4.6. The First Law of Thermodynamics for a Control Volume

The analyses in this section are written in a manner more appropriate for a fluid mechanics course rather than the current thermodynamics course.

The reader should review the Introductory Thermodynamics chapter (Chapter 3) before continuing with this section.

To write the First Law for a control volume, we utilize the Reynolds Transport Theorem (RTT) to convert our system expression to a control volume expression. Let's first rewrite Eq. (3.32) using the Lagrangian derivative notation (we're interested in how things change with respect to time as we follow a particular system of fluid) and write the total energy of a system in terms of an integral,

$$\underbrace{\frac{D}{Dt}\int_{V_{\text{sys}}} e\rho dV}_{=E_{\text{sys}}} = \dot{Q}_{\text{into sys}} + \dot{W}_{\text{on sys}}. \tag{4.104}$$

Applying the Reynolds Transport Theorem and noting that the system and control volume are coincident at the time we apply the theorem gives,

$$\boxed{\frac{d}{dt}\int_{CV} e\rho dV + \int_{CS} e\left(\rho\mathbf{u}_{\text{rel}}\cdot d\mathbf{A}\right) = \dot{Q}_{\text{into CV}} + \dot{W}_{\text{on CV}}}. \tag{4.105}$$

This is the First Law of Thermodynamics for a control volume!

Notes:

(1) The specific total energy is $e = u + \frac{1}{2}V^2 + G$ where G is a conservative potential energy function with the specific gravitational force given by $\mathbf{f}_{\text{gravity}} = -\boldsymbol{\nabla}G$. For the remainder of these notes, G will be assumed to be $G = gz$ ($\implies \mathbf{f}_{\text{gravity}} = -g\hat{\mathbf{e}}_z$) where g is the acceleration due to gravity.

Now let's expand the rate of work (power) term into rate of pressure work (pdV power) and the power due to other effects such as shaft work, viscous work, electric work, etc.,

$$\dot{W}_{\text{on CV}} = \dot{W}_{p,\text{on CV}} + \dot{W}_{\text{other,on CV}}. \tag{4.106}$$

In particular, we can write the rate of pressure work term for fluid crossing the boundary in the following

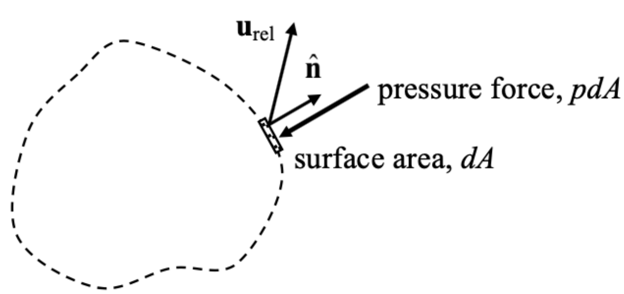


Figure 4.14. A schematic illustrating the rate of pressure work at the control surface.

way (Figure 4.14),

$$\begin{aligned} d\dot{W}_{p,\text{on CV}} &= d\mathbf{F}_{\text{p,on CV}}\cdot\mathbf{u}_{\text{rel}}, &(4.107)\\ &= (-pd\mathbf{A})\cdot\mathbf{u}_{\text{rel}}, &(4.108)\\ &= -p(\mathbf{u}_{\text{rel}}\cdot d\mathbf{A}), &(4.109)\\ &= -\frac{p}{\rho}\left(\rho\mathbf{u}_{\text{rel}}\cdot d\mathbf{A}\right). &(4.110)\end{aligned}$$

The rate of pressure work as fluid crosses the boundary over the entire CS is,

$$\boxed{\dot{W}_{p,\text{on CV}} = \int_{CS} -\frac{p}{\rho}\left(\rho\mathbf{u}_{\text{rel}}\cdot d\mathbf{A}\right)}. \tag{4.111}$$

Equation (4.111) is the rate at which pressure work is performed on the fluid flowing through the control surface.

