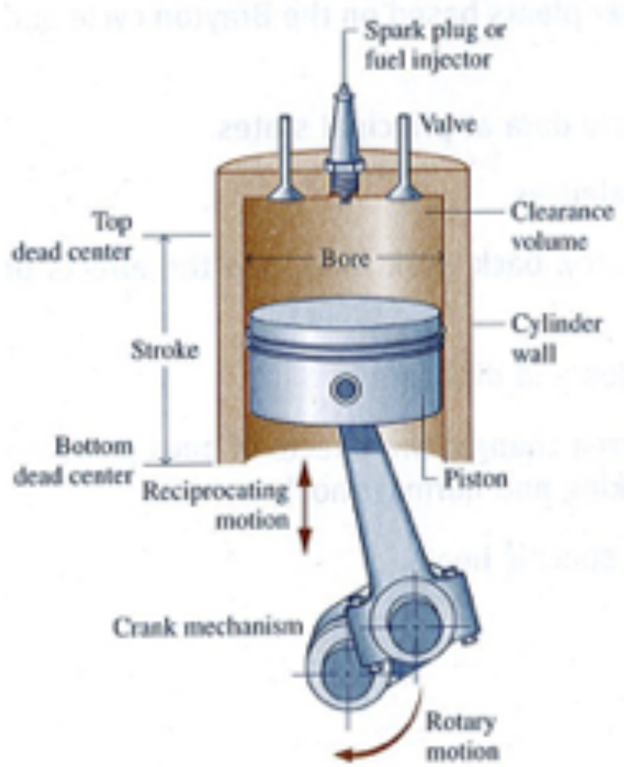


Figure 3.46. An illustration of the components in a typical IC engine piston-cylinder arrangement. This figure is from Moran, M.J., Shapiro, H.N., Boettner, D.D., and Bailey, M.B., *Fundamentals of Engineering Thermodynamics*, Wiley, 7th ed.

The processes involved in a four-stroke IC engine are shown in Figure 3.47. As shown in the figure, in the intake stroke the air/fuel mixture is drawn into the cylinder as the piston moves downward and an intake valve is opened (the exhaust valve is closed). Next, the air/fuel mixture is compressed within the cylinder during the compression stroke. During this process both valves are closed. Near the end of the compression stroke the spark plug fires in a spark-ignition engine causing rapid combustion of the fuel. If the engine instead operates via compression ignition, then combustion initiates a bit later in the compression stroke. The valves are closed during this process. Once ignition begins, the piston is forced downward by the rapid expansion of the combustion gases. This is the power stroke. After reaching the bottom dead center, the exhaust stroke begins in which the piston moves back upwards while the exhaust valve is opened (the intake valve is closed), which forces the combustion products out of the cylinder. The cycle then repeats after the piston reaches the top dead center. Note that the crankshaft completes two revolutions during this four stroke process.

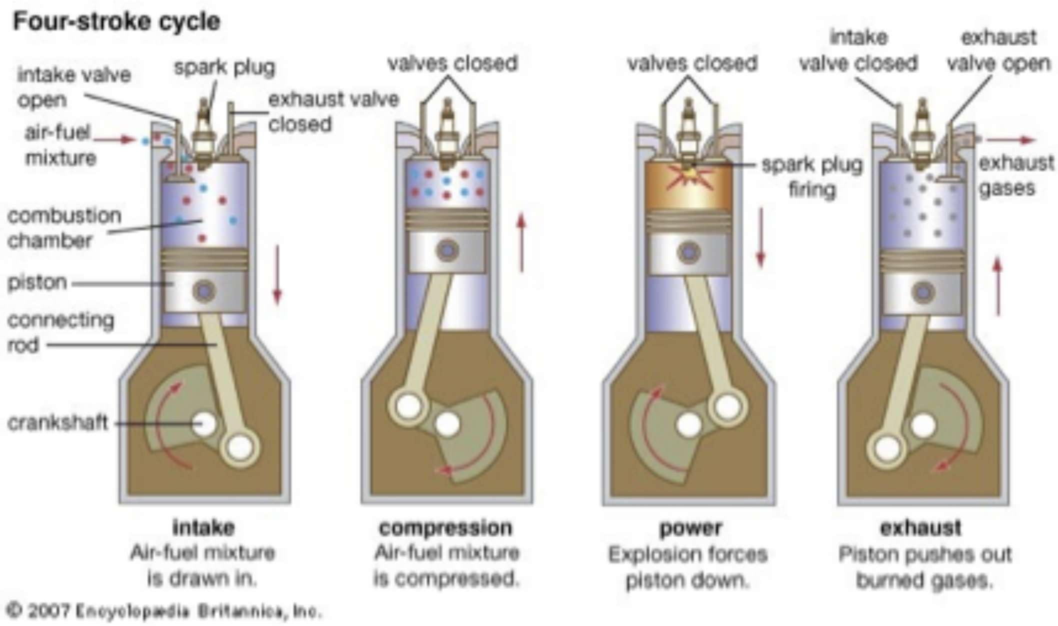


Figure 3.47. Illustrations of the different processes in a four stroke, internal combustion engine. Note that the crankshaft turns twice during the four-stroke process. This figure is from Encyclopedia Britannica, https://www.britannica.com/technology/four-stroke-cycle.

