### 4.7.1. Component Efficiencies

The Entropy Equation can be used to determine the entropy change expected during a given process. In particular, it can be used to calculate the entropy change expected for a system undergoing a process that is internally reversible, i.e., a process that is ideal and, thus, having the largest efficiency. For example, we can apply the Entropy Equation and the First Law together to calculate the efficiencies for various thermo-fuid components, such as compressors, turbines, and nozzles.

#### 4.7.1.1. Compressor Efficiency

The efficiency of a compressor is defined as,

$$\eta_{\text{comp}} \coloneqq \frac{(\dot{W}_{\text{on comp}})_{\text{rev}}}{(\dot{W}_{\text{on comp}})_{\text{actual}}}, \tag{4.143}$$

where the subscripts "rev" and "actual" indicate internally reversible and the actual processes, respectively. Note that the actual power required to operate a compressor will always be larger than or equal to the power required to operate the compressor if the process is internally reversible and, thus, the efficiency will be $\eta_{\text{comp}} \leq 1$.

#### 4.7.1.2. Turbine Efficiency

The efficiency of a turbine is defined as,

$$\eta_{\text{turb}} \coloneqq \frac{(\dot{W}_{\text{by turb}})_{\text{actual}}}{(\dot{W}_{\text{by turb}})_{\text{rev}}}. \tag{4.144}$$

Note that the actual power generated by a turbine will always be less than or equal to the power generated by an internally reversible turbine and, thus, $\eta_{\text{turb}} \leq 1$.

#### 4.7.1.3. Nozzle Efficiency

Since the purpose of a nozzle is to speed up a flow, it is reasonable to define the nozzle efficiency as the ratio of the specific kinetic energy actually produced by the nozzle to the specific kinetic energy that would be produced under internally reversible conditions,

$$\eta_{\text{nozzle}} \coloneqq \frac{(\frac{1}{2}V_{\text{exit}}^2)_{\text{actual}}}{(\frac{1}{2}V_{\text{exit}}^2)_{\text{rev}}}. \tag{4.145}$$

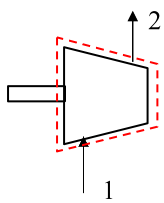
Nozzle efficiencies of 95% or more are common in practice.

Nitrogen ($N_2$) enters an insulated compressor operating at steady state at 1 bar (abs) and 37 °C with a mass flow rate of 1000 kg/h and exits at10 bar (abs). Kinetic and potential energy changes through the compressor are negligible. The nitrogen can be modeled as an ideal gas with a specific heat ratio of 1.391 and a specific heat at constant pressure of 1.056 kJ/(kg.K).

a. Determine the minimum theoretical power input required to operate the compressor and the corresponding exit temperature.
b. If the exit temperature is 397 °C, determine the power input and the compressor efficiency.

SOLUTION:

To find the power required to operate the compressor, apply the First Law to a control volume surrounding the compressor as shown in the following figure.




$$\frac{dE_{CV}}{dt} = \sum_{\text{in}}\left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} - \sum_{\text{out}}\left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} + \dot{Q}_{\substack{\text{into}\\ \text{CV}}} + \dot{W}_{\substack{\text{other,on}\\ \text{CV}}}, \tag{1}$$

where,

$$\frac{dE_{CV}}{dt} = 0 \quad \text{(steady flow assumed)}, \tag{2}$$

$$\sum_{\text{in}}\left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} - \sum_{\text{out}}\left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} = \dot{m}_1 h_1 - \dot{m}_2 h_2 = \dot{m}\left(h_1 - h_2\right), \tag{3}$$

(changes in KE and PE neglected; from COM, the mass flow rates are the same)

$$\dot{Q}_{\substack{\text{into}\\ \text{CV}}} = 0 \quad \text{(adiabatic operation since insulated).} \tag{4}$$

Solving for the power added into the compressor,

$$\dot{W}_{\substack{\text{other,on}\\ \text{CV}}} = \dot{m}\left(h_2 - h_1\right). \tag{5}$$

If we further assume that the nitrogen behaves as a perfect gas, i.e., it has constant specific heats, which is a reasonable assumption if the temperature change is only a few hundred degrees, then,

$$\boxed{\dot{W}_{\substack{\text{other,on}\\ \text{CV}}} = \dot{m}c_p\left(T_2 - T_1\right)}. \tag{6}$$

To calculate the <u>minimum</u> power required to operate the compressor, assume reversible operation. Since the flow is then adiabatic and reversible, it will also be isentropic. For isentropic operation of a perfect gas,

$$\frac{p_2}{p_1} = \left(\frac{T_{2s}}{T_1}\right)^{\frac{k}{k-1}} \Rightarrow T_{2s} = T_1\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}, \tag{7}$$

where the subscript "*s*" has been added to the temperature at state 2 to indicate isentropic conditions. Using the given parameters,