Substituting Eqs. (4.111) and (4.106) into Eq. (4.105), expanding the specific total energy term in the surface integral, and bringing the rate of pressure work term to the left-hand side gives,

$$\frac{d}{dt}\int_{CV} e\rho dV + \int_{CS}\left(u + \frac{p}{\rho} + \frac{1}{2}V^2 + gz\right)(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}) = \dot{Q}_{\text{into CV}} + \dot{W}_{\text{other,on CV}}. \tag{4.112}$$

The quantity $(u + p/\rho)$ appears often in thermal-fluid systems and is given the special name of specific enthalpy, h,

$$h \coloneqq u + \frac{p}{\rho} = u + pv, \tag{4.113}$$

where $v = 1/\rho$ is the specific volume. Note that just as with internal energy, tables of thermodynamic properties typically list the value of the specific enthalpy for various substances at various conditions.

Substituting Eq. (4.113) into Eq. (4.112) gives,

$$\boxed{\frac{d}{dt}\int_{CV} e\rho dV + \int_{CS}\left(h + \frac{1}{2}V^2 + gz\right)(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}) = \dot{Q}_{\text{into CV}} + \dot{W}_{\text{other,on CV}}}. \tag{4.114}$$

Notes:

(1) An alternate way to write Eq. (4.114) is,

$$\frac{dE_{CV}}{dt} = \sum_{\text{all inlets}} \dot{m}\left(h + \frac{1}{2}V^2 + gz\right) - \sum_{\text{all outlets}} \dot{m}\left(h + \frac{1}{2}V^2 + gz\right) + \dot{Q}_{\text{into CV}} + \dot{W}_{\text{other,on CV}}. \tag{4.115}$$

The previous equation can be integrated in time,

$$\Delta E_{CV} = \sum_{\text{all inlets}} m\left(h + \frac{1}{2}V^2 + gz\right) - \sum_{\text{all outlets}} m\left(h + \frac{1}{2}V^2 + gz\right) + Q_{\text{into CV}} + W_{\text{other,on CV}}, \tag{4.116}$$

where m is the total mass entering/leaving the control volume. It has been assumed that the specific enthalpies, kinetic energy, and potential energies at the inlets and outlets don't change with time. This form of the First Law is useful for evaluating conditions at the end of an unsteady process. Note that if there are no inlets and outlets, then Eq. (4.116) simplifies to the system form of the First Law (Eq. (3.32)).

(2) The specific enthalpy term in Eq. (4.114) accounts for the rate of pressure work as fluid crosses the control surface, e.g., at inlets and outlets of the control volume. If there is pressure work caused by a moving, solid boundary through which no fluid flows, e.g., a moving piston, then that work would be included in the $\dot{W}_{\text{other,on CV}}$ term.

(3) During problem solving, we often must estimate the relative magnitudes of the terms in the total specific enthalpy term, i.e., $h_T = h + \frac{1}{2}V^2 + gz$. For example, consider a simple system operating at steady state with a single inlet and a single outlet. The inlet and outlet mass flow rates will be the same. The change in the total enthalpy between the inlet and outlet is (refer to Eq. (4.115)),

$$\dot{m}\Delta h_T = \dot{m}\left[\Delta h + \Delta\left(\frac{1}{2}V^2\right) + g\Delta z\right]. \tag{4.117}$$

Let's assume that $\Delta h \sim 1\,\text{kJ}\,\text{kg}^{-1}$. To have an equivalent change in the kinetic energy, we would need $\Delta V \sim 45\,\text{m}\,\text{s}^{-1}$. An equivalent change in the potential energy would require $\Delta z \sim 100\,\text{m}$. Thus, it is often reasonable to neglect changes in kinetic and potential energies if the change in specific enthalpy is large and the changes in velocity and elevation are small.

(4) Let's examine the "other" work term more closely. This term includes work due to anything other than pressure work, such as work due to viscous forces, shaft work, electrical work, etc. In this note, let's examine the work done by viscous stresses. Consider the rate of viscous work done on the CV shown in Figure 4.15,

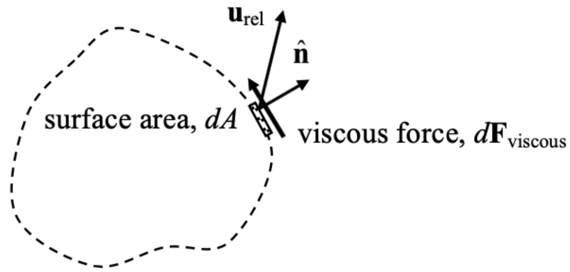


Figure 4.15. A schematic illustrating the rate of viscous work at the control surface.