A representative p-v plot for a four-stroke cycle is shown in Figure 3.48. The intake stroke starts at top dead center, with the inlet valve opened and exhaust valve closed, and moves at nearly constant pressure to bottom dead center, at which point the intake valve closes. The compression stroke compresses the air/fuel mixture until reaching the top dead center, moving from the bottom right to the top left in the plot (decreasing volume and increasing pressure). Combustion initiates near the end of the compression stroke. Note that the crankshaft has now completed one rotation. Next, the power stroke begins as the combustion gases push the piston downward, with the piston moving (and cylinder volume increasing) from top dead center to bottom dead center (increasing volume and decreasing pressure). Near the end of the power stroke the exhaust valve opens. Lastly, the exhaust stroke occurs and the piston moves back upwards at nearly constant pressure to push the combustion gases out through the exhaust valve. When the piston reaches top dead center another crankshaft rotation has occurred. The cycle then repeats.

We'll use an air standard analysis to study the Otto, Diesel, and dual cycles. An air standard analysis is a highly simplified model to provide qualitative understanding of cycles that use air as the working fluid. The numerical values in an air standard analysis generally won't be accurate when compared to experimental values; however, the air standard model can still provide valuable insight into the influence of various parameters on performance measures.

The assumptions made in an internal combustion engine air standard analysis include:

(1) The air mass in the system remains constant.

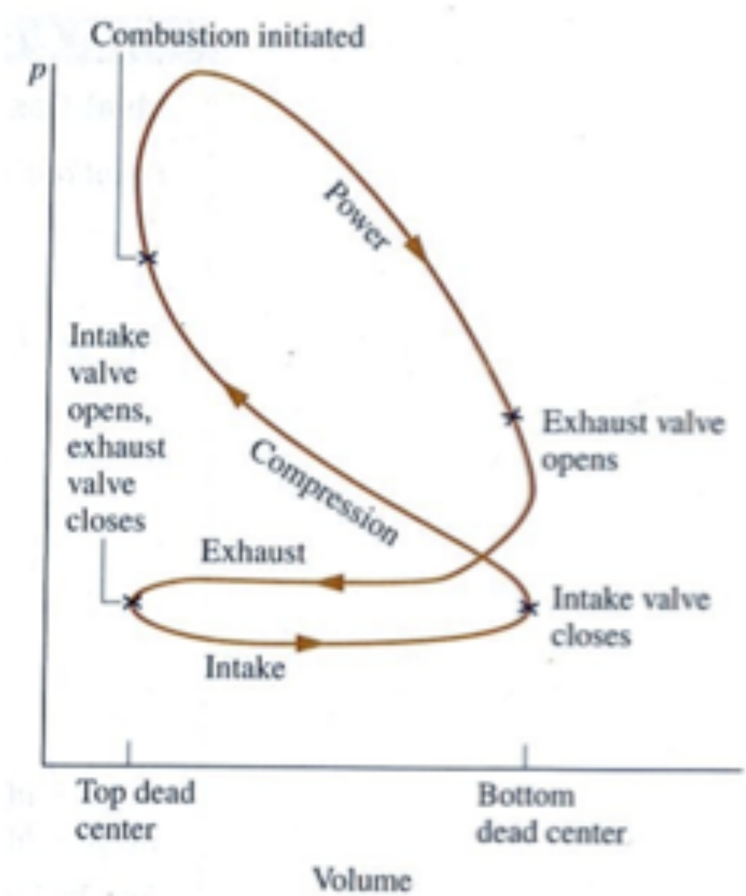


Figure 3.48. A representative p-v plot for a four-stroke, IC engine. This figure is from Moran, M.J., Shapiro, H.N., Boettner, D.D., and Bailey, M.B., *Fundamentals of Engineering Thermodynamics*, Wiley, 7th ed.

(2) There are no intake or exhaust processes.
(3) Air is modeled as an ideal gas.
(4) Combustion is modeled as a heat addition process and the working fluid remains as air.
(5) The exhaust process is modeled as a constant volume heat removal process.
(6) All processes are internally reversible.

In a cold air standard analysis, we further assume constant specific heats, i.e., a perfect gas assumption.

Two definitions used in the analysis of IC cycles include:

- The mean effective pressure, mep, defined as
$$\text{mep} := \frac{W_{\text{out,net}}}{V_{\text{bdc}} - V_{\text{tdc}}} \tag{3.198}$$
The mep can be used to compare the work output between different cycles having the same working volume.
- The compression ratio, r,
$$r := \frac{V_{\text{bdc}}}{V_{\text{tdc}}} = \frac{v_{\text{bdc}}}{v_{\text{tdc}}} > 1. \tag{3.199}$$
As is shown in the following analyses, increasing the compression ratio increases the thermal efficiency of the cycle.

3.8.4.1. Otto Cycle

The Otto cycle is an idealization of the cycle shown in Figure 3.48 for a spark ignition IC engine. The processes in an air-standard Otto cycle include (refer to Figure 3.49):

- *Process 1 - 2*: isentropic compression of the working fluid as the piston moves from bottom dead center to top dead center (compression stroke),
- *Process 2 - 3*: *constant volume* heat addition to the working fluid while the piston is at top dead center (combustion)
- *Process 3 - 4*: isentropic expansion of the working fluid as the piston moves from top dead center to bottom dead center (power stroke)
- *Process 4 - 1* constant volume heat removal from the working fluid while the piston is at bottom dead center (exhaust and intake strokes)

