# 3.8. Common Thermodynamic Cycles

Important: The reader should read the Conservation of Mass, First Law, and Second Law sections of Chapter 4 prior to reading this section since this section involves applying these relations to control volumes, which have mass crossing the control volume boundaries.

In this section we review several thermodynamic cycles encountered in practice, including:

- the Carnot cycle (not common in practice, but real cycles are often compared to this ideal cycle),
- the Rankine cycle and improvements to the Rankine cycle,

These are covered later in the semester.

the Otto, Diesel, and dual cycles, and
the Brayton cycle and improvements to the Brayton cycle.

# 3.8.1. Carnot Cycle

The Carnot cycle is one particular type of internally reversible cycle and serves as a point of comparison for other real and internally reversible cycles. Carnot cycles include power, refrigeration, and heat pump cycles. This section, however, focuses specifically on Carnot power cycles. The Carnot refrigeration and heat pump cycles are similar to the power cycle, but operate in the opposite direction.

The Carnot power cycle consists of the following four internally reversible processes:

- (1) Process 1 2: Isothermal expansion at  $T_H$ .
- (2) Process 2 3: Adiabatic expansion  $(Q_{23} = 0)$ .
- (3) Process 3 4: Isothermal compression at  $T_C$ .
- (4) Process 4 1: Adiabatic compression  $(Q_{41} = 0)$ .

As mentioned previously, a Carnot refrigeration or heat pump cycle would operate in reverse, i.e., Process 1 - 2 would be an isothermal compression, Process 2 - 3 would be an adiabatic compression, etc.

Notes:

- (1) Figure 3.39 illustrates the four Carnot power cycle processes for a piston-cylinder arrangement. The corresponding processes are shown on the p-v and T-s diagrams for a substance remaining entirely in the vapor phase throughout the cycle. Note that Carnot cycles can also involve working fluids that change phase.
- (2) One can also have a Carnot cycle involving a system of components, as shown in Figure 3.40. The same four processes described at the beginning of this section occur in this system. In the particular case shown in the  $p \cdot v$  and  $T \cdot s$  plots, the working fluid is a saturated liquid vapor mixture.
- (3) Since each of the processes in a Carnot cycle is internally reversible, the entire Carnot cycle is also internally reversible. Furthermore, in a Carnot cycle, the absolute temperatures at which the energy is added to or removed from the working fluid via heat transfer are  $T_H$  and  $T_C$ . Hence, the efficiency and coefficients of performance of a Carnot cycle are given by,

$$\eta_{\rm rev} = 1 - \frac{T_C}{T_H},\tag{3.189}$$

$$COP_{\rm ref, rev} = \frac{T_C}{T_H - T_C},\tag{3.190}$$

$$COP_{\rm HP,rev} = \frac{T_H}{T_H - T_C}.$$
(3.191)

(4) In order to be internally reversible, the heat addition and removal processes with the thermal reservoirs must not occur over a finite temperature gradient, i.e., the temperature of the system must equal the temperature of the thermal reservoir during the heat addition and heat removal processes. In a real system, there must be some finite temperature difference to drive the heat transfer process, which is one reason a real cycle would not be reversible. Similarly, in order to be internally reversible, the working fluid must have no viscosity, there should be no friction in the system, and all of the processes must be in quasi-equilibrium.



FIGURE 3.39. A sketch of the four processes of a Carnot power cycle for a piston-cylinder system. The corresponding processes are sketched on p - v and T - s plots for a working fluid that remains entirely in the vapor phase throughout the cycle.

(5) It's possible for a system to have the same set of processes as the Carnot cycle (isothermal expansion, adiabatic expansion, isothermal compression, adiabatic compression), but not be reversible. For example, the heat transfer may occur over a finite temperature difference or the working fluid may be viscous. In that case, the efficiency of the cycle will be less than the Carnot efficiency.



Carnot Power Cycle shown for a substance that is a SLVM

FIGURE 3.40. A sketch of a Carnot power cycle and p - v, and T - s plots for a simple vapor power cycle in which the working fluid remains as a saturated liquid-vapor mixture.