$$d\dot{W}_{\text{viscous,on CV}} = d\mathbf{F}_{\text{viscous,on CV}} \cdot \mathbf{u}_{\text{rel}}, \tag{4.118}$$

so that the total rate of viscous work acting on the CS is,

$$\dot{W}_{\text{viscous,on CV}} = \int_{CS} d\mathbf{F}_{\text{viscous,on CV}} \cdot \mathbf{u}_{\text{rel}}. \tag{4.119}$$

(a) Note that at a solid boundary, $\mathbf{u}_{\text{rel}} = \mathbf{0}$ due to the no-slip condition so that the rate of viscous work is zero at solid surfaces. If the flow is inviscid, then $\mathbf{u}_{\text{rel}} \neq \mathbf{0}$, but $d\mathbf{F}_{\text{viscous,on CV}} = \mathbf{0}$ and so the rate of viscous work is zero for that case too.

(b) If the control volume is oriented such that the velocity vectors are perpendicular to the normal vectors of the CS, then the rate of viscous work done on the CV will be zero,

$$d\mathbf{F}_{\text{viscous,on CV}} \cdot \mathbf{u}_{\text{rel}} = 0, \tag{4.120}$$

since the viscous force will be perpendicular to the velocity vector. Thus, orienting the control surface so that it cuts perpendicularly across streamlines eliminates viscous work on the control volume.

(c) The rate of viscous work may not be negligible if the control volume is chosen as shown in Figure 4.16. Viscous forces along streamline surfaces may be significant if the shear stress there isn't negligible.

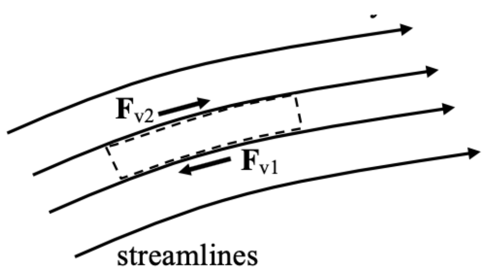


Figure 4.16. A schematic illustrating the viscous forces at the control surface if the control surface is tangential to the streamlines.

For the remainder of these notes, it will be assumed that the work on the CV due to viscous stresses is zero since our control surfaces will be chosen such that the surfaces are along solid boundaries or boundaries where viscous stresses are negligible (e.g., negligible velocity gradients), or with normal vectors perpendicular to the flow velocities.

(5) For a flow where the total energy within the CV does not change with time (steady state), Eq. (4.114) simplifies to,

$$\int_{CS} \left(h + \frac{1}{2}V^2 + gz \right) (\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}) = \dot{Q}_{\text{into CV}} + \dot{W}_{\text{other,on CV}}. \tag{4.121}$$

Note that flows may be unsteady at the local level, e.g., the localized flow within a pump, but may be steady at a larger scale, e.g., the average conditions within the pump housing.

(6) For a steady state, steady flow (meaning that the mass flow rate remains constant) with a single inlet (call it state 1) and outlet (call it state 2), we can write Eq. (4.121) as,

$$\left(h + \alpha\frac{1}{2}\bar{V}^2 + gz\right)_2 - \left(h + \alpha\frac{1}{2}\bar{V}^2 + gz\right)_1 = \dot{q}_{\text{into CV}} + \dot{w}_{\text{other,on CV}}. \tag{4.122}$$

where $q = \dot{Q}/\dot{m}$ and $w = \dot{W}/\dot{m}$ are the specific heat, i.e., the heat transfer per unit mass, and the specific work, i.e., the work per unit mass, respectively. Note that from COM the mass flow rate into the CV equals the mass flow rate out of the CV, i.e., $\dot{m}_{\text{in}} = \dot{m}_{\text{out}} = \dot{m}$.