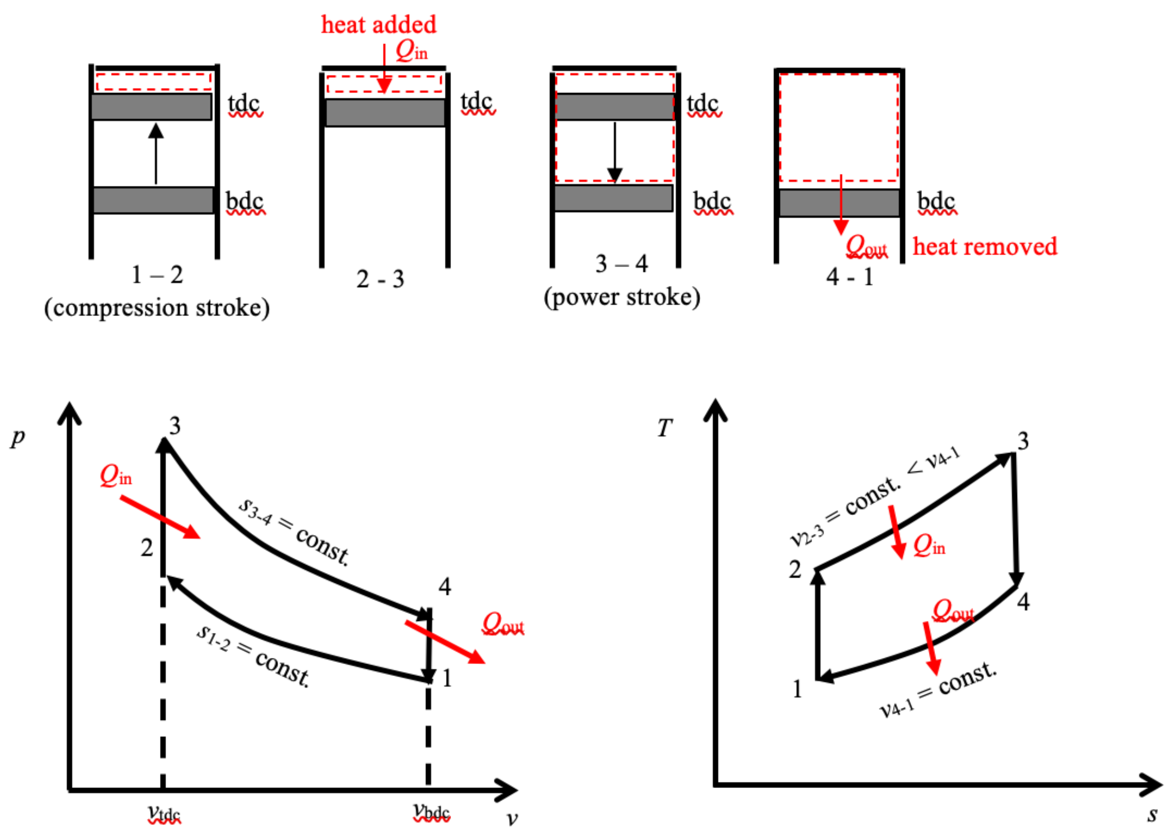


FIGURE 3.49. Illustrations of the processes involved in an air standard Otto cycle including p-v and T-s plots. Compare this p-v plot to the one shown in Figure 3.48.

The cycle can be analyzed using the First Law applied to the air in the cylinder, neglecting changes in kinetic and potential energy. For Process 1 - 2,

$$\Delta U_{12} = m(u_2 - u_1) = Q_{\text{in},12} + W_{\text{on},12}, \tag{3.200}$$

$$\implies W_{\text{on},12} = m(u_2 - u_1). \tag{3.201}$$

where $Q_{\text{in},12} = 0$ since the process is isentropic and internally reversible.

For Process 2 - 3,

$$\Delta U_{23} = m(u_3 - u_2) = Q_{\text{in},23} - W_{\text{by},23}, \tag{3.202}$$

$$\implies Q_{\text{in},23} = m(u_3 - u_2). \tag{3.203}$$

where $W_{\text{by},23} = 0$ since the process is at constant volume.

For Process 3 - 4,

$$\Delta U_{34} = m(u_4 - u_3) = Q_{\text{in},34} - W_{\text{by},34}, \tag{3.204}$$

$$\implies W_{\text{by},34} = m(u_3 - u_4). \tag{3.205}$$

where $Q_{\text{in},34} = 0$ since the process is isentropic and internally reversible.

For Process 4 - 1,

$$\Delta U_{41} = m(u_1 - u_4) = -Q_{\text{out},41} - W_{\text{by},41}, \tag{3.206}$$

$$\implies Q_{\text{out},41} = m(u_4 - u_1). \tag{3.207}$$

where $W_{\text{by},41} = 0$ since the process is at constant volume.

The thermal efficiency of the cycle is,

$$\eta_{\text{Otto}} = \frac{W_{\text{by,net}}}{Q_{\text{in}}} = \frac{W_{\text{by},34} - W_{\text{on},12}}{Q_{\text{in},23}} = \frac{(u_3 - u_4) - (u_2 - u_1)}{(u_3 - u_2)} = \frac{(u_3 - u_2) - (u_4 - u_1)}{(u_3 - u_2)}, \tag{3.208}$$

or, re-writing further,

$$\eta_{\text{Otto}} = 1 - \frac{u_4 - u_1}{u_3 - u_2} = 1 - \frac{Q_{\text{out},41}}{Q_{\text{in},23}}. \tag{3.209}$$

The compression ratio for the cycle (Eq. (3.199)) is,

$$r = \frac{v_{\text{bdc}}}{v_{\text{tdc}}} = \frac{v_1}{v_2} = \frac{v_4}{v_3}. \tag{3.210}$$