An ideal gas within a piston-cylinder assembly undergoes a Carnot refrigeration cycle. The isothermal compression occurs at 325 K from 2 bar (abs) to 4 bar (abs). The isothermal expansion occurs at 250 K. Determine:

- a. the coefficient of performance,
- b. the heat transfer to the gas during the isothermal compression, in kJ per kmol of gas,
- b. the heat transfer to the gas during the isothermal expansion, in kJ per kmol of gas, and
- c. the magnitude of the net work input over the cycle, in kJ per kmol of gas.



Since a Carnot cycle is reversible, the coefficient of performance is,

$$\operatorname{COP}_{\operatorname{ref}} = \frac{T_C}{T_H - T_C} , \qquad (1)$$

with 
$$T_H$$
 = 325 K and  $T_C$  = 250 K =>  $COP_{ref, rev}$  = 3.33.

The heat transfer to the gas during isothermal compression (process 3-4) can be found using the 1<sup>st</sup> Law for the process. From the 1<sup>st</sup> Law applied to the system identified in the figure, specifically for the process from 3 to 4,

$$\Delta E_{\text{sys},34} = Q_{\text{into sys},34} + W_{\text{on sys},34} \Longrightarrow Q_{\text{into sys},34} = W_{\text{by sys},34} , \qquad (2)$$

where  $\Delta E_{sys,34} = 0$  since we're assuming no change in kinetic or potential energy during the process, and since the process is isothermal and we're working with an ideal gas in which u = u(T),  $\Delta U_{34} = 0$ . The work done by the system during this compression is,

$$W_{\text{by sys,34}} = \int_{V_3}^{V_4} p \, dV = \int_{V_3}^{V_4} \frac{mRT_H}{V} \, dV = mRT_H \int_{V_3}^{V_4} \frac{dV}{V} = mRT_H \ln\left(\frac{V_4}{V_3}\right) = mRT_H \ln\left(\frac{P_3}{P_4}\right), \tag{3}$$

where the ideal gas law has been used a couple of times (pV = mRT) couple with the fact that the process is isothermal with  $T = T_H$  and m = constant. Since we're asked to find the heat in kJ per kmol, re-write Eq. (3) on a per mole basis (recall m = n/M and  $R = MR_u$ , where M is molecular weight),

$$W_{\rm by\,sys,34} = \left(n/M\right) \left(M\overline{R}_u\right) T_H \ln\left(\frac{p_3}{p_4}\right) \Longrightarrow \frac{W_{\rm by\,sys,34}}{n} = \overline{R}_u T_H \ln\left(\frac{p_3}{p_4}\right). \tag{4}$$

Combining Eqs. (4) and (2) gives,

$$\frac{Q_{\text{into sys,34}}}{n} = \overline{R}_u T_H \ln\left(\frac{p_3}{p_4}\right).$$
(5)

Using the given data,

 $R_u = 8.314 \text{ kJ/(kmol.K)}$ 

 $p_3 = 2$  bar (abs)

 $p_4 = 4$  bar (abs)

=>  $Q_{into sys,34}/n = -1870 \text{ kJ/kmol}$  (heat is leaving the system during this compression process)

To determine the heat transfer during isothermal expansion, make use of the definition of the absolute temperature scale,

$$\frac{Q_C}{Q_H}\Big|_{\substack{\text{rev.}\\\text{cycle}}} = \frac{T_C}{T_H} \Longrightarrow Q_C = Q_H \frac{T_C}{T_H} \Longrightarrow Q_{12} = -Q_{34} \frac{T_C}{T_H} , \qquad (6)$$

where  $Q_C = Q_{into,12}$  and  $Q_H = -Q_{into,34}$ . Using the given data, =>  $Q_{into,12}/n = 1440 \text{ kJ/kmol.}$ 

The magnitude of the net work input over the cycle may be found using the definition of the coefficient of performance for a refrigeration cycle,

$$COP_{\rm ref} \equiv \frac{Q_C}{W_{\rm on \ sys, net}} \Longrightarrow W_{\rm on \ sys, net} = \frac{Q_C}{COP_{\rm ref}} \,. \tag{7}$$

Using the given data, =>  $W_{\text{on sys,net}}/n = 433 \text{ kJ/kmol}$ .