(a) The average velocity through an area is,

$$\bar{V} := \frac{1}{A}\int_A (\mathbf{V} \cdot d\mathbf{A}). \tag{4.123}$$

(b) The quantity, α, is known as the kinetic energy correction factor. It is a correction factor accounting for the fact that an average velocity profile, $\bar{V}$, may not contain the same kinetic energy as a non-uniform velocity profile. For example, consider the kinetic energy contained in the two flow profiles shown in Figure 4.17. The average flow rate of specific kinetic energy is,

$$\overline{ke} = \int_A \frac{1}{2}V^2 \left(\rho\mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) \neq \frac{1}{2}\dot{m}\bar{V}^2, \tag{4.124}$$

in general. We define the kinetic energy correction factor, α, as,

$$\boxed{\alpha := \frac{\int_A \frac{1}{2}V^2 \left(\rho\mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right)}{\frac{1}{2}\dot{m}\bar{V}^2}}. \tag{4.125}$$

so that,

$$\overline{ke} = \alpha\frac{1}{2}\dot{m}\bar{V}^2. \tag{4.126}$$

For a laminar flow in a circular pipe, the velocity profile is parabolic (discussed in a different chapter) resulting in $\alpha = 2$. For a turbulent flow, $\alpha \to 1$ as increasing turbulent mixing causes the velocity profile to become more uniform.

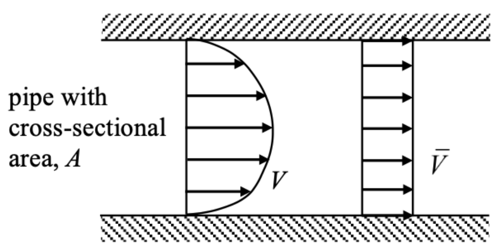


Figure 4.17. A schematic of a pipe flow with two different velocity profiles.

(c) The quantity,

$$h_T = h_0 := h + \alpha\frac{1}{2}\bar{V}^2 + gz, \tag{4.127}$$

is referred to as the total specific enthalpy, h_T or the stagnation specific enthalpy, h_0. Note that for gases, the gz term is much smaller than the other terms and, thus, is often neglected.

(d) If the flow is adiabatic ($q = 0$) and the rate of work by forces other than pressure can be neglected ($w_{\text{other}} = 0$), then,

$$h_T = h_0 = \text{constant}. \tag{4.128}$$

(7) Now let's re-write Eq. (4.122) but expand the specific enthalpy terms,

$$\left(u + \frac{p}{\rho} + \alpha\frac{1}{2}\bar{V}^2 + gz\right)_2 - \left(u + \frac{p}{\rho} + \alpha\frac{1}{2}\bar{V}^2 + gz\right)_1 = q_{\text{into CV}} + w_{\text{other,on CV}}. \tag{4.129}$$

Re-arranging terms and dividing through by the gravitational acceleration gives,

$$\left(\frac{p}{\rho g} + \alpha\frac{\bar{V}^2}{2g} + z\right)_2 = \left(\frac{p}{\rho g} + \alpha\frac{\bar{V}^2}{2g} + z\right)_1 - \frac{u_2 - u_1 - q_{\text{into CV}}}{g} + \frac{\dot{W}_{\text{other,on CV}}}{\dot{m}g}. \tag{4.130}$$

Each term in this equation is referred to as a <u>head quantity</u> and has the dimensions of length:

$$\frac{p}{\rho g} \coloneqq \text{pressure head} \tag{4.131}$$

$$\frac{\bar{V}^2}{2g} \coloneqq \text{velocity head} \tag{4.132}$$

$$z \coloneqq \text{elevation head} \tag{4.133}$$

$$\frac{u_2 - u_1 - q_{\text{into CV}}}{g} = H_L \coloneqq \text{head loss} \tag{4.134}$$

$$\frac{\dot{W}_{\text{shaft,on CV}}}{\dot{m}g} = H_S \coloneqq \text{shaft head} \tag{4.135}$$

The head loss is the head lost due to mechanical energy being converted to thermal energy and energy lost via heat transfer to the surroundings. The "other" work term frequently only includes shaft work, particularly in pipe flow systems, and so the shaft head is a convenient definition. It is the head added to the flow due to shaft work.

