3.8.2. The Rankine Cycle and Improvements

The Rankine Cycle and its variations are commonly used vapor power cycles for large-scale power generation, such as in natural gas and coal-fired power plants, nuclear power plants, and solar power plants. The standard Rankine cycle consists of the following four processes (refer to Figure 3.41):

- *Process 1 – 2*: Expansion of the working fluid from saturated vapor through the turbine.
- *Process 2 – 3*: Heat transfer from the working fluid as it flows at constant pressure through the condenser to a saturated liquid state.
- *Process 3 - 4*: Compression of the working fluid in the pump in the compressed liquid region.
- *Process 4 - 1*: Heat transfer to the working fluid as it flows at constant pressure through the boiler.

If the turbine and pump have 100% isentropic efficiencies, then the cycle is referred to as an "ideal standard Rankine cycle", state 2 is identified as state "2s", and state 4 is identified as state "4s".

Notes:

(1) Using Eq. (3.189) *as a guide*, we observe that the thermal efficiency of a power cycle generally increases as the *average* temperature at which the heat is added in the boiler increases and the *average* temperature at which the heat is rejected in the condenser decreases.

(a) An (internally reversible) Carnot cycle has a larger thermal efficiency than an ideal (internally reversible) Rankine cycle operating between the same two thermal reservoirs since the *average* temperature at which heat is added in the boiler is smaller for the Rankine cycle due to the portion of the path from States 4 - 1 in the condensed liquid phase.

(b) Increasing the average temperature at which heat is added may be achieved by increasing the boiler pressure, thus shifting the path from 4 - 1 to a larger temperature isotherm, or by moving State 1 into a superheated vapor (SHV) phase along the same isotherm. Increasing the boiler pressure can be costly due to the increased stress on the pipe system; however, moving State 1 into a SHV phase while at the same pressure is relatively easy. Moving into the SHV region is known as a Rankine Cycle with Superheat and is discussed in a following note.

(c) The smallest possible condenser temperature corresponds to just larger than the temperature of the surroundings since the surroundings are where the heat is being rejected. Recall that heat is transferred from a hotter object to a colder one so the working fluid temperature would need to be slightly larger than the surrounding temperature. In practice, the cold reservoir usually corresponds to the atmospheric air or a large body of water, such as the ocean, a lake, or a river.

(d) The typical thermal efficiency of a standard Rankine cycle is on the order of 20 - 40%.

(e) The ratio of the power required by the pump to the power generated by the turbine is known as the back work ratio, *bwr*,

$$bwr := \frac{\dot{W}_P}{\dot{W}_T}. \tag{3.192}$$

The *bwr* in a typical Rankine cycle is generally small - on the order of 1 - 3%, for example.

(f) The vibration and mechanical stress generated when a pump impeller encounters alternating regions of vapor (small density) and liquid (large density) can damage the pump. Thus, states 3 and 4 are always either in a saturated liquid or compressed liquid phase. In practice, state 3 is often in the compressed liquid region rather than in a saturated liquid phase to provide a margin of safety to keep the working fluid liquid.

(g) Assuming internally reversible, adiabatic pump operation, i.e., 100% isentropic efficiency, the power required by the pump is (refer to Eq. (4.152), assuming negligible changes in kinetic and potential energies across the pump),

$$\dot{W}_P = \dot{m}v(p_4 - p_3). \tag{3.193}$$

This power assumes that the working fluid can be modeled as an incompressible substance through the pump, which is reasonable since, as the previous note states, the working fluid is a compressed liquid through the pump.

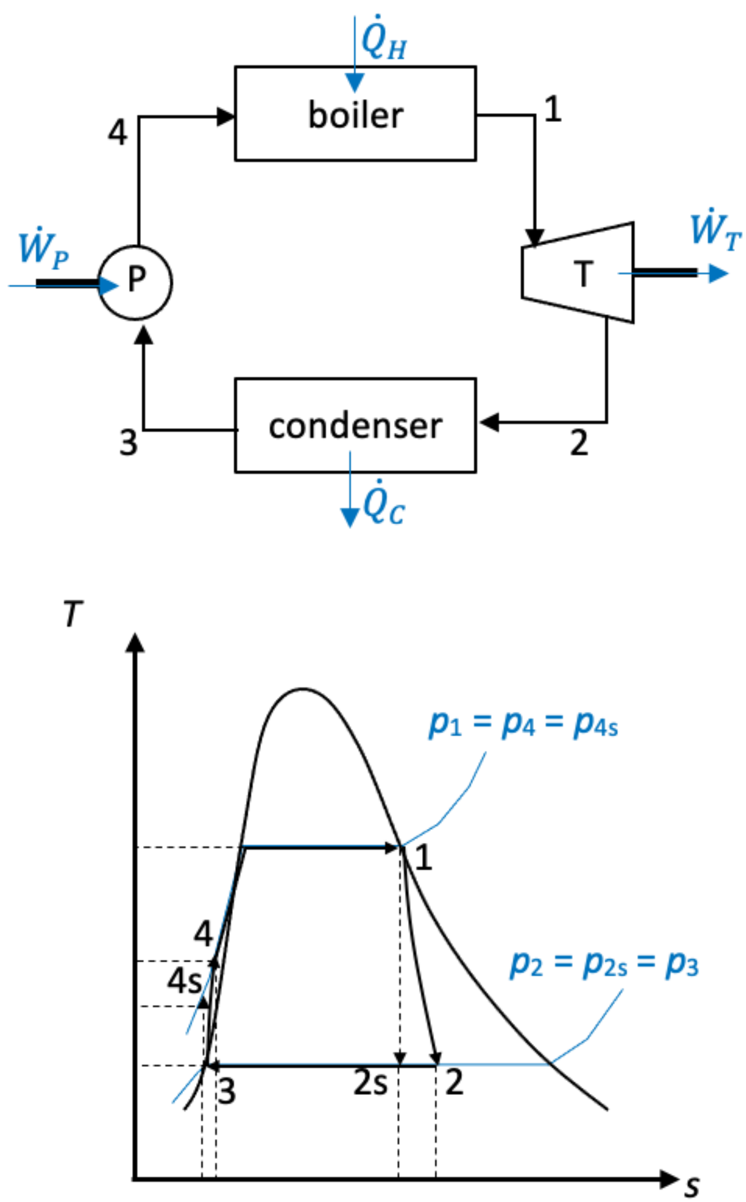


Figure 3.41. A sketch showing the components of a standard vapor power Rankine cycle. The corresponding processes are sketched on a T - s plot. The "2s" and "4s" states correspond to, respectively, flow through the turbine and pump with 100% isentropic efficiencies.