Two kilograms of air within a piston-cylinder assembly execute a Carnot power cycle with maximum and minimum temperatures of 750 K and 300 K, respectively. The heat transfer to the air during the isothermal expansion is 60 kJ. At the end of the isothermal expansion the volume is 0.4 m<sup>3</sup>. Assuming the ideal gas model for the air, determine:

- a. the thermal efficiency,
- b. the pressure and volume at the beginning of the isothermal expansion, in kPa (abs) and m<sup>3</sup>, respectively,
- c. the work and heat transfer for each of the four processes, in kJ, and
- d. sketch the cycle on a *p*-*V* diagram.

SOLUTION:



Since the cycle is a Carnot power cycle, which is reversible, the efficiency is given by,

$$\eta_{\rm rev} = 1 - \frac{T_C}{T_H} \,. \tag{1}$$

Using the given temperatures ( $T_c$  = 300 K and  $T_H$  = 750 K) =>  $\eta_{rev}$  = 0.6.

The pressure and volume at the beginning of the isothermal expansion (state 1) may be found combining the  $1^{st}$  Law applied to the system shown in the figure, with the given heat transfer, and the definition of the power cycle efficiency. Applying the  $1^{st}$  Law to the system for the process from 1 to 2,

$$\Delta E_{12} = Q_{\text{into},12} + W_{\text{on},12} \Longrightarrow W_{\text{by},12} = Q_{\text{into},12} , \qquad (2)$$

where  $W_{by,12} = -W_{on,12}$ . Furthermore,  $\Delta E_{12} = 0$  since the change in kinetic and potential energy in going from state 1 to state 2 is zero, and since the process is isothermal and the system is an ideal gas (u = u(T)) =>  $\Delta U_{12} = 0$ . The heat added to the system in going from 1 to 2 is given (= 60 kJ) and thus the work done by the system is also known. The work done by the system may also be found using,

$$W_{\rm by,12} = \int_{V_1}^{V_2} p \, dV = \int_{V_1}^{V_2} \frac{mRT_H}{V} \, dV = mRT_H \int_{V_1}^{V_2} \frac{dV}{V} = mRT_H \ln\left(\frac{V_2}{V_1}\right),\tag{3}$$

where the ideal gas law has been used (pV = mRT) along with the fact that the process from 1 to 2 is isothermal with  $T = T_{H}$ . The mass is constant too. Since the volume at the end of the isothermal expansion process is given ( $V_2 = 0.4 \text{ m}^3$ ), we can solve for  $V_1$ ,

$$W_{\rm by,12} = mRT_H \ln\left(\frac{V_2}{V_1}\right) \Longrightarrow V_1 = V_2 \exp\left(-\frac{W_{\rm by,12}}{mRT_H}\right),\tag{4}$$

with,

 $V_2 = 0.4 \text{ m}^3,$   $W_{by,12} = 60 \text{ kJ},$  m = 2 kg,  $R_{air} = 0.287 \text{ kJ/(kg.K)},$   $T_H = 750 \text{ K},$  $=> V_1 = 0.348 \text{ m}^3.$  The pressure at state 1 may be found using the ideal gas law,

$$p_{1} = \frac{mRT_{H}}{V_{1}} \quad (T_{1} = T_{H}),$$

$$\Rightarrow p_{1} = 1.24 \text{ MPa (abs)}.$$
(5)

The work and heat for process 1-2 have already been calculated. The heat for processes 2-3 and 4-1 are zero, i.e.,  $Q_{23} = Q_{41} = 0$ , since these processes are adiabatic.