The equation in this form is known as the <u>Extended Bernoulli Equation</u>,

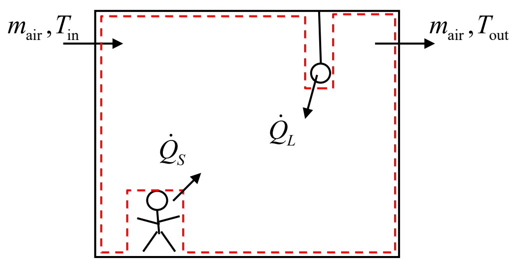
$$\boxed{\left(\frac{p}{\rho g} + \alpha\frac{\bar{V}^2}{2g} + z\right)_2 = \left(\frac{p}{\rho g} + \alpha\frac{\bar{V}^2}{2g} + z\right)_1 - H_L + H_S}, \tag{4.136}$$

where it has been assumed that the only form of "other" work is shaft work.

Let's consider some examples to see how the First Law is applied to control volumes.

Consider a large classroom on a hot summer day with 150 students, each dissipating 60 W of sensible heat. All the lights, with 4.0 kW of rated power, are kept on. The room has no external walls, and thus heat gain through the walls and the roof is negligible. Chilled air is available at 15 °C and the temperature of the return air is not to exceed 25 °C. Determine the required flow rate of air, in kg/s, that needs to be supplied to the room to keep the average temperature of the room constant.

SOLUTION:



Apply the First Law to the CV shown. Assume the conditions in the CV are steady and uniform and that the inlet and outlet flows are also steady and uniform,

$$\frac{d}{dt}\int_{CV} e\rho dV + \int_{CS}\left(h + \tfrac{1}{2}V^2 + gz\right)\left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = \dot{Q}_{\substack{\text{into}\\ \text{CV}}} + \dot{W}_{\substack{\text{other,}\\ \text{on CV}}} \tag{1}$$

where,

$$\frac{d}{dt}\int_{CV} e\rho dV = 0 \quad \text{(steady state in the CV)} \tag{2}$$

$$\int_{CS}\left(h + \tfrac{1}{2}V^2 + gz\right)\left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = \dot{m}\left(h_{\text{out}} - h_{\text{in}}\right) \tag{3}$$

The differences in the inlet and outlet kinetic and potential energies are assumed negligible and, since the flow is steady, the inlet and outlet mass flow rates are identical.

$$\dot{Q}_{\substack{\text{into}\\ \text{CV}}} = \dot{Q}_{\text{L}} + \dot{Q}_{\text{S}} \quad \text{(The influx of heat is due to the lights and the students.)} \tag{4}$$

$$\dot{W}_{\substack{\text{other,}\\ \text{on CV}}} = 0 \quad \text{(There is no work performed on the CV.)} \tag{5}$$

Substitute and simplify,

$$\dot{m}\left(h_{\text{out}} - h_{\text{in}}\right) = \dot{Q}_{\text{L}} + \dot{Q}_{\text{S}} \Rightarrow \dot{m} = \frac{\dot{Q}_{\text{L}} + \dot{Q}_{\text{S}}}{h_{\text{out}} - h_{\text{in}}} \tag{6}$$

Given that,

$$\dot{Q}_{\text{L}} = 4.0 \text{ kW} \tag{7}$$

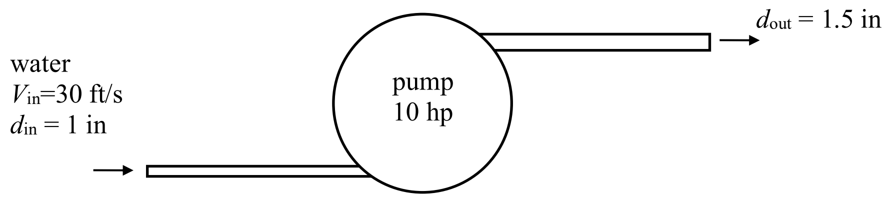
$$\dot{Q}_{\text{S}} = \text{(150 students)(60 W/student)} = 9.0 \text{ kW} \tag{8}$$

$$h_{\text{out}} = 298.18 \text{ kJ/kg} \quad \text{(thermodynamics tables for air at } T_{\text{out}} = 25\ ^\circ\text{C} = 298 \text{ K)} \tag{9}$$

$$h_{\text{in}} = 288.15 \text{ kJ/kg} \quad \text{(thermodynamics tables for air at } T_{\text{in}} = 15\ ^\circ\text{C} = 288 \text{ K)} \tag{10}$$

$$\boxed{\therefore \dot{m} = 1.30 \ \text{kg/s}} \tag{11}$$

Determine the maximum pressure increase across the 10 hp pump shown in the figure.