Recall that the compression (Process 1-2) and power (Process 3-4) strokes are modeled in the air standard analysis as isentropic processes involving an ideal gas. Thus,

$$r = \frac{v_1}{v_2} = \frac{v_r(T_1)}{v_r(T_2)}, \tag{3.211}$$

and,

$$r = \frac{v_4}{v_3} = \frac{v_r(T_4)}{v_r(T_3)}. \tag{3.212}$$

Thus,

$$\frac{v_r(T_1)}{v_r(T_2)} = \frac{v_r(T_4)}{v_r(T_3)}. \tag{3.213}$$

For a cold air standard analysis, which assumes perfect gas behavior, the previous two relations may be written as,

$$\frac{1}{r} = \frac{v_2}{v_1} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{1-k}} \implies \frac{T_2}{T_1} = r^{k-1}. \tag{3.214}$$

and,

$$r = \frac{v_4}{v_3} = \left(\frac{T_4}{T_3}\right)^{\frac{1}{1-k}} \implies \frac{T_4}{T_3} = r^{1-k} \tag{3.215}$$

where k is the specific heat ratio. Combining,

$$r^{k-1} = \frac{T_2}{T_1} = \frac{T_3}{T_4} \implies \frac{T_4}{T_1} = \frac{T_3}{T_2}. \tag{3.216}$$

Again, assuming perfect gas behavior, Eq. (3.209) may be written as,

$$\eta_{\text{Otto}} = 1 - \frac{c_v(T_4 - T_1)}{c_v(T_3 - T_2)} = 1 - \frac{T_1}{T_2}\left(\frac{\frac{T_4}{T_1} - 1}{\frac{T_3}{T_2} - 1}\right). \tag{3.217}$$

Using Eq. (3.216),

$$\eta_{\text{Otto}} = 1 - \frac{T_1}{T_2} = 1 - \frac{1}{r^{k-1}}. \tag{3.218}$$

Notes:

(1) Typical values for the compression ratio for a spark ignition IC engine are $r = 8$ to 10 with engine thermal efficiencies of $\eta = 30$ to 35% (these are real efficiencies, not reversible efficiencies).
(2) As the compression ratio r increases, the thermal efficiency η increases. In practice, the compression ratio is limited by auto-ignition, which causes engine knock. Auto-ignition occurs when the temperature reaches a sufficiently high value during compression that the air/fuel mixture ignites prior to spark ignition. Higher octane fuels can go to higher compression ratios before knocking occurs.
(3) As the specific heat ratio k increases, the thermal efficiency η increases. The specific heat ratio is determined by the type of fuel used. In addition, the specific heat ratio increases as the temperature decreases.

3.8.4.2. Diesel Cycle

The Diesel cycle is an idealization of the cycle shown in Figure 3.48 for a compression ignition IC engine. The processes in an air-standard Diesel cycle include (refer to Figure 3.50):

- *Process 1 - 2*: isentropic compression of the working fluid as the piston moves from bottom dead center to top dead center (compression stroke),
- *Process 2 - 3*: *constant pressure* heat addition to the working fluid starting from the top dead center (combustion and start of power stroke)
- *Process 3 - 4*: isentropic expansion of the working fluid as the piston continues to move to bottom dead center (power stroke)
- *Process 4 - 1* constant volume heat removal from the working fluid while the piston is at bottom dead center (exhaust and intake strokes)

The cycle can be analyzed using the First Law applied to the air in the cylinder, neglecting changes in kinetic and potential energy. For Process 1 - 2,

$$\Delta U_{12} = m(u_2 - u_1) = Q_{\text{in},12} + W_{\text{on},12}, \tag{3.219}$$

$$\implies W_{\text{on},12} = m(u_2 - u_1). \tag{3.220}$$

where $Q_{\text{in},12} = 0$ since the process is isentropic and internally reversible.

For Process 2 - 3,

$$\Delta U_{23} = m(u_3 - u_2) = Q_{\text{in},23} - W_{\text{by},23}, \tag{3.221}$$

$$\implies Q_{\text{in},23} = m(u_3 - u_2) + p_{23}(v_3 - v_2) = m(h_3 - h_2). \tag{3.222}$$

Note that this process is different than the corresponding process for an Otto cycle. For the Diesel cycle, Process 2 - 3 is at constant pressure while in the Otto cycle the process is at constant volume. Thus, there is some work done during this process for a Diesel cycle.

For Process 3 - 4,

$$\Delta U_{34} = m(u_4 - u_3) = Q_{\text{in},34} - W_{\text{by},34}, \tag{3.223}$$

$$\implies W_{\text{by},34} = m(u_3 - u_4). \tag{3.224}$$

where $Q_{\text{in},34} = 0$ since the process is isentropic and internally reversible.

For Process 4 - 1,

$$\Delta U_{41} = m(u_1 - u_4) = -Q_{\text{out},41} - W_{\text{by},41}, \tag{3.225}$$

$$\implies Q_{\text{out},41} = m(u_4 - u_1). \tag{3.226}$$

where $W_{\text{by},41} = 0$ since the process is at constant volume.

