Not part of this reading.

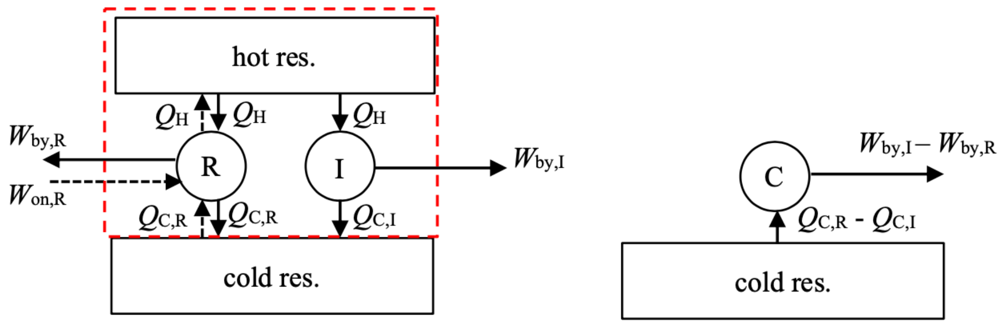


Figure 3.33. Schematic used to prove the first and second corollaries to the Second Law.

there is no net energy transfer to/from it since Q_H is removed by the irreversible system, but then replaced by the reversible one. The energy received by the combined system from the cold reservoir is $Q_{\text{C,R}} - Q_{\text{C,I}} > 0$ and the net work done by the combined system is $W_{\text{by,I}} - W_{\text{by,R}} > 0$. This combined system is shown in the right-hand schematic of Figure 3.33. Since the combined system operates over a cycle and interacts with a single thermal reservoir, from the Kelvin-Plank Statement of the Second Law (Eq. (3.103)),

$$W_{\text{by,I}} - W_{\text{by,R}} < 0 \implies W_{\text{by,I}} < W_{\text{by,R}}. \tag{3.113}$$

Note that the inequality has been used since the combined system includes irreversibilities. Since the thermal efficiency is given by,

$$\eta = \frac{W_{\text{by}}}{Q_H}, \tag{3.114}$$

and Q_H is the same for the irreversible and reversible systems, we conclude that,

$$\eta_R > \eta_I. \tag{3.115}$$

Thus, we have proven the first corollary to the Second Law.

The second corollary can be proven by replacing the irreversible system in Figure 3.33 with a reversible one so that there are two reversible systems (call these "R1" and "R2"). Following the same arguments as before, we will arrive at the following statement using the Kelvin-Plank Statement of the Second Law,

$$W_{\text{by,R2}} - W_{\text{by,R1}} = 0 \implies W_{\text{by,R2}} = W_{\text{by,R1}}. \tag{3.116}$$

Here, the equals sign is used since the combined system is reversible. Since the works and heat transfers are the same, the efficiencies of the two reversible systems must be identical.

Proofs for the third and fourth corollaries are not provided here, but follow similar arguments.

3.6.5. Kelvin Absolute Temperature Scale

Note that from the Second Law Corollaries, the reversible cycle performance measures depend solely on the interaction with the thermal reservoirs, namely (Q_C/Q_H) in Eqs. (3.104) - (3.106) since all reversible cycle efficiencies are identical. Since it is the temperature difference between the reservoirs that drives this heat transfer, we can conclude that,

$$\left.\frac{Q_C}{Q_H}\right|_{\text{rev. cycle}} = fcn\left(\frac{T_C}{T_H}\right), \tag{3.117}$$

where T_C and T_H are the temperatures of the cold and hot reservoirs, respectively. Note that since the left hand side of the equation is dimensionless, the right hand side must also be dimensionless. The function fcn is determined by how we define temperature. In the Kelvin absolute temperature scale, we define the temperatures such that the function is a simple linear one, i.e.,

$$\left.\frac{Q_C}{Q_H}\right|_{\text{rev. cycle}} = \frac{T_C}{T_H}. \tag{3.118}$$

