3.6.4. Performance Measures for Cycles

Recall that the power cycle thermal efficiency is given by,

$$\eta := \frac{W_{\text{by sys}}}{Q_H} = \frac{Q_H - Q_C}{Q_H} = 1 - \frac{Q_C}{Q_H}. \tag{3.104}$$

The refrigeration cycle coefficient of performance is,

$$COP_{\text{ref}} := \frac{Q_C}{W_{\text{on sys}}} = \frac{Q_C}{Q_H - Q_C} = \frac{1}{Q_H/Q_C - 1}, \tag{3.105}$$

and the heat pump cycle coefficient of performance is,

$$COP_{\text{HP}} := \frac{Q_H}{W_{\text{on sys}}} = \frac{Q_H}{Q_H - Q_C} = \frac{1}{1 - Q_C/Q_H}. \tag{3.106}$$

Notes:

- The subscript "cycle" has been removed in the previous equations for convenience; however, the evaluations for work and heat are still over a cycle.
- From the Kelvin-Planck statement of the Second Law of Thermodynamics, in the power cycle, we cannot have $Q_C = 0$, which implies that $\eta < 1$.
- From the Clausius statement of the Second Law, in the refrigeration and heat pump cycles, we cannot have $W_{\text{on sys}} = 0$, implying that COP_{ref} and COP_{HP} must remain finite.

Corollaries to the Second Law of Thermodynamics
It can be shown (refer to the proof at the end of this section) that the following corollaries to the Second Law are also true.

(1) The thermal efficiency of an irreversible power cycle will always be less than the thermal efficiency of a reversible power cycle when the two power cycles operate between the same two thermal reservoirs, i.e.,

$$\eta_{\text{irreversible}} < \eta_{\text{reversible}} \quad \text{(same thermal reservoirs)}. \tag{3.107}$$

(2) All reversible power cycles operating between the same two thermal reservoirs have the same thermal efficiency, i.e.,

$$\eta_{\text{reversible,1}} = \eta_{\text{reversible,2}} \quad \text{(same thermal reservoirs)}. \tag{3.108}$$

(3) The coefficient of performance for a reversible refrigeration cycle (or heat pump cycle) will be larger than the coefficient of performance or an irreversible refrigeration cycle (or heat pump cycle) when operating between the same two thermal reservoirs, i.e.,

$$COP_{\text{irreversible}} < COP_{\text{reversible}} \quad \text{(same thermal reservoirs)}. \tag{3.109}$$

(4) All reversible refrigeration (or heat pump) cycles operating between the same two reservoirs will have the same coefficient of performance, i.e.,

$$COP_{\text{reversible,1}} = COP_{\text{reversible,2}} \quad \text{(same thermal reservoirs)}. \tag{3.110}$$

The proof for the first corollary is presented now. Consider the situation shown in the left-hand schematic of Figure 3.33. A reversible and irreversible system receive the same energy Q_{H} from a hot reservoir. The irreversible system produces work $W_{\text{by,I}}$ and discharges energy $Q_{\text{C,I}}$ into a cold reservoir while the reversible system produces work $W_{\text{by,R}}$ and discharges energy $Q_{\text{C,R}}$ into the same cold reservoir. From the First Law applied separately to the irreversible and reversible systems and assuming both operate over a cycle,

$$0 = (Q_{\text{H}} - Q_{\text{C,I}}) - W_{\text{by,I}} \implies W_{\text{by,I}} = Q_{\text{H}} - Q_{\text{C,I}}, \tag{3.111}$$

$$0 = (Q_{\text{H}} - Q_{\text{C,R}}) - W_{\text{by,R}} \implies W_{\text{by,R}} = Q_{\text{H}} - Q_{\text{C,R}}, \tag{3.112}$$

where the change in the total energy of each of the two systems is zero since both are operating over a cycle. Choose the reversible system such that $Q_{\text{C,R}} > Q_{\text{C,I}}$ and, thus, $W_{\text{by,R}} < W_{\text{by,I}}$. Now switch the direction of the reversible system (indicated by the dashed arrows in the figure) and consider the combined system indicated by the dashed, red line in the figure. Note that this combined system includes the hot reservoir since