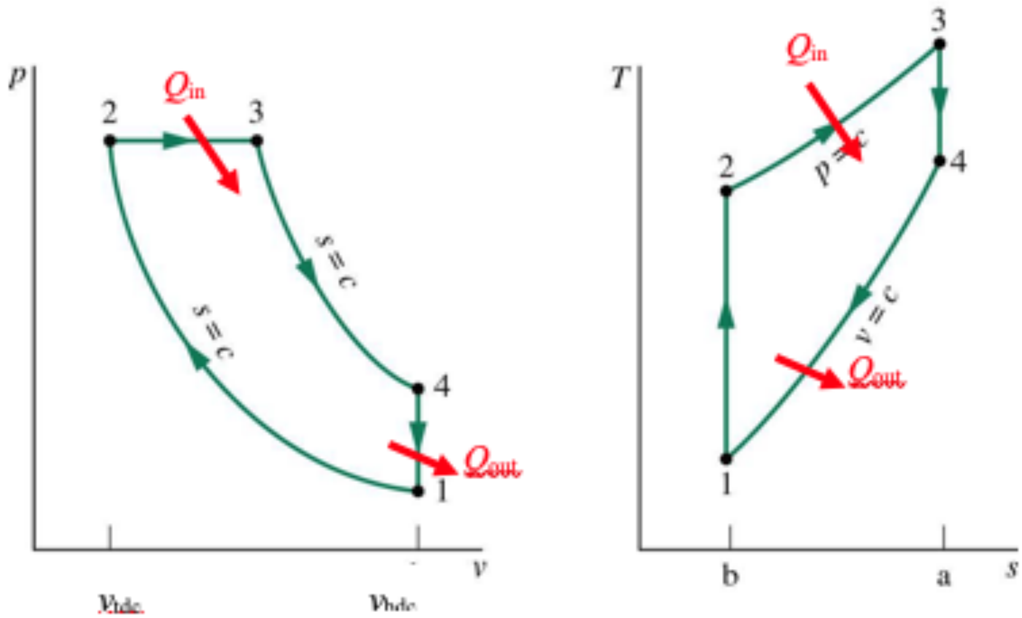


Figure 3.50. Sketches of the p - v and T - s plots for a Diesel cycle. Compare this p - v plot to the one shown in Figure 3.48. These plots are originally from from Moran, M.J., Shapiro, H.N., Boettner, D.D., and Bailey, M.B., *Fundamentals of Engineering Thermodynamics*, Wiley, 7th ed.

The thermal efficiency of the cycle is,

$$\eta_{\text{Diesel}} = 1 - \frac{Q_{\text{out},41}}{Q_{\text{in},23}} = 1 - \frac{u_4 - u_1}{h_3 - h_2}. \tag{3.227}$$

The compression ratio for the cycle (Eq. (3.199)) is,

$$r = \frac{v_{\text{bdc}}}{v_{\text{tdc}}} = \frac{v_1}{v_2}, \tag{3.228}$$

and, since Processes 1 - 2 and 3 - 4 are isentropic processes involving an ideal gas,

$$r = \frac{v_1}{v_2} = \frac{v_r(T_1)}{v_r(T_2)} \quad \text{and} \quad \frac{v_3}{v_4} = \frac{v_r(T_3)}{v_r(T_4)}. \tag{3.229}$$

Note also that $v_4 = v_1 = v_{\text{tdc}}$.

For a cold air standard analysis,

$$r = \frac{v_1}{v_2} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{k-1}} \implies \frac{T_2}{T_1} = r^{k-1}, \tag{3.230}$$

$$\frac{v_3}{v_4} = \left(\frac{T_4}{T_3}\right)^{\frac{1}{k-1}} \implies \frac{T_4}{T_3} = \left(\frac{v_3}{v_4}\right)^{k-1}. \tag{3.231}$$

For a Diesel cycle, we also define a cut-off ratio, r_c, which is the ratio of specific volumes during Process 2 - 3, i.e.,

$$r_c := \frac{v_3}{v_2} = \frac{T_3}{T_2}, \tag{3.232}$$

where the temperature ratio follows from the Ideal Gas Law since the pressure remains constant during Process 2 - 3. Combining Eqs. (3.231) and (3.232),

$$\frac{T_4}{T_3} = \left(\frac{v_3}{v_4}\right)^{k-1} = \left(\frac{v_3}{v_1}\right)^{k-1} = \left(\frac{v_3}{v_2}\frac{v_2}{v_1}\right)^{k-1}, \tag{3.233}$$

$$\therefore \frac{T_4}{T_3} = \left(\frac{r_c}{r}\right)^{k-1}. \tag{3.234}$$

For the cold air standard analysis, Eq. (3.227) can be written as,

$$\eta_{\text{Diesel}} = 1 - \frac{c_v(T_4 - T_1)}{c_p(T_3 - T_2)}, \tag{3.235}$$

$$= 1 - \frac{1}{k}\frac{T_1}{T_2}\left(\frac{\frac{T_4}{T_1} - 1}{\frac{T_3}{T_2} - 1}\right), \tag{3.236}$$

$$= 1 - \frac{1}{k}\frac{1}{r^{k-1}}\left(\frac{\frac{T_4}{T_3}\frac{T_3}{T_2}\frac{T_2}{T_1} - 1}{r_c - 1}\right), \tag{3.237}$$

$$= 1 - \frac{1}{k}\frac{1}{r^{k-1}}\left[\frac{\left(\frac{r_c}{r}\right)^{k-1} r_c r^{k-1} - 1}{r_c - 1}\right], \tag{3.238}$$

$$\therefore \eta_{\text{Diesel}} = 1 - \frac{1}{r^{k-1}}\left[\frac{r_c^k - 1}{k(r_c - 1)}\right]. \tag{3.239}$$