$T_1$ = 37 °C = 310 K,
$p_2$ = 10 bar (abs),
$p_1$ = 1 bar (abs),
$k$ = 1.391,
$\Rightarrow T_{2s}$ = 592 K (= 319 °C)

Substituting into Eq. (6) gives,

$$\boxed{\left(\dot{W}_{\substack{\text{other,into}\\ \text{CV}}}\right)_{\min} = 82.8\text{ kW}},$$

with $c_p$ = 1.056 kJ/(kg.K).

Using the actual measured temperature of $T_2$ = 397 °C = 670 K,

$$\boxed{\dot{W}_{\substack{\text{other,into}\\ \text{CV}}} = 106\text{ kW}.}$$

The efficiency of the compressor is given by,

$$\boxed{\eta = \frac{\left(\dot{W}_{\substack{\text{other,into}\\ \text{CV}}}\right)_{\min}}{\dot{W}_{\substack{\text{other,into}\\ \text{CV}}}} = 0.78.}$$

If we assume ideal, rather than perfect, gas behavior, then outlet temperature corresponding to an isentropic process is found using,

$$s_2 - s_1 = 0 = s_2^0\left(T_{2s}\right) - s_1^0\left(T_1\right) - R\ln\left(\frac{p_2}{p_1}\right) \Rightarrow \bar{s}_2^0\left(T_{2s}\right) = \bar{s}_1^0\left(T_1\right) + \bar{R}_u \ln\left(\frac{p_2}{p_1}\right), \tag{8}$$

with,

$\bar{s}_1^0$ ($T_1$ = 310 K) = 192.638 kJ/(kmol.K) (Table A-23 in Moran et al., 8th ed.),

$p_2/p_1$ = (10 bar)/(1 bar) = 10,

$\bar{R}_u$ = 8.314 kJ/(kmol.K),

=> $\bar{s}_2^0$ = 211.78 kJ/(kmol.K) => $\boxed{T_{2s} = 594\text{ K}}$ (interpolating in Table A-23).

This result is less than 1% different from the one found earlier assuming perfect gas behavior.

Water vapor at 10 MPa (abs) and 600 °C enters a turbine operating at steady state with a volumetric flow rate of 0.36 m$^3$/s and exits at 0.1 bar (abs) and a quality of 92%. Stray heat transfer and kinetic and potential energy changes across the turbine are negligible. Determine for the turbine:

a. the mass flow rate,
b. the power developed by the turbine,
c. the rate at which entropy is produced, and
d. the isentropic turbine efficiency.

SOLUTION:

The mass flow rate through the turbine is given by,

$$\dot{m} = \rho Q \,, \tag{1}$$

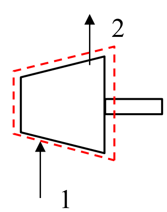
where $\rho$ is the density of the water vapor and $Q$ is the given volumetric flow rate. The water vapor density may be found from the inlet conditions,

$$\rho = 1/v = 26.1 \text{ kg/m}^3, \tag{2}$$

where $v = 0.03837$ m$^3$/kg @ 10 MPa (abs), 600 °C ⇒ superheated vapor (using Table A-4 in Moran et al., 7$^{\text{th}}$ ed., for example). Hence, the mass flow rate is,

$\boxed{\dot{m} = 9.38 \text{ kg/s}}$.

The power generated by the turbine may be found by applying the First Law to a control volume surrounding the turbine as shown in the following figure.




$$\frac{dE_{CV}}{dt} = \sum_{\text{in}} \left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} - \sum_{\text{out}} \left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} + \dot{Q}_{\substack{\text{into}\\ \text{CV}}} + \dot{W}_{\substack{\text{other,on}\\ \text{CV}}} \,, \tag{3}$$

where,

$$\frac{dE_{CV}}{dt} = 0 \quad \text{(steady flow assumed)}, \tag{4}$$

$$\sum_{\text{in}} \left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} - \sum_{\text{out}} \left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} = \dot{m}_1 h_1 - \dot{m}_2 h_2 = \dot{m}\left(h_1 - h_2\right), \tag{5}$$

(changes in KE and PE neglected; from COM, the mass flow rates are the same)

$$\dot{Q}_{\substack{\text{into}\\ \text{CV}}} = 0 \quad \text{(adiabatic operation since insulated).} \tag{6}$$

Solving for the power generated by the turbine,

$$\dot{W}_{\substack{\text{other,on}\\ \text{CV}}} = \dot{m}\left(h_2 - h_1\right). \tag{7}$$

The specific enthalpies may be found using thermodynamic property tables,

$h_1 = 3625.3$ kJ/kg (@ 10 MPa (abs), 600 °C ⇒ superheated vapor; using Table A-4 in Moran et al., 7$^{\text{th}}$ ed.)