SOLUTION:

Apply the First Law to a control volume surrounding the pump,

$$\frac{d}{dt}\int_{CV} e\rho\, dV + \int_{CS}\left(h+\tfrac{1}{2}V^2+gz\right)\rho\mathbf{u}_{\text{rel}}\cdot d\mathbf{A} = \dot{Q}_{\text{into CV}} + \dot{W}_{\substack{\text{on CV,}\\ \text{other}}}, \quad (1)$$

where,

$$\frac{d}{dt}\int_{CV} e\rho\, dV = 0 \quad \text{(steady flow)}, \quad (2)$$

$$\int_{CS}\left(h+\tfrac{1}{2}V^2+gz\right)\rho\mathbf{u}_{\text{rel}}\cdot d\mathbf{A} = \dot{m}\left[\left(h_{\text{out}}-h_{\text{in}}\right)+\tfrac{1}{2}\left(V_{\text{out}}^2-V_{\text{in}}^2\right)\right] \quad \text{(neglecting elevation differences)}, \quad (3)$$

$$\dot{Q}_{\text{into CV}} = 0, \quad (4)$$

$$\dot{W}_{\substack{\text{on CV,}\\ \text{other}}} = \text{given} \quad (5)$$

Assuming the water is an incompressible fluid, re-write the change in specific enthalpy as,

$$h_{\text{out}} - h_{\text{in}} = c\left(T_{\text{out}}-T_{\text{in}}\right)+\frac{1}{\rho}\left(p_{\text{out}}-p_{\text{in}}\right). \quad (6)$$

Substitute into Eq. (1) and simplify,

$$\dot{m}\left[c\left(T_{\text{out}}-T_{\text{in}}\right)+\frac{1}{\rho}\left(p_{\text{out}}-p_{\text{in}}\right)+\tfrac{1}{2}\left(V_{\text{out}}^2-V_{\text{in}}^2\right)\right] = \dot{W}_{\substack{\text{on CV,}\\ \text{other}}} \quad (7)$$

In this particular case we're asked to find the maximum pressure rise across the pump, which would correspond to no temperature change, i.e., $T_{\text{out}} = T_{\text{in}}$. Thus,

$$\dot{m}\left[\frac{1}{\rho}\left(p_{\text{out}}-p_{\text{in}}\right)+\tfrac{1}{2}\left(V_{\text{out}}^2-V_{\text{in}}^2\right)\right] = \dot{W}_{\substack{\text{on CV,}\\ \text{other}}}. \quad (8)$$

The velocity at the outlet may be found by applying conservation of mass to the same control volume,

$$V_{\text{out}} = V_{\text{in}}\left(\frac{d_{\text{in}}}{d_{\text{out}}}\right)^2. \quad (9)$$

Substitute and simplify Eq. (8),

$$\dot{m}\left\{\frac{1}{\rho}\left(p_{\text{out}}-p_{\text{in}}\right)+\tfrac{1}{2}V_{\text{in}}^2\left[\left(\frac{d_{\text{in}}}{d_{\text{out}}}\right)^4-1\right]\right\} = \dot{W}_{\substack{\text{on CV,}\\ \text{other}}}, \quad (10)$$

$$p_{\text{out}} - p_{\text{in}} = \frac{\rho\dot{W}_{\substack{\text{on CV,}\\ \text{other}}}}{\dot{m}}+\frac{1}{2}\rho V_{\text{in}}^2\left[1-\left(\frac{d_{\text{in}}}{d_{\text{out}}}\right)^4\right]. \quad (11)$$

Using the given parameters,

ρ = 62.4 $\text{lb}_\text{m}/\text{ft}^3$ (approximate density of liquid water at standard conditions)

V_{in} = 30 ft/s

d_{in} = 1 in. = (1/12) ft

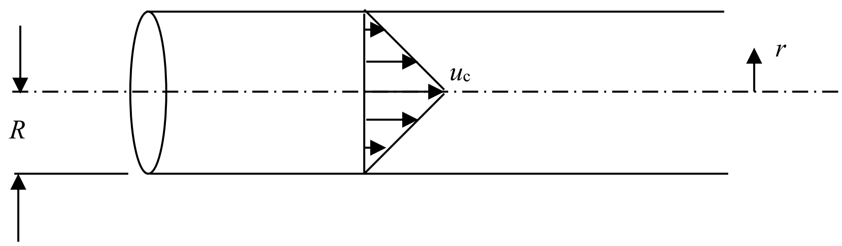
d_{out} = 1.5 in.