Notes:

(1) Typical values for the compression ratio for a compression ignition IC engine are $r = 12$ to 24 with engine thermal efficiencies of $\eta = 40$ to 45% (these are real efficiencies, not reversible efficiencies).
(2) As the compression ratio r increases, the thermal efficiency η increases. Diesel engines are not limited by engine knock.
(3) Since they rely on compression ignition, Diesel cycle engines are built for larger pressures. They tend to last longer than spark ignition engines.
(4) The quantity in square brackets in Eq. (3.239) is greater than one for $k > 1$ and $r_c > 1$. Thus, comparing Eq. (3.239) to Eq. (3.218), we observe that the thermal efficiency of the Diesel cycle is less than the thermal efficiency for the Otto cycle *at the same compression ratio* r. Furthermore, the thermal efficiency of the Diesel cycle decreases as r_c increases. As stated previously, however, in practice the compression ratios for compression ignition engines are larger than the compression ratios for spark ignition engines and, thus, the actual efficiencies tend to be larger for compression ignition engines. Additional non-air standard cycle factors, such as air/fuel combustion chemistry, also factor into why compression ignition engines are more efficient in practice.
(5) As the specific heat ratio k increases, the thermal efficiency η increases. The specific heat ratio is determined by the type of fuel used. In addition, the specific heat ratio increases as the temperature decreases.

3.8.4.3. Dual Cycle

The dual cycle combines elements of the combustion processes of the Otto and Diesel cycles to better approximate a real engine cycle. The processes in a dual cycle are (refer to Figure 3.51):

- *Process 1 - 2*: isentropic compression of the working fluid as the piston moves from bottom dead center to top dead center (compression stroke),
- *Process 2 - 3*: *constant volume* heat addition to the working fluid while the piston is at top dead center (beginning of combustion)
- *Process 3 - 4*: *constant pressure* heat addition to the working fluid starting from the top dead center (remaining combustion and start of power stroke)

- *Process 4 - 5*: isentropic expansion of the working fluid as the piston continues to move to bottom dead center (power stroke)
- *Process 5 - 6* constant volume heat removal from the working fluid while the piston is at bottom dead center (exhaust and intake strokes)

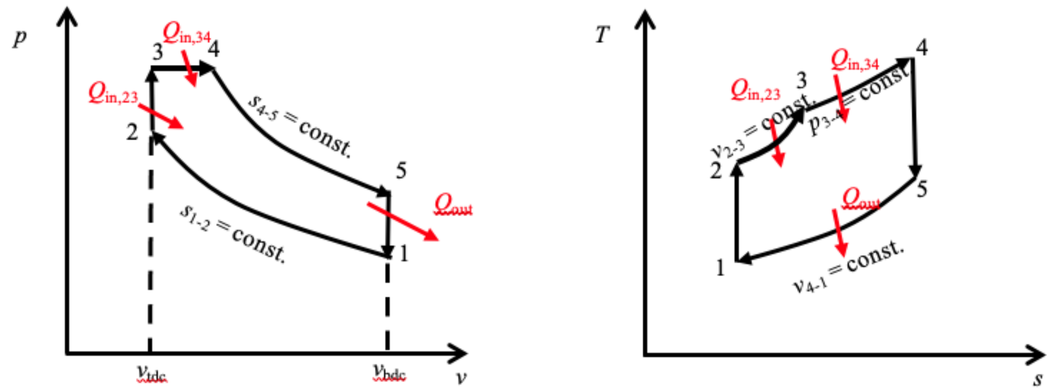


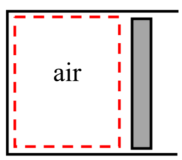
FIGURE 3.51. Sketches of the p-v and T-s plots for a dual cycle. Compare this p-v plot to the one shown in Figure 3.48.

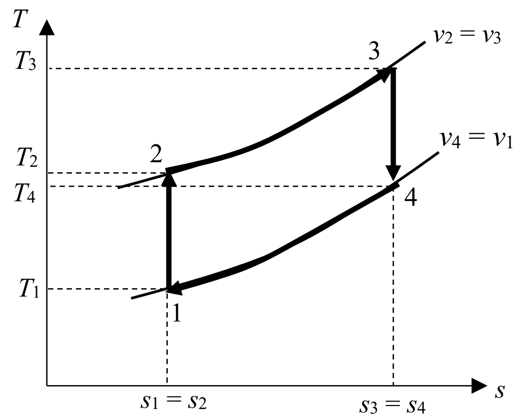
Analysis of the dual cycle won't be described here, but it follows closely the First Law analyses that have already been presented for Otto and Diesel cycles.

An air-standard Otto cycle has a compression ratio of 10. At the beginning of compression, the pressure is 100 kPa (abs) and temperature is 27 ºC. The mass of air is 5 g and the maximum temperature in the cycle is 727 ºC. Determine:

a. the heat rejection, in kJ,
b. the net work, in kJ,
c. the thermal efficiency of the cycle,
d. the mean effective pressure, in kPa (abs), and
e. sketch the process on a *T-s* plot, clearly indicating states, paths, and lines of constant specific volume.

SOLUTION:





Note: $T_1 = 27$ ºC = 300 K and $T_3 = 727$ ºC = 1000 K.