(h) Again for mechanical reasons, turbines perform best with superheated vapor or a saturated liquid vapor mixture at a large quality. Liquid droplets impacting high speed turbine blades can cause damage.

(2) As discussed previously, one improvement to the standard Rankine cycle is the Rankine Cycle with Superheat. The components of this cycle are identical to the standard Rankine cycle, but the T - s diagram is different (Figure 3.42). Specifically, in order to raise the average temperature at which energy is added via heat transfer from the hot reservoir, the working fluid leaves the boiler in a superheated vapor phase (State 3).

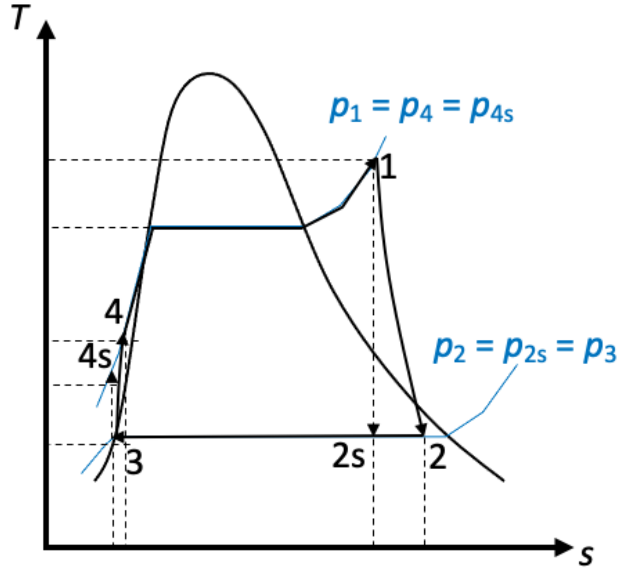


Figure 3.42. A T-s plot for a Rankine cycle with superheating.

(a) The larger average temperature at which heat is added in a Rankine cycle with superheating results in a larger thermal efficiency as compared to a standard Rankine cycle.
(b) Another advantage of superheating is that the working fluid passes through the turbine either as a superheated vapor or a high quality saturated liquid-vapor mixture.

(3) An illustration of a Rankine Cycle with Reheat and the corresponding T-s plot are shown in Figure 3.43). In this cycle, the working fluid leaves the boiler (also often called a steam generator) at State 1, passes through a first-stage turbine (State 2), then re-enters the boiler for additional heating (State 3; hence, the name "reheat"). The re-energized working fluid then passes through a second-stage turbine (State 4) to complete the remainder of the cycle.
(a) The reheating process increases the average temperature at which heat is added in the cycle, thus, increasing the thermal efficiency as compared to a standard Rankine cycle.
(b) The first stage turbine typically exits in the superheated vapor phase (State 2). In addition, the quality at the exit of the second stage turbine (State 4) is larger than that in a standard Rankine cycle and may even be in a superheated vapor phase.

(4) The Rankine Cycle with Supercritical Reheat has the same components as the reheat cycle, but heating in the steam generator occurs in the supercritical phase (above the critical point on the vapor dome). A representative T-s plot is shown in Figure 3.44).
(a) The larger average temperature at which heat is added in a Rankine cycle with supercritical reheating results in a larger efficiency as compared to a standard Rankine cycle.
(b) The first stage turbine typically exits in the superheated phase (State 2). In addition, the quality at the exit of the second stage turbine (State 4) is larger than that in a standard Rankine cycle and may even be in a superheated vapor phase.
(c) The large pressures and temperatures in a Rankine cycle with supercritical reheat requires the use of more expensive components, including high pressure piping and steam generator, and turbine materials that can withstand high temperatures. Thus, the capitol cost of this type of facility is much higher than it would be for a standard Rankine cycle facility. However, a Rankine cycle with supercritical reheating can achieve thermodynamic efficiencies of up to nearly 50%.

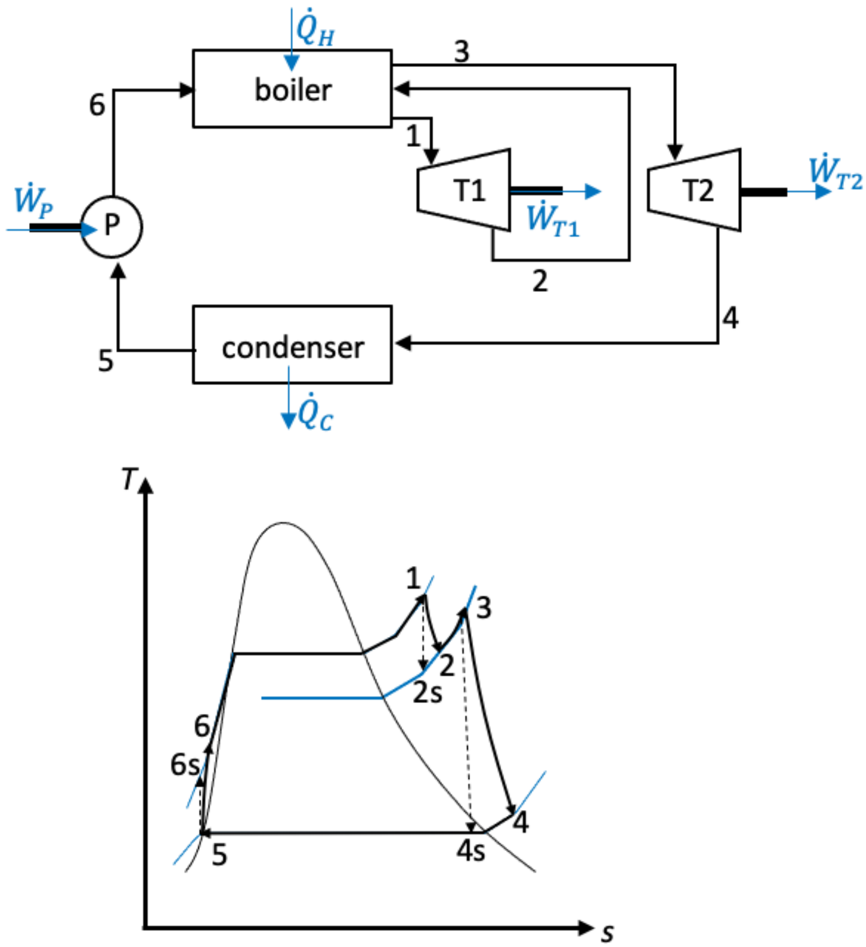


Figure 3.43. An illustration of a Rankine cycle with reheat and the corresponding T - s plot.