The work done on the system during process 2-3 may be found using the 1<sup>st</sup> Law,

$$\Delta E_{23} = Q_{into,23} + W_{on,23} \Longrightarrow W_{on,23} = \Delta U_{23} = m(u_3 - u_2),$$
(6)

where  $u_2 = u_2(T_2 = T_H)$  and  $u_3 = u_3(T_3 = T_C)$  since the air is being treated as an ideal gas. These specific internal energies may be looked up in a table, e.g., Table A-22 from Moran et al., 8<sup>th</sup> ed.,

$$u_2 = 551.99 \text{ kJ/kg}$$
 and  $u_3 = 214.07 \text{ kJ/kg} => W_{on,23} = \Delta U_{23} = 676 \text{ kJ}$ . (7)  
If a perfect gas model is used, then,

$$\Delta U_{23} = mc_v(T_3 - T_2) \Rightarrow \Delta U_{23} = 673 \text{ kJ},$$
(8)

where  $c_v = 0.748 \text{ kJ/(kg.K)}$  (Table A-20, Moran et al., 8<sup>th</sup> ed.). This result is less than 1% different than the ideal gas result.

The work done on the system during process 4-1 will be identical in magnitude, but opposite in sign to the work done on the system during process 2-3 since both processes are adiabatic and operate between the same temperatures. Hence,  $W_{on,41} = -676$  kJ.

For process 3-4, the heat transferred into the system may be found using the definition of the absolute temperature scale (since the process is assumed reversible),

$$\frac{Q_C}{Q_H}\Big|_{\substack{\text{rev.}\\\text{cycle}}} = \frac{T_C}{T_H} \Rightarrow Q_C = Q_H\left(\frac{T_C}{T_H}\right) \Rightarrow Q_{\text{out of, 34}} = Q_{\text{into, 12}}\left(\frac{T_C}{T_H}\right),\tag{9}$$

Using the given data,  $Q_{out of, 34} = 24 \text{ kJ}$ .

Since the process from 3 to 4 is isothermal, we're dealing with an ideal gas, and there is no change in kinetic or potential energy, the 1<sup>st</sup> Law gives,

$$\Delta E_{34} = Q_{\text{into},34} + W_{\text{on},34} \Longrightarrow W_{\text{on},34} = Q_{\text{out of},34} ,$$

$$=> W_{\text{on},34} = 24 \text{ kJ}.$$
(10)

As a check, we can substitute the heat and work values into the definition for the power cycle efficiency,

$$\eta = \frac{W_{\rm by\,sys,net}}{Q_{\rm into\,sys,12}},$$
(11)

where  $W_{by sys,net} = W_{by,12} + W_{by,23} + W_{by,34} + W_{by,41} = (60 \text{ kJ}) + (676 \text{ kJ}) + (-24 \text{ kJ}) + (-676 \text{ kJ}) = 36 \text{ kJ},$ =>  $\eta = 0.6$ , which is exactly the value expected from part (a).

Also, from the 1<sup>st</sup> Law applied to the entire cycle (recall  $\Delta E_{cycle} = 0$ ), we must have,

$$Q_{\rm into,net} = W_{\rm by,net}$$

where  $Q_{into,net} = Q_{into,12} + Q_{into,23} + Q_{into,34} + Q_{into,41} = (60 \text{ kJ}) + (0) + (-24 \text{ kJ}) + (0) = 36 \text{ kJ}$ , which is precisely  $W_{by,net}$  found previously.



FIGURE 3.34. Schematic used in proving the Clausius Inequality.

## 3.6.6. Clausius Inequality

It can be shown using the Second Law (given later in this section) that for a system undergoing a thermodynamic cycle,

$$\left(\int_{b} \frac{\delta Q}{T}\right)_{\text{cycle}} \le 0, \tag{3.123}$$

where  $\delta Q_{into}$  is the heat into a system over a small boundary area and T is the absolute temperature at that part of the boundary. The integral over "b" refers to an integral over the system's boundary surface. The equality (=) holds for internally reversible processes (no irreversibilities in the system) and the inequality applies when internal irreversibilities are present (< 0).