$h_2 = x_2 h_{2v} + (1 - x_2)h_{2l} = 2393.3$ kJ/kg with $h_{2v} = 2584.7$ kJ/kg and $h_{2l} = 191.83$ kJ/kg (@ 0.1 bar (abs), $x_2 = 0.92$ ⇒ two-phase, liquid-vapor state; using Table A-3 in Moran et al., 7$^{\text{th}}$ ed.)

Using the given parameters,

$\boxed{\dot{W}_{\substack{\text{other,on}\\ \text{CV}}} = \text{-11.6 MW}}$ (work is being done by the turbine).

The rate at which entropy is produced is found by applying the entropy equation to the same control volume,

$$\frac{dS_{\text{CV}}}{dt} = \sum_{\text{in}} s\dot{m} - \sum_{\text{out}} s\dot{m} + \int_b \frac{\dot{Q}_{\text{into}}}{T} + \dot{\sigma} \,, \tag{8}$$

where,

$$\frac{dS_{CV}}{dt} = 0 \quad \text{(steady flow)}, \tag{9}$$

$$\sum_{\text{in}} s\dot{m} - \sum_{\text{out}} s\dot{m} = \dot{m}\left(s_1 - s_2\right) \quad \text{(the mass flow rate is constant from COM)}, \tag{10}$$

$$\int_b \frac{\dot{Q}_{\text{into}}}{T} = 0 \quad \text{(adiabatic operation)}, \tag{11}$$

$$\Rightarrow \quad \dot{\sigma} = \dot{m}\left(s_2 - s_1\right). \tag{12}$$

The specific entropies may be found using thermodynamic property tables,

$s_1 = 6.9029$ kJ/(kg.K) (@ 10 MPa (abs), 600 °C ⇒ superheated vapor; using Table A-4 in Moran et al., 7th ed.)

$s_2 = x_2 s_{2v} + (1 - x_2)s_{2l} = 7.5501$ kJ/(kg.K) with $s_{2v} = 8.1502$ kJ/(kg.K) and $s_{2l} = 0.6493$ kJ/(kg.K) (@ 0.1 bar (abs), $x_2 = 0.92$ ⇒ two-phase, liquid-vapor state; using Table A-4 in Moran et al., 7th ed.)

Using the given parameters,

$\boxed{\dot{\sigma} = 6.07 \text{ kW/K}}$. Note that the positive value indicates that the process is internally irreversible.

The isentropic efficiency of the turbine is defined as,

$$\eta \equiv \frac{\dot{W}_{\substack{\text{other,into}\\ \text{CV}}}}{\left(\dot{W}_{\substack{\text{other,into}\\ \text{CV}}}\right)_{\max}} \,, \tag{13}$$

where the maximum power generated by the turbine may be found assuming isentropic operation,

$$s_2 = s_1 = 6.9029 \text{ kJ/(kg.K)}. \tag{14}$$

Since this specific entropy falls between $s_{2l} = 0.6493$ kJ/(kg.K) and $s_{2v} = 8.1502$ kJ/(kg.K) at $p_2 = 0.1$ bar (abs), state 2 for the isentropic case is in the two-phase, liquid-vapor region with a quality given by,

$$s_{2s} = x_{2s}s_{2v} + \left(1 - x_{2s}\right)s_{2l} \Rightarrow x_{2s} = \frac{s_{2s} - s_{2l}}{s_{2v} - s_{2l}} \,, \tag{15}$$

$\Rightarrow x_{2s} = 0.833$,

where the subscript "$s$" indicates the conditions for the isentropic case.

The specific enthalpy for this case is,

$$h_{2s} = x_{2s}h_{2v} + \left(1 - x_{2s}\right)h_{2l} \,, \tag{16}$$

$\Rightarrow h_{2s} = 2186.8$ kJ/kg,

$$\Rightarrow \left(\dot{W}_{\substack{\text{other,into}\\ \text{CV}}}\right)_{\max} = -13.5 \text{ MW}, \quad \text{(from Eq. (7))}$$

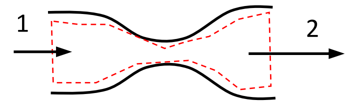
$\Rightarrow \boxed{\eta = 0.86}$.