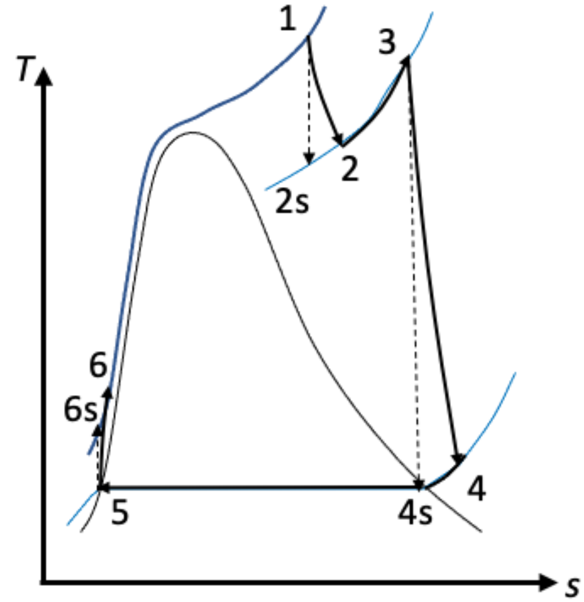
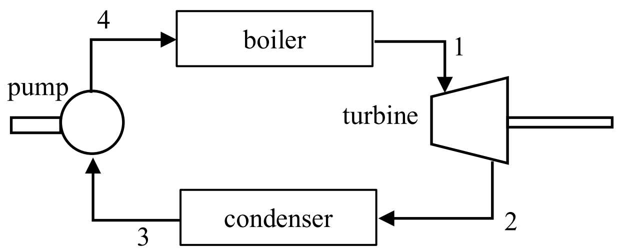


FIGURE 3.44. A T - s plot for a Rankine cycle with supercritical reheating.

Consider a steam-power plant cycle in which saturated water vapor enters the turbine at 12.0 MPa (abs) and saturated liquid exits the condenser at a pressure of 0.012 MPa (abs). The net power output of the cycle is 122 MW.

a. Assuming that the isentropic efficiencies of the turbine and pump are 80%, determine the following:
   i. the mass flow rate of the water, in kg/h,
   ii. the rate of heat transfer into the boiler, in MW
   iii. the rate of heat transfer from the condenser, in MW, and
   iv. the thermal efficiency of the power plant cycle.
b. Draw a *T-s* diagram for the cycle, clearly indicating the process paths, states, and isobar values.



SOLUTION:
First determine the properties at each of the states.

At State 1:

We're given that the water is in a saturated vapor phase and p_1 = 12.0 MPa (abs) = 120 bar (abs). Using the Saturated Property Tables for water,

$T_1 = T_{1,sat}$ = 324.68 °C, $h_1 = h_{1g}$ = 2685.4 kJ/kg, and $s_1 = s_{1g}$ = 5.4939 kJ/(kg.K)

At State 2:

We're given that the turbine has an isentropic efficiency of 80%. In addition, since the pressure is assumed to remain constant across the condenser, $p_2 = p_3$ = 0.012 MPa (abs) = 0.12 bar (abs). At this pressure, interpolating from the Saturated Property Tables for water,

$T_2 = T_{2,sat}$ = 48.66 °C, h_{2f} = 203.73 kJ/kg, h_{2g} = 2588.9 kJ/kg, s_{2f} = 0.68576 kJ/(kg.K), and s_{2g} = 8.10048 kJ/(kg.K).

The isentropic efficiency of the turbine is given by,

$$\eta_{turbine,isen} \equiv \frac{\dot{W}_{by\ CV}}{\dot{W}_{by\ CV,isen}} = \frac{h_1-h_2}{h_1-h_{2s}}, \quad (1)$$

Thus,

$$h_2 = h_1 - \eta_{turbine,isen}(h_1 - h_{2s}), \quad (2)$$

To find h_{2s}, assume the turbine operates isentropically from 1 to 2, so that $s_{2s} = s_1$ = 5.4939 kJ/(kg.K). Thus,

$$x_{2s} = \frac{h_{2s}-h_{2f}}{h_{2g}-h_{2f}} = \frac{s_{2s}-s_{f2}}{s_{g2}-s_{f2}} \Rightarrow h_{2s} = h_{2f} + \left(h_{2g} - h_{2f}\right)\left(\frac{s_{2s}-s_{2f}}{s_{2g}-s_{2f}}\right). \quad (3)$$

Using the values found previously, h_{2s} = 1748.33 kJ/kg. Substituting into Eq. (2) gives,

h_2 = 1937.41 kJ/kg.

The quality for this state is,

$$x_2 = \frac{h_2-h_{2f}}{h_{2g}-h_{2f}} \Rightarrow x_2 = 0.7269. \quad (4)$$

The specific entropy at state 2 is then,

$$\Rightarrow s_2 = (1-x_2)s_{2f} + x_2 s_{2g} \Rightarrow s_2 = 6.07521 \text{ kJ/(kg.k)}. \quad (5)$$

Note that $s_2 > s_1$, as expected for adiabatic operation of the turbine.

At State 3:

We're given that the water is in a saturated liquid phase and $p_3 = 0.012$ MPa (abs) = 0.12 bar (abs). Using the Saturated Property Tables for water and interpolating,

$T_3 = 48.66$ °C, $h_3 = 203.73$ kJ/kg, $v_3 = 0.0010012$ m^3/kg, and $s_3 = 0.68576$ kJ/(kg.K)

At State 4:

We're given that the pump has an isentropic efficiency of 80%. In addition, since the pressure is assumed to remain constant across the boiler, $p_1 = p_4 = 12.0$ MPa (abs) = 120 bar (abs).