Notes:

(1) Equation (3.123) may be also written as,

$$\left(\int_{b} \frac{\delta Q_{\text{into}}}{T}\right)_{\text{cycle}} = -\sigma_{\text{cycle}},\tag{3.124}$$

where,

$$\sigma_{\text{cycle}} > 0$$
 internally irreversible system, (3.125)

 $\sigma_{\text{cycle}} = 0$  internally reversible system, (3.126)

$$\sigma_{\rm cycle} < 0$$
 impossible (violates the Second Law). (3.127)

The parameter  $\sigma_{\text{cycle}}$  is the entropy produced by irreversibilities in the system over the cycle.

The proof for the Clausius Inequality is as follows. Consider Figure 3.34, which consists of a reservoir with absolute temperature  $T_R$ , an intermediate system, and a main system. The main system receives energy with magnitude  $\delta Q$  via heat transfer at a location on the system boundary where the absolute temperature is T. The main system uses this energy to perform work  $\delta W$ . In order to avoid potential irreversibility caused by heat transfer across a finite temperature difference (from  $T_R$  to T), the energy transfer occurs through an intermediate system, which operates reversibly. This intermediate system receives energy  $\delta Q'$  from the reservoir, produces work  $\delta W'$ , and discharges the energy  $\delta Q$ , which goes into the main system. Since this intermediate system operates reversibly, we have from the definition of the absolute temperature scale (refer to Section 3.6.5),

$$\frac{\delta Q'}{\delta Q} = \frac{T_R}{T} \implies \delta Q' = T_R \frac{\delta Q}{T}.$$
(3.128)

Although the figure only shows a single intermediate system and a single transfer of energy into the main system, we can imagine additional intermediate systems and energy transfers into the main system at various locations on the main system boundary. Now apply the First Law of Thermodynamics to the combined system shown by the red dashed line in the figure,

$$dE_C = \int_b \delta Q' - \delta W_C, \qquad (3.129)$$

where  $\delta W_C$  is the sum of all the intermediate system works and the main system's work. The boundary integral on the heat transfer term refers to all of the heat transfers from the reservoir into the intermediate systems. Let the main system and the intermediate systems operate over a cycle so that  $dE_C = 0$  and, thus,

$$0 = \int_{b} \delta Q' - \delta W_C. \tag{3.130}$$

Note that the subscript "cycle" has not been included for convenience. Substituting Eq. (3.128) and rearranging,

$$\delta W_C = \int_b T_R \frac{\delta Q}{T} = T_R \int_b \frac{\delta Q}{T}.$$
(3.131)

where the reservoir temperature  $T_R$  is moved outside the integral since it is a constant. The combined system operates over a cycle (since its components operate over a cycle) and interacts with a single reservoir and, thus, from the Kelvin-Plank Statement of the Second Law (Eq. (3.103)),

$$W_C \le 0 \implies \int_b \frac{\delta Q}{T} \le 0,$$
 (3.132)

which is the same as Eq. (3.123). Recall that the "<" occurs when the combined system has internal irreversibilities and the "=" occurs when the combined system is internally reversible. Since the intermediate systems are reversible, if there are any irreversibilities, they must occur in the main system.

A system executes a power cycle while receiving 1050 kJ by heat transfer at a temperature of 525 K and discharging 700 kJ by heat transfer at 350 K. There are no other heat transfers.

- a. Determine if the cycle is internally reversible, irreversible, or impossible.
- b. Determine the thermal efficiency. Compare this value with the maximum possible efficiency.

#### SOLUTION:

To determine if the cycle is internally reversible, irreversible, or impossible, consider the Clausius Inequality,

$$\int_{A_{\rm sys}} \frac{\delta Q_{\rm into,cycle}}{T} \le 0, \qquad (1)$$

where,

$$\begin{aligned} Q_{H}/T_{H} &= (1050 \text{ kJ})/(525 \text{ K}) = 2 \text{ kJ/kg}, \\ Q_{C}/T_{C} &= (700 \text{ kJ})/(350 \text{ K}) = 2 \text{ kJ/kg}, \\ \Rightarrow \left(\int_{b} \frac{\delta Q_{\text{into}}}{T}\right)_{\text{cycle}} = \frac{Q_{H}}{T_{H}} - \frac{Q_{C}}{T_{C}} = 2 \text{ kJ/K} - 2 \text{ kJ/kg} = 0. \end{aligned}$$



(Note  $Q_C < 0$  since heat leaves the system. We're also assuming that the temperature at the boundaries to the system where the heat is added/removes is the same as the adjacent reservoir.)