Assume air is an ideal gas so that,

$$\frac{V_2}{V_1} = \frac{v_r(T_2)}{v_r(T_1)} \Rightarrow v_r(T_2) = v_r(T_1)\frac{V_2}{V_1} \quad (1)$$

where,

$V_2/V_1 = 1/10 = 0.1$ (given),
$v_r(T_1 = 300\text{ K}) = 621.2$ (using the Ideal Gas Table for air),
⇨ $v_r(T_2) = 62.12$.

Using the Ideal Gas Table for air,
$T_2 = 730$ K and $u_2 = 536.1$ kJ/kg.
In addition, $u_1 = 214.1$ kJ/kg.

Similarly,

$$\frac{V_4}{V_3} = \frac{v_r(T_4)}{v_r(T_3)} \Rightarrow v_r(T_4) = v_r(T_3)\frac{V_4}{V_3} \quad (2)$$

where,

$V_4/V_3 = 10/1 = 10$ (given),
$v_r(T_3 = 1000\text{ K}) = 25.17$ (using the Ideal Gas Table for air),
⇨ $v_r(T_4) = 251.7$.

Using the Ideal Gas Table for air,
$T_4 = 430$ K and $u_4 = 308.0$ kJ/kg.
In addition, $u_3 = 758.9$ kJ/kg.

Apply the 1st Law to the system (i.e., the air), for process 2-3,

$$\Delta E_{sys,23} = Q_{into\ sys,23} - W_{by\ sys,23}, \quad (3)$$

where,

$$\Delta E_{sys,23} = \Delta U_{sys,23} = m(u_3 - u_2) \text{ (Neglecting changes in kinetic and potential energies.)}, \quad (4)$$

$$W_{by\ sys,23} = 0 \text{ (Constant volume process.)} \quad (5)$$

Substitute and simplify,

$$Q_{into\ sys,23} = m(u_3 - u_2). \quad (6)$$

Using the previously determined and given values,
$Q_{into\ sys,23} = 1.114$ kJ.

Similarly, for process 4-1,

$$\Delta E_{sys,41} = Q_{into\ sys,41} - W_{by\ sys,41}, \quad (7)$$

where,

$$\Delta E_{sys,41} = \Delta U_{sys,41} = m(u_1 - u_4) \text{ (Neglecting changes in kinetic and potential energies.)}, \quad (8)$$

$$W_{by\ sys,41} = 0 \text{ (Constant volume process.)} \quad (9)$$

Substitute and simplify,

$Q_{into\ sys,41} = m(u_1 - u_4)$. (10)

Using the previously determined and given values,

$Q_{\text{into sys,41}}$ = -0.4695 kJ. Thus, 0.470 kJ of energy is rejected via heat transfer from the system.

The net work for the cycle may be found by applying the 1st Law to the system over the entire cycle,

$\Delta E_{sys,cycle} = Q_{into\ sys,cycle} - W_{by\ sys,cycle}$, (11)

where,

$\Delta E_{sys,cycle} = 0$ (The net change in properties over a cycle is zero.) (12)

$Q_{into\ sys,cycle} = Q_{into\ sys,23} + Q_{into\ sys,41}$ (No heat is added in processes 1-2 and 3-4.) (13)

Substitute and simplify,

$W_{by\ sys,cycle} = Q_{into\ sys,23} + Q_{into\ sys,41}$. (14)

Using the previously calculated values,

$W_{by\ sys,cycle}$ = 0.645 kJ.

Alternately, we could have found the net work by applying the 1st Law to the compression and power strokes of the cycle separately,

$m(u_2 - u_1) = -W_{by\ sys,12}$, (15)

$m(u_4 - u_3) = -W_{by\ sys,34}$, (16)

⇨ $W_{by\ sys,12}$ = -1.61 kJ and $W_{by\ sys,34}$ = 2.2545 kJ,

⇨ $W_{by\ sys,cycle} = W_{by\ sys,12} + W_{by\ sys,34}$ = 0.645 kJ, which is the same answer found previously.

The thermal efficiency for the cycle is,

$\eta \equiv \frac{W_{by\ sys,cycle}}{Q_{into\ sys}}$, (17)

Using the previously calculated values,

η = 0.578.

The mean effective pressure is given by,

$mep \equiv \frac{W_{by\ sys,cycle}}{V_1 - V_2}$ => $mep \equiv \frac{W_{by\ sys,cycle}}{V_1(1 - V_2/V_1)}$ (18)

where,

$V_1 = \frac{m R_{air} T_1}{p_1}$, (19)

with,

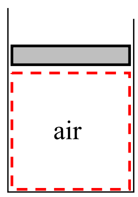
R_{air} = 0.287 kJ/(kg.K), p_1 = 100 kPa (abs), and V_2/V_1 = (1/10) = 0.1,

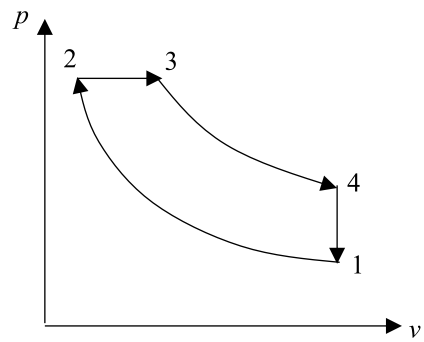
⇨ $V_1 = 4.31*10^{-3}$ m^3 => mep = 166 kPa (abs)

The displacement volume of an internal combustion engine is 3 L. The processes within each cylinder of the engine are modeled as an air-standard Diesel cycle with a cutoff ratio of 2.5. The state of the air at the beginning of compression is fixed by p_1 = 95 kPa (abs), T_1 = 22°C, and V_1 = 3.17 L. Determine:

a. the net work per cycle,
b. the power developed by the engine if the cycle repeats 1000 times per minute,
c. and the thermal efficiency of the cycle.