The isentropic efficiency of the pump is given by,

$$\eta_{pump,isen} \equiv \frac{\dot{W}_{into\ CV,isen}}{\dot{W}_{into\ CV}} = \frac{h_{4s}-h_3}{h_4-h_3}, \quad (6)$$

$$h_4 = h_3 + \frac{(h_{4s}-h_3)}{\eta_{pump,isen}}, \quad (7)$$

Assuming an isentropic process from State 3 to State 4s (i.e., $s_{4s} = s_3 = 0.68576$ kJ/(kg.K)) and since the water can be treated as an incompressible substance at State 4s,

$$Tds = dh - vdp \Rightarrow dh = vdp \Rightarrow h_{4s} - h_3 = v_3(p_{4s} - p_3), \quad (8)$$

Using the parameters calculated previously, along with $p_{4s} = 120$ bar (abs),

$h_{4s} = 215.86$ kJ/kg.

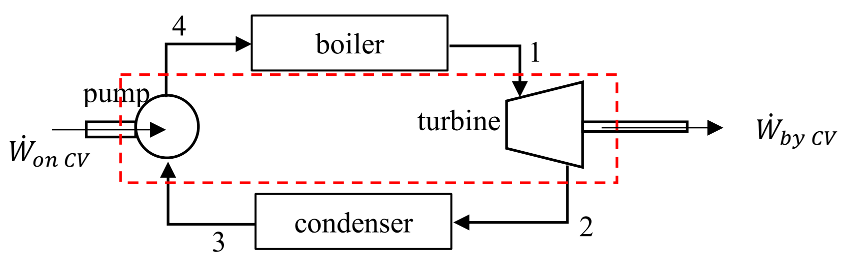
Using Eq. (7),

$h_4 = 218.89$ kJ/kg.

Applying Conservation of Mass to individual control volumes surrounding each component and assuming steady flow gives,

$$\dot{m} = \dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \dot{m}_4. \quad (9)$$

Apply the 1st Law to a control volume surrounding the turbine and pump.



$$\frac{dE_{CV}}{dt} = \dot{Q}_{into\ CV} - \dot{W}_{net,by\ CV} + \sum_{in}\dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out}\dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right), \quad (10)$$

where,

$$\frac{dE_{CV}}{dt} = 0 \text{ (Assuming steady state operation.)}, \quad (11)$$

$$\dot{Q}_{into\ CV} = 0 \text{ (Assuming adiabatic operation.)}, \quad (12)$$

$$\dot{W}_{net,by\ CV} = \dot{W}_{by\ CV} - \dot{W}_{on\ CV}, \quad (13)$$

$$\sum_{in}\dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out}\dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) = \dot{m}(h_3 - h_4 + h_1 - h_2). \quad (14)$$

(The changes in kinetic and potential energies are assumed to be negligibly small.)

Substitute and simplify,

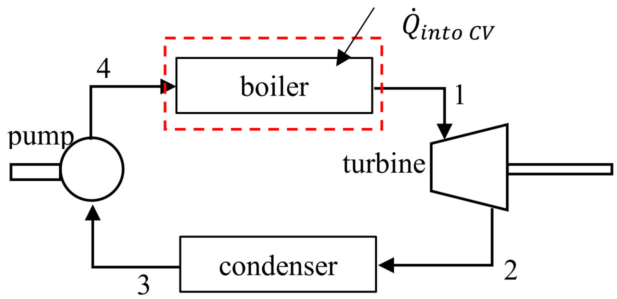
$$\dot{W}_{net,by\ CV} = \dot{m}(h_3 - h_4 + h_1 - h_2), \quad (15)$$

$$\boxed{\dot{m} = \frac{\dot{W}_{net,by\ CV}}{(h_3-h_4+h_1-h_2)}}. \quad (16)$$

Using the parameters calculated previously in addition to the given net power output of 122 MW,

$\boxed{\dot{m} = 599 \times 10^3 \text{ kg/h}}$.

The rate of heat transfer in the boiler is found by applying the 1st Law to a control volume surrounding the boiler.



$$\frac{dE_{CV}}{dt} = \dot{Q}_{into\ CV} - \dot{W}_{by\ CV} + \sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right), \quad (17)$$

where,

$$\frac{dE_{CV}}{dt} = 0 \text{ (Assuming steady state operation.)}, \quad (18)$$

$$\dot{W}_{by\ CV} = 0 \text{ (A boiler is a passive device.)}, \quad (19)$$

$$\sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) = \dot{m}(h_4 - h_1). \quad (20)$$

(The changes in kinetic and potential energies are assumed to be negligibly small.)

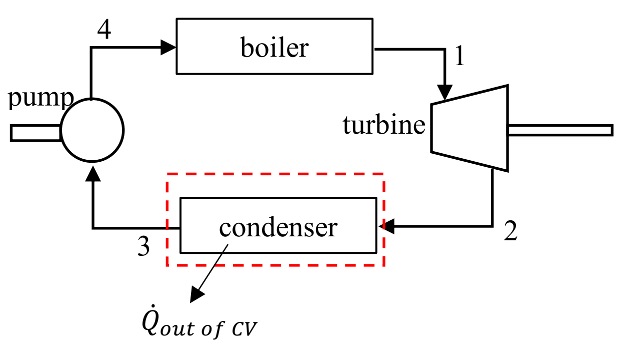
Substitute and simplify,

$$\boxed{\dot{Q}_{into\ CV} = \dot{m}(h_1 - h_4)}, \quad (21)$$

Using the parameters calculated previously in addition to the given net power output of 122 MW,

$$\boxed{\dot{Q}_{into\ CV} = 411 \text{ MW}}.$$

To find the heat transfer from the condenser, apply the 1st Law to a control volume surrounding the condenser.



$$\frac{dE_{CV}}{dt} = -\dot{Q}_{out\ of\ CV} - \dot{W}_{by\ CV} + \sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right), \quad (22)$$

where,

$$\frac{dE_{CV}}{dt} = 0 \text{ (Assuming steady state operation.)}, \quad (23)$$

$$\dot{W}_{by\ CV} = 0 \text{ (A condenser is a passive device.)}, \quad (24)$$

$$\sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) = \dot{m}(h_2 - h_3). \quad (25)$$

(The changes in kinetic and potential energies are assumed to be negligibly small.)