Thus, we see that the cycle is internally reversible.

The thermal efficiency is,

$$\eta = 1 - \frac{Q_c}{Q_H},$$

$$\Rightarrow \boxed{\eta = 0.33}.$$
The maximum possible efficiency is,
$$(2)$$

$$\eta_{\max} = 1 - \frac{T_c}{T_H},$$

$$\Rightarrow \overline{\eta_{\max} = 0.33}.$$
(3)

The cycle is operating at the maximum possible efficiency since it is internally reversible.

Consider the vapor power plant cycle shown in the figure. The working fluid is water. Water flows through the boiler and condenser at constant pressure and through the turbine and pump adiabatically. Kinetic and potential energy effects can be ignored. The process data are:

- Process 4 1: constant pressure at 1 MPa (abs) from saturated liquid to saturated vapor,
- Process 2 3: constant pressure at 20 kPa (abs) from  $x_2 = 0.88$  to  $x_3 = 0.18$ .
- a. Determine if the cycle is internally reversible, irreversible, or impossible.
- b. Determine the thermal efficiency of the cycle.
- c. Compare the thermal efficiency from (b) to the maximum possible efficiency.



### SOLUTION:

First sketch the cycle on a p-v diagram for convenience.



 $T_{41} = 179.9^{\circ}\text{C} = 453.05 \text{ K}$  (*a*) 1 MPa = 10 bar (abs),

Using the thermodynamic property tables for water in a saturated state (e.g., Table A-3 in Moran et al., 7<sup>th</sup> ed.):

 $T_{23} = 60.06^{\circ}\text{C} = 333.21 \text{ K}$  @ 20 kPa = 0.2 bar (abs). In addition, also using Table A-3 in Moran et al., 7<sup>th</sup> ed. at the given saturation temperature,  $h_4 = h_l = 762.81 \text{ kJ/kg}$  (saturated liquid state) State 4:  $h_1 = h_y = 2778.1 \text{ kJ/kg}$  (saturated vapor state) State 1: State 2:  $h_2 = x_2 h_{2\nu} + (1 - x_2) h_{2l} = (0.88)(2609.7 \text{ kJ/kg}) + (1 - 0.88)(251.4 \text{ kJ/kg}) = 2326.7 \text{ kJ/kg}$  $h_3 = x_3 h_{3v} + (1 - x_3) h_{3l} = (0.18)(2609.7 \text{ kJ/kg}) + (1 - 0.18)(251.4 \text{ kJ/kg}) = 675.89 \text{ kJ/kg}$ State 3:  $\dot{Q}_{in,41}$ hot res. (a)  $T_H$ boiler  $\dot{Q}_H = \dot{Q}_{in,41}$ turbine pump sys  $\dot{W}_{in,34}$  $\dot{W}_{out,net} = \dot{W}_{out,12} - \dot{W}_{in,34}$ Wout.12  $\dot{Q}_{C} = \dot{Q}_{out,23}$ condenser 3 cold res. (a)  $T_C$ ===  $Q_{out,23}$ 

Applying the 1<sup>st</sup> Law to a control volume surrounding the boiler, assuming steady flow, negligible changes in kinetic and potential energy across the control volume, and no work other than pressure,

$$\dot{Q}_{in,41} = \dot{m}(h_1 - h_4) \Longrightarrow \frac{\dot{Q}_{in,41}}{\dot{m}} = h_1 - h_4.$$
(1)

Similarly, for the condenser,

$$\dot{Q}_{out,23} = \dot{m}(h_2 - h_3) \Longrightarrow \frac{\dot{Q}_{out,23}}{\dot{m}} = h_2 - h_3.$$
 (2)