SOLUTION:





First, determine the mass of air in the cylinder using the ideal gas law,

$$m = \frac{p_1 V_1}{R T_1}, \quad (1)$$

Using the given values with R = 0.287 kJ/(kg.K),

$m = 3.5570 \times 10^{-3}$ kg.

Now determine the properties at each state:

State 1:

p_1 = 95 kPa (abs), T_1 = 22°C = 295 K, and V_1 = 3.17 L
=> u_1 = 210.5 kJ/kg and $v_r(T_1 = 295\text{ K}) = 647.9$ (from the Ideal Gas Table (IGT) for air)

State 2:

$V_2 = V_1 - 3.0$ L = 0.17 L (given that the displacement volume is 3 L), (2)

$$\frac{v_2}{v_1} = \frac{V_2}{V_1} = \frac{v_r(T_2)}{v_r(T_1)} \Rightarrow v_r(T_2) = v_r(T_1)\left(\frac{V_2}{V_1}\right), \quad (3)$$

where V_1 = 3.17 L, V_2 = 0.17 L,

⇨ $v_r(T_2) = 34.745$ => T_2 = 896.15 K, u_2 = 671.405 kJ/kg, h_2 = 928.59 kJ/kg (interpolating in the IGT)

The pressure may be found using the ideal gas law,

⇨ $p_2 = \frac{mRT_2}{V_2}$ => p_2 = 5381.37 kPa. (4)

State 3:

The cut-off ratio is given as $r_c = 2.5 = V_3/V_2 = T_3/T_2$ => T_3 = 2240.4 K, V_3 = 0.425 L, (5)
=> h_3 = 2553.87 kJ/kg, u_3 = 1911.76 kJ/kg, $v_r(T_3) = 1.8925$ (interpolating in the IGT)

State 4:

$$\frac{v_4}{v_3} = \frac{V_4}{V_3} = \frac{v_r(T_4)}{v_r(T_3)} \Rightarrow v_r(T_4) = v_r(T_3)\left(\frac{V_4}{V_3}\right) = v_r(T_3)\left(\frac{V_4}{V_1}\cdot\frac{V_1}{V_2}\cdot\frac{V_2}{V_3}\right), \quad (6)$$

where $V_4 = V_1$, V_1 = 3.17 L (given), V_2 = 0.17 L (Eq. (2)), and $V_2/V_3 = 1/r_c = 1/2.5$ (Eq. (5)),
=> $v_r(T_4) = 14.1157$ => T_4 = 1209.8 K and u_4 = 942.17 kJ/kg (interpolating in the IGT)

The work into the air during the compression stroke is found by applying the 1st Law to the air (assuming negligible changes in KE and PE and an adiabatic process),

$$m(u_2 - u_1) = W_{in,12} \tag{7}$$

Using the previously calculated values,

$W_{in,12} = 1.6394$ kJ.

Now calculate the work done by the air during the heat addition and power strokes using the 1st Law,

$$W_{out,23} = p_2(V_3 - V_2), \tag{8}$$

$$m(u_4 - u_3) = -W_{out,34} \tag{9}$$

Using the previously calculated values,

$W_{out,23} = 1.3722$ kJ and $W_{out,34} = 3.449$ kJ

The net work out is,

$$W_{out,net} = W_{out,23} + W_{out,34} - W_{in,12}, \tag{10}$$

$\boxed{W_{out,net} = 3.18 \text{ kJ} \text{ (This is the work over one cycle.)}}$

Alternately, we could apply the 1st Law over the whole cycle, keeping in mind that the total energy does not change over the cycle,

$$0 = Q_{in,23} - Q_{out,41} + W_{in,12} - W_{out,23} - W_{out,34}, \tag{11}$$

$$0 = Q_{in,23} - Q_{out,41} - W_{out,net}, \tag{12}$$

$$W_{out,net} = Q_{in,23} - Q_{out,41}. \tag{13}$$

The heat transfer into the system during the combustion process is,

$$m(u_3 - u_2) = Q_{in,23} - p_2(V_3 - V_2), \text{ (noting that } p_3 = p_2\text{)}, \tag{14}$$

$$Q_{in,23} = m(u_3 - u_2) + p_2(V_3 - V_2) = m(h_3 - h_2). \tag{15}$$

Using the previously calculated values,

$Q_{in,23} = 5.7811$ kJ.

The heat transfer out of the system is,

$$m(u_4 - u_1) = -Q_{out,41}. \tag{16}$$

$Q_{out,41} = 2.6025$ kJ.

Using the calculated heat values and Eq. (13),

$W_{out,net} = 3.18$ kJ, which is the same value found previously.

The power is,

$$\dot{W}_{out,net} = \left(\frac{W_{out,net}}{1 \text{ cycle}}\right)\left(\frac{1000 \text{ cycle}}{1 \text{ min}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right), \tag{17}$$

$\boxed{\dot{W}_{out,net} = 53.0 \text{ kJ/s} = 53.0 \text{ kW}}$.

The thermal efficiency is,

$$\eta = \frac{W_{out,net}}{Q_{in}}, \tag{18}$$

Using $W_{out,net} = 3.18$ kJ and $Q_{in} = 5.7811$ kJ,

⇨ $\boxed{\eta = 0.550 = 55.0\%}$.