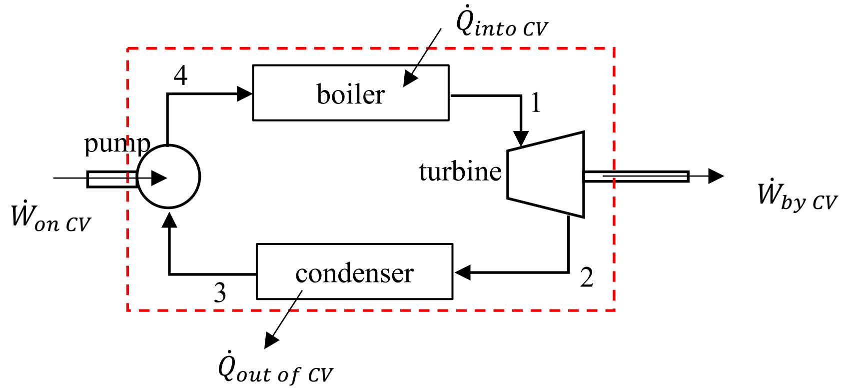
Substitute and simplify,

$$\boxed{\dot{Q}_{out\ of\ CV} = \dot{m}(h_2 - h_3)}, \quad (26)$$

Using the parameters calculated previously in addition to the given net power output of 122 MW,

$$\boxed{\dot{Q}_{out\ of\ CV} = 289 \text{ MW}}.$$

Alternately, the rate of heat transfer out from the boiler could be found by applying the 1st Law to a control volume that surrounds the entire cycle,



$$\frac{dE_{CV}}{dt} = \dot{Q}_{net,into\ CV} - \dot{W}_{net,by\ CV} + \sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right), \quad (27)$$

where,

$$\frac{dE_{CV}}{dt} = 0 \text{ (Assuming steady state operation.)}, \quad (28)$$

$$\dot{Q}_{net,into\ CV} = \dot{Q}_{into\ CV} - \dot{Q}_{out\ of\ CV}, \quad (29)$$

$$\sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) = 0, \quad (30)$$

(Since there is no mass transfer across the CV surface.)

Substitute and simplify,

$$\dot{W}_{net,by\ CV} = \dot{Q}_{into\ CV} - \dot{Q}_{out\ of\ CV}, \quad (31)$$

$$\boxed{\dot{Q}_{out\ of\ CV} = \dot{Q}_{into\ CV} - \dot{W}_{net,by\ CV}}. \quad (32)$$

Using the value found previously for the heat transfer into the control volume and given net power done by the cycle,

$$\boxed{\dot{Q}_{out\ of\ CV} = 289 \text{ MW}},$$

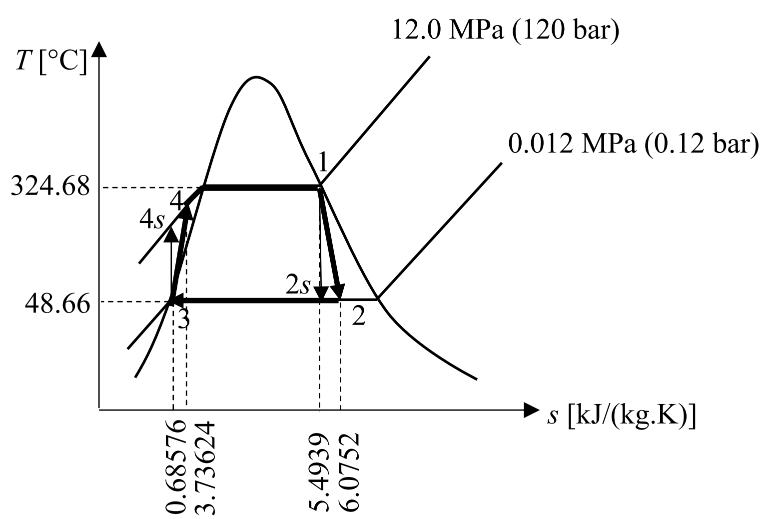
which is the same value found previously.

The thermal efficiency of the power plant is,

$$\boxed{\eta \equiv \frac{\dot{W}_{net,by\ CV}}{\dot{Q}_{into\ CV}}}, \quad (33)$$

Using the parameters found previously,

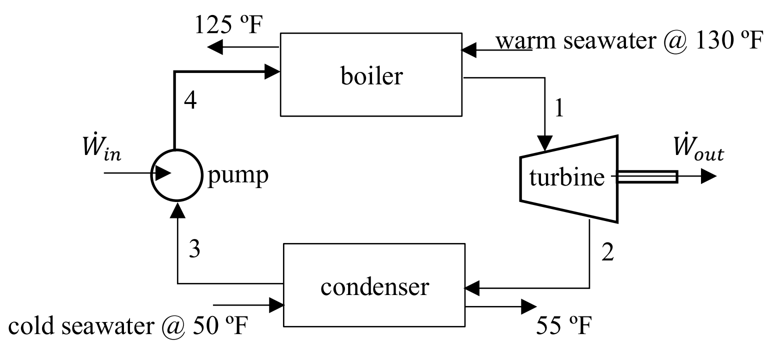
$$\boxed{\eta = 0.297}.$$



On the island of Hawaii lava flows continuously into the ocean. It is proposed to anchor a floating power plant offshore of the lava flow that uses ammonia as the working fluid. The plant would exploit the temperature variation between the warm water near the surface at 130 °F and seawater at 50 °F from a depth of 500 ft to produce power. Using the properties of pure water for the seawater and modeling the power plant as a Rankine cycle, determine:

a. the plant's thermal efficiency, and
b. the mass flow rate of ammonia in lb_m/min, for a net power output of 300 hp.
c. the mass flow rates of seawater through the boiler and condenser, in lb_m/min.