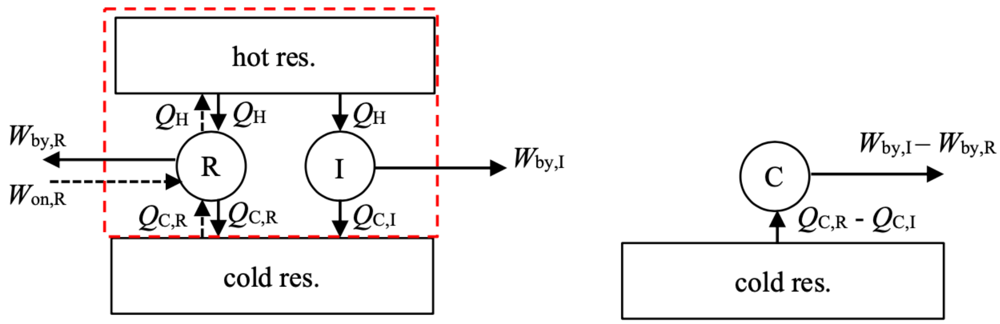

Figure 3.33. Schematic used to prove the first and second corollaries to the Second Law.

there is no net energy transfer to/from it since Q_H is removed by the irreversible system, but then replaced by the reversible one. The energy received by the combined system from the cold reservoir is $Q_{\text{C,R}} - Q_{\text{C,I}} > 0$ and the net work done by the combined system is $W_{\text{by,I}} - W_{\text{by,R}} > 0$. This combined system is shown in the right-hand schematic of Figure 3.33. Since the combined system operates over a cycle and interacts with a single thermal reservoir, from the Kelvin-Plank Statement of the Second Law (Eq. (3.103)),

$$W_{\text{by,I}} - W_{\text{by,R}} < 0 \implies W_{\text{by,I}} < W_{\text{by,R}}. \tag{3.113}$$

Note that the inequality has been used since the combined system includes irreversibilities. Since the thermal efficiency is given by,

$$\eta = \frac{W_{\text{by}}}{Q_H}, \tag{3.114}$$

and Q_H is the same for the irreversible and reversible systems, we conclude that,

$$\eta_R > \eta_I. \tag{3.115}$$

Thus, we have proven the first corollary to the Second Law.

The second corollary can be proven by replacing the irreversible system in Figure 3.33 with a reversible one so that there are two reversible systems (call these "R1" and "R2"). Following the same arguments as before, we will arrive at the following statement using the Kelvin-Plank Statement of the Second Law,

$$W_{\text{by,R2}} - W_{\text{by,R1}} = 0 \implies W_{\text{by,R2}} = W_{\text{by,R1}}. \tag{3.116}$$

Here, the equals sign is used since the combined system is reversible. Since the works and heat transfers are the same, the efficiencies of the two reversible systems must be identical.

Proofs for the third and fourth corollaries are not provided here, but follow similar arguments.

Not part of this reading.

3.6.5. Kelvin Absolute Temperature Scale

Note that from the Second Law Corollaries, the reversible cycle performance measures depend solely on the interaction with the thermal reservoirs, namely (Q_C/Q_H) in Eqs. (3.104) - (3.106) since all reversible cycle efficiencies are identical. Since it is the temperature difference between the reservoirs that drives this heat transfer, we can conclude that,

$$\left.\frac{Q_C}{Q_H}\right|_{\text{rev. cycle}} = fcn\left(\frac{T_C}{T_H}\right), \tag{3.117}$$

where T_C and T_H are the temperatures of the cold and hot reservoirs, respectively. Note that since the left hand side of the equation is dimensionless, the right hand side must also be dimensionless. The function fcn is determined by how we define temperature. In the Kelvin absolute temperature scale, we define the temperatures such that the function is a simple linear one, i.e.,

$$\left.\frac{Q_C}{Q_H}\right|_{\text{rev. cycle}} = \frac{T_C}{T_H}. \tag{3.118}$$