By definition, the ratio of temperatures on the Kelvin scale is equal to the ratio of the heat fluxes. Equation (3.118) only provides a ratio of temperatures; it doesn't actually set a value for the temperature. To complete the thermodynamic scale, we arbitrarily set the value of T on the Kelvin scale at the triple point of water to be,

$$T_{\text{triple pt of H2O}} = 273.16\,\text{K}. \tag{3.119}$$

Notes:

(1) Since the performance of a reversible cycle is independent of the details of the cycle, e.g., working fluid, cycle components, etc., it also means that $(Q_C/Q_H)_{\text{rev,cycle}}$ and thus T_C/T_H are also independent of the details of the cycle. This means that the Kelvin absolute temperature scale is independent of any substance or cycle details.

(2) Since $Q_C > 0$ (to satisfy the Kelvin-Planck statement of the Second Law), it also means that $T_C > 0$. Thus, the minimum temperature limit on the Kelvin scale is zero Kelvin, which can never be reached as stipulated by the Second Law.

(3) We can substitute Eq. (3.118) into Eqs. (3.104) - (3.106) to determine reversible, i.e., ideal, cycle performance measures,

   (a) Power cycle *reversible* thermal efficiency,

$$\eta_{\text{rev}} = 1 - \frac{T_C}{T_H}. \tag{3.120}$$

   (b) Refrigeration cycle *reversible* coefficient of performance,

$$COP_{\text{ref,rev}} = \frac{T_C}{T_H - T_C}. \tag{3.121}$$

   (c) Heat pump cycle *reversible* coefficient of performance,

$$COP_{\text{HP,rev}} = \frac{T_H}{T_H - T_C}. \tag{3.122}$$

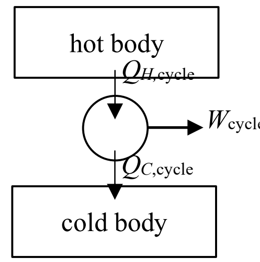
Interestingly, the maximum performance of these cycles is independent of the details of the cycle (design, working materials, etc.). The only factors that matter are the (absolute) temperatures of the thermal reservoirs.

For a power cycle, the maximum efficiency increases as T_H increases or T_C decreases. For example, if a combustion process is used to supply heat to the system, the hotter the combustion gases (T_H), the more efficient the reversible cycle. In most practical power cycles, the cycle discharges heat to the environment so there is often less control over T_C. Similar arguments may be made for refrigeration and heat pump cycles.

(4) We can still calculate the efficiency and COPs of any cycle, reversible or irreversible, using Eqs. (3.104) - (3.106). However, for a reversible cycle, we can also make use of Eqs. (3.120) - (3.122).

An internally reversible power cycle with a thermal efficiency of 40% receives 50 kJ of energy by heat transfer from a hot reservoir at 600 K and rejects energy by heat transfer to a cold reservoir at a temperature T_C. Determine the energy rejected and the temperature T_C.

SOLUTION:



We can determine the heat transfer to the cold reservoir using the power cycle thermal efficiency in terms of the heat transfers,

$$\eta = 1 - \frac{Q_{C,\text{cycle}}}{Q_{H,\text{cycle}}} \Rightarrow Q_{C,\text{cycle}} = (1-\eta) Q_{H,\text{cycle}}. \tag{1}$$

Using the given data,

η = 0.40,

$Q_{H,\text{cycle}}$ = 50 kJ,

$\Rightarrow$ $\boxed{Q_{C,\text{cycle}} = 30\text{ kJ}}$.

The temperature of the reservoir can be found by noting that for a reversible cycle,

$$\left.\frac{Q_H}{Q_C}\right|_{\substack{\text{rev,}\\\text{cycle}}} = \frac{T_H}{T_C} \Rightarrow T_C = T_H \left.\frac{Q_C}{Q_H}\right|_{\substack{\text{rev,}\\\text{cycle}}}. \tag{2}$$

Using the parameters given above in addition to T_H = 600 K,

$\boxed{T_C = 360\text{ K}}$.