For a related story, see:
https://www.scientificamerican.com/article/hawaii-first-to-harness-deep-ocean-temperatures-for-power/



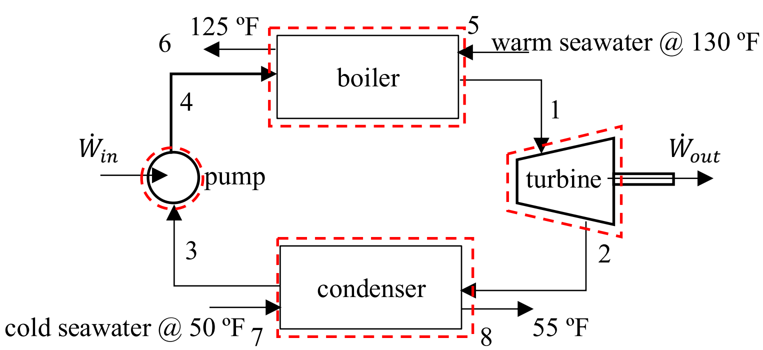
Working fluid: ammonia
State 1:
T_1 = 120 °F
saturated vapor

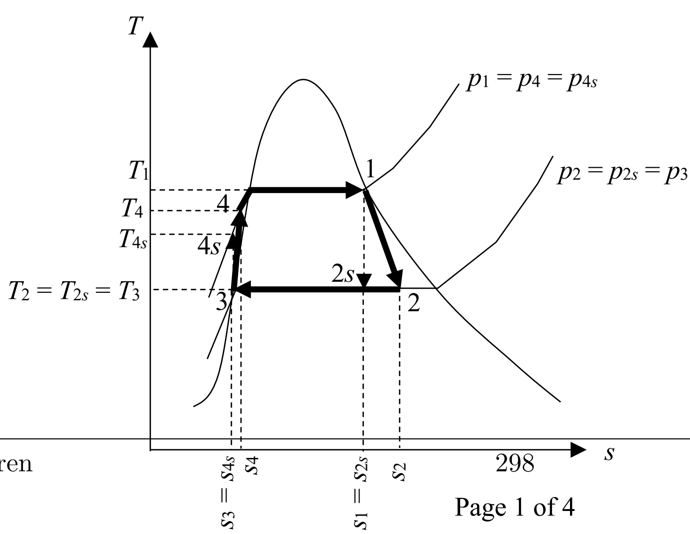
State 2:
T_2 = 60 °F

State 3:
$p_3 = p_2$
saturated liquid

isentropic turbine efficiency = 0.80
isentropic pump efficiency = 0.85

SOLUTION:





First find the temperatures, specific enthalpies, and specific entropies at each of the states using the property tables for ammonia.

State	*p* [psia]	*T* [ºF]	Phase	*x* [-]	*h* [Btu/lbm]	*s* [Btu/(lbm. ºR)]
1	286.47 (= p_{sat})	120	sat. vapor	1	632.95	1.1405
2	107.66 (= p_3)	60	SLVMº,⁼	0.932429	591.626	1.16038
2*s*	107.66 (= p_2)	60	SLVMº	0.912476	581.295	1.1405 (= s_1)
3	107.66 (= p_{sat})	60	sat. liquid	0	108.87	0.2314
4	286.47 (= p_1)	61.08	CL⁺	N/A	109.881	0.23374
4*s*	286.47 (= p_4)	60	CL⁺	N/A	109.729	0.2314 (= s_3)

ºFor a SLVM,

$$x = \frac{s - s_f}{s_g - s_f}, \quad (1)$$

$$h = (1 - x)h_f + xh_g. \quad (2)$$

State 2*s*:

T_{2s} = 60 ºF, $s_{2s} = s_1$ = 1.1405 Btu/(lbm.ºR)]; s_{f2s} = 0.2314 Btu/(lbm.ºR), s_{g2s} = 1.2277 Btu/(lbm.ºR)
=> x_{2s} = 0.912476.
h_{f2s} = 108.87 Btu/lbm, h_{g2s} = 626.61 Btu/lbm => h_{2s} = 581.295 Btu/lbm.

State 2:

T_2 = 60 ºF, h_2 = 591.626 Btu/lbm (see below); h_{f2} = 108.87 Btu/lbm, h_{g2} = 626.61 Btu/lbm
=> x_2 = 0.932429.
s_{f2} = 0.2314 Btu/(lbm.ºR), s_{g2} = 1.2277 Btu/(lbm.ºR) => s_2 = 1.16038 Btu/(lbm.ºR).

⁼To find the conditions at State 2, make use of the turbine isentropic efficiency,

$$\eta_{turb.,isen.} = \frac{\dot{W}_{out}}{\dot{W}_{out,isen}} = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_{turb.,isen.}(h_1 - h_{2s}) = 591.626 \text{ Btu/lbm}, \quad (3)$$

where h_1 = 632.95 Btu/lbm, h_{2s} = 581.295 Btu/lbm, and $\eta_{turb.,isen.}$= 0.80.

⁺ For a compressed liquid,

$$h_{CL}(T,p) \approx h_f(T) + [p - p_{sat}(T)]v_f(T) \quad \text{and} \quad s_{CL}(T,p) \approx s_f(T) \quad (4)$$

State 4*s*:

p_{4s} = 286.47 psia, $s_{4s} = s_3$ = 0.2314 Btu/(lbm.ºR) =>
T_{4s} = 60 ºF, $p_{sat,4s}$ = 107.66 psia, v_{f4s} = 0.02597 ft^3/lbm, h_{f4s} = 108.87 Btu/lbm => h_{4s} = 109.729 Btu/lbm.

⁼To find the conditions at State 4, make use of the pump isentropic efficiency,

$$\eta_{pump,isen.} = \frac{\dot{W}_{in,isen}}{\dot{W}_{in}} = \frac{h_{4s} - h_3}{h_4 - h_3} \Rightarrow h_4 = h_3 + \frac{h_{4s} - h_3}{\eta_{pump,isen.}} = 109.881 \text{ Btu/lbm}, \quad (5)$$

where h_3 = 108.87 Btu/lbm, h_{4s} = 109.729 Btu/lbm, and $\eta_{pump,isen.}$= 0.85. The temperature corresponding to this specific enthalpy is, after some linear interpolation, T_4 = 61.08 ºF, and the specific entropy is, s_4 = 0.23374 Btu/(lbm.ºR).