Apply the 1<sup>st</sup> Law to control volumes surrounding the turbine and pump, assuming steady flow, negligible changes in kinetic and potential energy across the control volume, and adiabatic conditions,

$$\dot{W}_{out,12} = \dot{m}(h_1 - h_2) \Longrightarrow \frac{\dot{W}_{out,12}}{\dot{m}} = h_1 - h_2.$$
 (3)

$$\dot{W}_{in,34} = \dot{m}(h_4 - h_3) \Longrightarrow \frac{\dot{W}_{in,34}}{\dot{m}} = h_4 - h_3.$$
 (4)

Substituting the specific enthalpy values found previously,

$$\frac{\frac{Q_{in,41}}{\dot{m}}}{\frac{\dot{Q}_{out,23}}{\dot{m}}} = 2015.29 \text{ kJ/kg}$$

$$\frac{\dot{Q}_{out,23}}{\dot{m}} = 1650.81 \text{ kJ/kg}$$

$$\frac{\dot{W}_{out,12}}{\dot{m}} = 451.4 \text{ kJ/kg}$$

$$\frac{\dot{W}_{in,34}}{\dot{m}} = 86.92 \text{ kJ/kg}$$

To determine if the cycle is internally reversible, irreversible, or impossible, consider the Clausius Inequality applied to the entire cycle (or, alternately, the Entropy Equation with dS/dt = 0 because the cycle is at steady state),

$$\int_{b} \frac{\delta Q_{into,cycle}}{T} = -\dot{\sigma},\tag{5}$$

where,

$$\int_{b} \frac{\delta \dot{Q}_{into,cycle}}{T} = -\dot{\sigma} = \dot{m} \left( \frac{\dot{Q}_{in,41}/\dot{m}}{T_{H}} - \frac{\dot{Q}_{out,23}/\dot{m}}{T_{C}} \right), \tag{6}$$

$$\frac{\dot{\sigma}}{\dot{m}} = -\left(\frac{2015.29 \,\text{k}]/\text{kg}}{453.05 \,\text{K}} - \frac{1650.81 \,\text{k}]/\text{kg}}{333.21 \,\text{K}}\right),\tag{7}$$

$$\frac{\dot{\sigma}}{\dot{\sigma}} = 0.507 \,\frac{\text{kJ}}{333.21 \,\text{K}}$$

$$\frac{\sigma}{m} = 0.507 \frac{\kappa_{\rm s}}{k_{\rm g,K}}.$$
(8)

Thus, we see that the cycle is internally irreversible. Note that in this analysis it is assumed that the temperatures at which the heat enters and leaves the CV are  $T_H = T_{41}$  and  $T_C = T_{23}$  since we're not given any information about their temperatures. In reality, this would not be the case since there must be some temperature difference between the hot/cold reservoir and the boiler/condenser to drive the heat transfer, i.e.,  $T_H > T_{41}$  and  $T_C < T_{23}$ . If there was a temperature difference between  $T_H$  and  $T_{41}$  and  $T_C$  an  $T_{23}$ , then this would create even more irreversibility due to the larger temperature gradient.

The thermal efficiency of the cycle is,

$$\eta = 1 - \frac{\dot{q}_{c}}{\dot{q}_{H}} = 1 - (1650.81 \text{ kJ/kg})/(2015.29 \text{ kJ/kg})$$

$$\Rightarrow \boxed{\eta = 0.18 = 18\%}.$$
(9)

We could have also calculated the efficiency using the work generated,

$$\eta = \frac{\dot{W}_{out,net}/\dot{m}}{\dot{Q}_{H}/\dot{m}} = \frac{(W_{out,12} - W_{in,34})/\dot{m}}{\dot{Q}_{in,41}/\dot{m}} = \frac{(451.4 \text{ kJ/kg} - 86.92 \text{ kJ/kg})}{2015.29 \text{ kJ/kg}} = 0.18$$
(10)

which is the same result as found previously.

The maximum possible efficiency is,  

$$\eta_{max} = \eta_{int.rev} = 1 - (333.21 \text{ K})/(453.05 \text{ K}),$$

$$\Rightarrow \boxed{\eta_{max} = 0.26 = 26\%}.$$
(11)

Since the cycle is irreversible, the actual efficiency is less than the maximum possible efficiency.