Helium gas at 810 °R, 45 psia, and a speed of 10 ft/s enters an insulated nozzle operating at steady state and exits at 670 °R, 25 psia. Modeling helium as an ideal gas with a specific heat ratio of 1.67, determine:

a. the speed at the nozzle exit, in ft/s,
b. the isentropic nozzle efficiency, and
c. the rate of entropy production within the nozzle, in Btu/(lb$_m$.°R).

SOLUTION:
Apply the First Law to a control volume surrounding the nozzle,

$$\frac{dE_{CV}}{dt} = \sum_{\text{in}}\left(h+\tfrac{1}{2}V^2+gz\right)\dot{m} - \sum_{\text{out}}\left(h+\tfrac{1}{2}V^2+gz\right)\dot{m} + \dot{Q}_{\text{into,CV}} + \dot{W}_{\text{other,on CV}}, \tag{1}$$




where,

$$\frac{dE_{CV}}{dt} = 0 \quad \text{(assumed steady flow)}, \tag{2}$$

$$\sum_{\text{in}}\left(h+\tfrac{1}{2}V^2+gz\right)\dot{m} - \sum_{\text{out}}\left(h+\tfrac{1}{2}V^2+gz\right)\dot{m} = \dot{m}\left[\left(h_1-h_2\right)+\tfrac{1}{2}\left(V_1^2-V_2^2\right)\right], \tag{3}$$

(since the flow is steady, conservation of mass states that the mass flow rate will remain constant; also assuming that the change in potential energy for the gas is negligible when compared to the change in specific enthalpy and specific kinetic energy)

$$\dot{Q}_{\text{into,CV}} = 0 \quad \text{(assumed adiabatic)}, \tag{4}$$

$$\dot{W}_{\text{other,on CV}} = 0 \quad \text{(no work other than pressure work)}. \tag{5}$$

Thus,

$$\dot{m}\left[\left(h_1-h_2\right)+\tfrac{1}{2}\left(V_1^2-V_2^2\right)\right] = 0 \Rightarrow V_2 = \sqrt{V_1^2 + 2\left(h_1-h_2\right)}. \tag{6}$$

Since helium is a noble gas, its specific heat won't change with temperature. Hence, a perfect gas assumption can be used and Eq. (6) becomes,

$$V_2 = \sqrt{V_1^2 + 2c_p\left(T_1-T_2\right)}. \tag{7}$$

Using the given data,

$V_1$ = 10 ft/s,
$T_1$ = 810 °R,
$T_2$ = 670 °R,

$c_p = \dfrac{kR}{k-1} = \dfrac{k}{k-1}\left(\dfrac{\overline{R}_u}{M}\right)$ with $k$ = 1.67, $\overline{R}_u$ = 1545.4 ft.lb$_f$/(lbmol.°R), $M$ = 4.003 lb$_m$/lbmol

=> $R$ = 386.1 ft.lb$_f$/(lb$_m$.°R), $c_p$ = 962.3 ft.lb$_f$/(lb$_m$.°R) (Note: 1 lb$_f$ = 32.2 lb$_m$.ft/s$^2$) (8)

=> $\boxed{V_2 = 2950\text{ ft/s}}$.

The isentropic nozzle efficiency is defined as,

$$\eta_{\text{nozzle}} = \frac{\left(\tfrac{1}{2}V^2\right)_{\text{actual}}}{\left(\tfrac{1}{2}V^2\right)_{\text{ideal}}}. \tag{9}$$

The ideal specific kinetic energy may be found by assuming an internally reversible process. Since the flow is both internally reversible and adiabatic, it is also isentropic so $s_2 = s_1$. Since helium is a perfect gas,

$$\underbrace{\Delta s}_{=0} = c_p \ln\left(\frac{T_{2s}}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right) \Rightarrow \frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{R}{c_p}}, \tag{10}$$

=> $T_{2s}$ = 640 °R,
=> $V_{2s}$ = 3250 ft/s,
=> $\boxed{\eta_{\text{nozzle}} = 0.823}$.

The rate of entropy production is found from,

$$\dot{m}\Delta s = \underbrace{\int_b \frac{\dot{Q}_{\text{into}}}{T}}_{=0\ (\text{adiabatic})} + \dot{\sigma} \Rightarrow \frac{\dot{\sigma}}{\dot{m}} = \Delta s\ , \tag{11}$$

where, again, because helium is a perfect gas,

$$\Delta s = c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right), \tag{12}$$

=> $\boxed{\dot{\sigma}/\dot{m} = \Delta s = 44.3\ \text{ft}^2/(\text{s}^2.°\text{R})}$.