Now apply the 1st Law to a control volume surrounding the turbine,

$$\frac{dE_{CV}}{dt} = \sum_{in} \dot{m}(h + ke + pe) - \sum_{out} \dot{m}(h + ke + pe) + \dot{Q}_{in} - \dot{W}_{out}, \tag{6}$$

where,

$$\frac{dE_{CV}}{dt} = 0 \quad \text{(assuming steady state operation)}, \tag{7}$$

$$\sum_{in} \dot{m}(h + ke + pe) - \sum_{out} \dot{m}(h + ke + pe) = \dot{m}(h_1 - h_2), \tag{8}$$

(neglecting kinetic and potential energy changes; from COM $\dot{m}_1 = \dot{m}_2 = \dot{m}$)

$$\dot{Q}_{in} = 0 \quad \text{(assuming adiabatic operation)}, \tag{9}$$

$$\dot{W}_{out} = ?. \tag{10}$$

Substitute and solve for the power,

$$\frac{\dot{W}_{out}}{\dot{m}} = h_1 - h_2. \tag{11}$$

Using the data from the table and the given mass flow rate,

$$\frac{\dot{W}_{out}}{\dot{m}} = 41.324 \text{ Btu/lb}_m.$$

Apply the 1st Law to a control volume surrounding the pump,

$$\frac{dE_{CV}}{dt} = \sum_{in} \dot{m}(h + ke + pe) - \sum_{out} \dot{m}(h + ke + pe) + \dot{Q}_{in} + \dot{W}_{in}, \tag{12}$$

where,

$$\frac{dE_{CV}}{dt} = 0 \quad \text{(assuming steady state operation)}, \tag{13}$$

$$\sum_{in} \dot{m}(h + ke + pe) - \sum_{out} \dot{m}(h + ke + pe) = \dot{m}(h_3 - h_4), \tag{14}$$

(neglecting kinetic and potential energy changes; from COM $\dot{m}_3 = \dot{m}_4 = \dot{m}$)

$$\dot{Q}_{in} = 0 \quad \text{(assuming adiabatic operation)}, \tag{15}$$

$$\dot{W}_{in} = ?. \tag{16}$$

Substitute and solve for the power,

$$\frac{\dot{W}_{in}}{\dot{m}} = h_4 - h_3. \tag{17}$$

Using the data from the table and the given mass flow rate,

$$\frac{\dot{W}_{in}}{\dot{m}} = 1.011 \text{ Btu/lb}_m.$$

Apply the 1st Law to a CV surrounding the boiler,

$$\frac{dE_{CV}}{dt} = \sum_{in} \dot{m}(h + ke + pe) - \sum_{out} \dot{m}(h + ke + pe) + \dot{Q}_{in} - \dot{W}_{out}, \tag{18}$$

where,

$$\frac{dE_{CV}}{dt} = 0 \quad \text{(assuming steady state operation)}, \tag{19}$$

$$\sum_{in} \dot{m}(h + ke + pe) - \sum_{out} \dot{m}(h + ke + pe) = \dot{m}(h_4 - h_1), \tag{20}$$

(neglecting kinetic and potential energy changes; from COM $\dot{m}_1 = \dot{m}_4 = \dot{m}$)

$$\dot{Q}_{in} = ?, \tag{21}$$

$$\dot{W}_{out} = 0 \quad \text{(the steam generator is a passive device)}. \tag{22}$$

Substitute and solve for the rate of heat transfer,

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_1 - h_4. \tag{23}$$

Using the data from the table and the given mass flow rate,

$$\frac{\dot{Q}_{in}}{\dot{m}} = 523.069 \text{ Btu/lb}_m.$$

Using the power in and power out results,

$$\frac{\dot{W}_{out,net}}{\dot{m}} = \frac{\dot{W}_{out}}{\dot{m}} - \frac{\dot{W}_{in}}{\dot{m}} = 40.313 \text{ Btu/lb}_m \tag{24}$$

The thermal efficiency for the power cycle is,

$$\boxed{\eta = \frac{\dot{W}_{out,net}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = 0.0771 = 7.71\%} \tag{25}$$

This thermal efficiency is less than the Carnot cycle thermal efficiency of

$$\eta_{Carnot} = 1 - \frac{T_C}{T_H} = 0.136 = 13.6\%, \quad (26)$$

where T_C = 509.67 ºR (= 50 ºF) and T_H = 589.67 ºR (= 130 ºF). The Rankine cycle efficiency is smaller than the Carnot cycle efficiency because of irreversibilities in the cycle. Even if the Rankine cycle was ideal (isentropic conditions across the pump and turbine), it would still have a smaller efficiency than the Carnot cycle because the average temperature during heat addition is smaller than that for a Carnot cycle, i.e., the average temperature from State 4 to State 1 is smaller than the average temperature from State 4 to State 1 in a Carnot cycle.

The mass flow rate in the cycle can be determined using Eq. (24) and the given net power output of $\dot{W}_{out,net} = 300$ hp,

$$\frac{\dot{W}_{out,net}}{\dot{m}} = 40.313 \text{ Btu/lb}_m \Rightarrow \boxed{\dot{m} = \frac{300 \text{ hp}}{40.313 \text{ Btu/lbm}} = 316 \text{ lb}_m\text{/min}}. \quad (27)$$

Now determine the mass flow rate of the cooling water for the boiler. Apply a control volume around the boiler and apply the 1st Law,

$$\frac{dE_{CV}}{dt} = \sum_{in} \dot{m}(h + ke + pe) - \sum_{out} \dot{m}(h + ke + pe) + \dot{Q}_{in} + \dot{W}_{in}, \quad (28)$$

where,

$$\frac{dE_{CV}}{dt} = 0 \quad \text{(assuming steady state operation)}, \quad (29)$$

$$\sum_{in} \dot{m}(h + ke + pe) - \sum_{out} \dot{m}(h + ke + pe) = \dot{m}(h_4 - h_1) + \dot{m}_{b,sw}(h_5 - h_6), \quad (30)$$

(neglecting kinetic and potential energy changes; from COM $\dot{m}_4 = \dot{m}_1 = \dot{m}$ and $\dot{m}_5 = \dot{m}_6 = \dot{m}_{b,sw}$)

$$\dot{Q}_{in} = 0 \quad \text{(assuming adiabatic operation)}, \quad (31)$$

$$\dot{W}_{in} = 0 \quad \text{(the device is passive)}. \quad (32)$$

Substitute and solve for the boiler seawater mass flow rate,

$$0 = \dot{m}(h_4 - h_1) + \dot{m}_{b,sw}(h_5 - h_6), \quad (33)$$

$$\dot{m}_{b,sw} = \dot{m}\left(\frac{h_4 - h_1}{h_6 - h_5}\right). \quad (34)$$

The mass flow rate for the cycle was found in Eq. (27). The specific enthalpies for States 4 and 1 are given in the table at the start of this solution. Not enough information is given to determine the specific enthalpies for States 5 and 6 using the compressed liquid approximation; however, since the temperature is small and seawater can be reasonably assumed to be incompressible, let,

$$\Delta h = \Delta u + v\Delta p = \Delta u = c\Delta T. \quad (35)$$

where $\Delta p = 0$ since the pressure of the surrounding seawater at the inlet and outlet to the boiler is approximately the same. The specific heat for seawater is found from a property table to be c = 0.999 Btu/(lbm.ºR). Using T_5 = 130 ºF and T_6 = 125 ºF along with the previously determined values,

$$\boxed{\dot{m}_{b,sw} = 33000 \text{ lb}_m\text{/min}}.$$

Performing a similar 1st Law analysis for a CV surrounding the condenser, but with c = 1.005 Btu/(lbm.ºR),

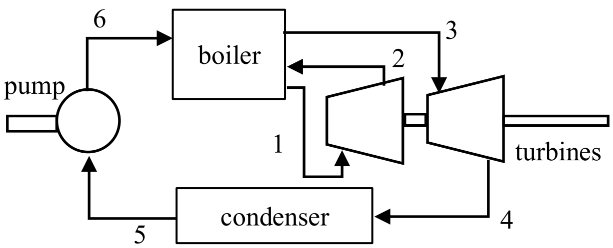
$$\dot{m}_{c,sw} = \dot{m}\left(\frac{h_2 - h_3}{h_8 - h_7}\right), \quad (36)$$

$$\boxed{\dot{m}_{c,sw} = 30300 \text{ lb}_m\text{/min}}.$$

The efficiency of this power plant is small (7.71%). Even for an ideal Carnot cycle the efficiency is small (13.6%). The small cycle thermal efficiency coupled with the large mass flow rates required for pumping the seawater (decreasing the net power out of the cycle even further) and the material and structural costs for operating in corrosive seawater make for a weak incentive to construct and operate this powerplant from a financial point of view.

Consider a vapor power cycle with reheat where the working fluid is water. The pump and turbines operate adiabatically. At the exit of both turbines, the water exits as saturated water vapor. The mass flow rate through the system is 2.1 kg/s.

a. Determine the net power developed by the cycle, in kW.
b. Determine the thermal efficiency of the power cycle.
c. Sketch the cycle on a *T-s* plot, indicating states, paths, and isobars. You needn't include numerical values for the properties.

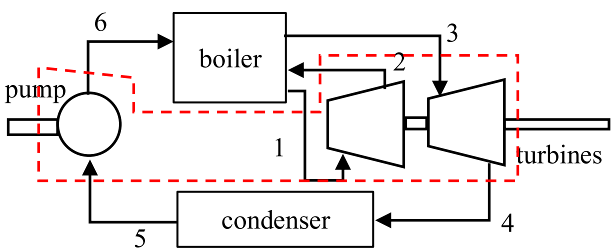


State	*p* [bar (abs)]	*h* [kJ/kg]	*x*
1	160	3355.6	-
2	15	2791.0	1
3	15	3169.8	-
4	1.5	2693.1	1
5	1.5	466.97	0
6	160	486.74	-

SOLUTION:
Applying Conservation of Mass to individual control volumes surrounding each component and assuming steady flow gives,

$$\dot{m} = \dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \dot{m}_4 = \dot{m}_5 = \dot{m}_6. \quad (1)$$

Apply the 1st Law to a control volume that surrounds both turbines and the pump.



$$\frac{dE_{CV}}{dt} = \dot{Q}_{into\ CV} - \dot{W}_{by\ CV} + \sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right), \quad (2)$$

where,

$$\frac{dE_{CV}}{dt} = 0 \text{ (Assuming steady state operation.)}, \quad (3)$$

$$\dot{Q}_{into\ CV} = 0 \text{ (Assuming adiabatic operation.)}, \quad (4)$$

$$\sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) = \dot{m}(h_5 - h_6 + h_1 - h_2 + h_3 - h_4). \quad (5)$$

(The changes in kinetic and potential energies are assumed to be negligibly small.)

Substitute and simplify,

$$\dot{W}_{by\ CV} = \dot{m}(h_5 - h_6 + h_1 - h_2 + h_3 - h_4), \quad (6)$$

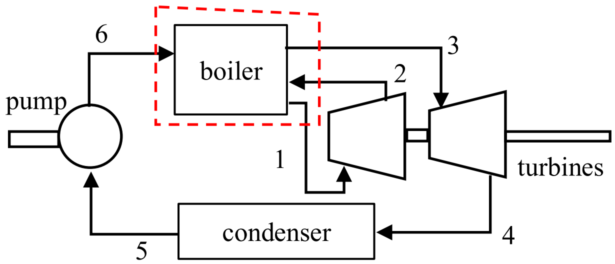
Using the given data,

$\boxed{\dot{W}_{by\ CV} = 2150 \text{ kW}}$.

The thermal efficiency of the power cycle is given by,

$$\eta \equiv \frac{\dot{W}_{net}}{\dot{Q}_{into}}. \quad (7)$$

To find the heat added to the power cycle, apply the 1st Law to a control volume that surrounds the boiler,



$$\frac{dE_{CV}}{dt} = \dot{Q}_{into\ CV} - \dot{W}_{by\ CV} + \sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right), \quad (8)$$

where,

$$\frac{dE_{CV}}{dt} = 0 \text{ (Assuming steady state operation.)}, \quad (9)$$

$$\dot{W}_{by\ CV} = 0 \text{ (The boiler is a passive device.)}, \quad (10)$$

$$\sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) = \dot{m}(h_6 - h_1 + h_2 - h_3). \quad (11)$$

(The changes in kinetic and potential energies are assumed to be negligibly small.)

Substitute and simplify,

$$\dot{Q}_{into\ CV} = \dot{m}(h_1 - h_6 + h_3 - h_2), \quad (12)$$

Using the given data,

$\underline{\dot{Q}_{into\ CV} = 6820\text{ kW}}$.

Substituting values into Eq. (7) gives the power cycle thermal efficiency,

$\boxed{\eta = 0.315}$.