$\dot{W}_{\text{on CV, other}}$ = 10 hp = 5500 $\text{ft.lb}_\text{f}/\text{s}$

$\dot{m}$ = $\rho V_{\text{in}}(\pi d_{\text{in}}^2/4)$ = 10.2 $\text{lb}_\text{m}/\text{s}$

$\Rightarrow \boxed{p_{\text{out}} - p_{\text{in}} = 34300\ \text{lb}_\text{f}/\text{ft}^2 = 238\ \text{lb}_\text{f}/\text{in}^2}$

The velocity profile for a particular pipe flow is linear from zero at the wall to a maximum of u_c at the centerline. Determine the average velocity and the kinetic energy correction factor.



SOLUTION:
The average velocity is found by setting the volumetric flow rate using the average velocity profile equal to the volumetric flow rate using the real profile,

$$Q_{\substack{\text{avg}\\\text{profile}}} = Q_{\substack{\text{real}\\\text{profile}}} \tag{1}$$

$$\bar{u}\pi R^2 = \int_{r=0}^{r=R} u_C\left(1-\frac{r}{R}\right)(2\pi r dr)$$

$$= 2\pi u_C \int_{r=0}^{r=R}\left(r-\frac{r^2}{R}\right)dr$$

$$= 2\pi u_C\left[\frac{1}{2}r^2-\frac{1}{3}\frac{r^3}{R}\right]_{r=0}^{r=R}$$

$$\boxed{\therefore \bar{u} = \tfrac{1}{3}u_C} \tag{2}$$

The kinetic energy correction factor, α, is found by equating the kinetic energy flow rate using the average velocity with the kinetic energy flow rate using the actual velocity profile,

$$\alpha\tfrac{1}{2}\underbrace{\left(\rho\bar{u}\pi R^2\right)}_{=\dot{m}}\bar{u}^2 = \int_{r=0}^{r=R}\tfrac{1}{2}\left[\underbrace{\rho u_C\left(1-\frac{r}{R}\right)(2\pi r dr)}_{=d\dot{m}}\right]\left[u_C\left(1-\frac{r}{R}\right)\right]^2$$

$$= \int_{r=0}^{r=R}\tfrac{1}{2}\rho u_C^3\left(1-\frac{r}{R}\right)^3(2\pi r dr) \tag{3}$$

$$= \pi\rho u_C^3\int_{r=0}^{r=R}\left(1-\frac{r}{R}\right)^3(r dr)$$

where $\bar{u} = \tfrac{1}{3}u_C$. Solving the previous equation for α gives,

$$\boxed{\alpha = \frac{27}{10} = 2.7} \tag{4}$$

Air at 10 °C and 80 kPa (abs) enters the diffuser of a jet engine steadily with a velocity of 200 m/s. The inlet area of the diffuser is 0.4 m^2. The air leaves the diffuser with a velocity that is very small compared with the inlet velocity. Determine
a. the mass flow rate of the air and
b. the temperature of the air leaving the diffuser.
You may assume adiabatic flow through the diffuser.

SOLUTION:

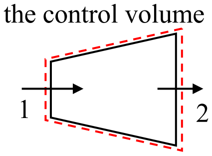
The mass flow rate of the air may be found from,

$$\dot{m} = \rho_1 V_1 A_1 \Rightarrow \dot{m} = \left(\frac{p_1}{RT_1}\right) V_1 A_1 \quad (1)$$

Using the given data,

p_1 = 80 kPa (abs)
R = 287 J/(kg·K)
T_1 = 10 °C = 283 K
V_1 = 200 m/s
A_1 = 0.4 m^2
$\Rightarrow \boxed{\dot{m} = 78.8 \text{ kg/s}}$

Now apply the First Law to the control volume shown below.



$$\frac{d}{dt}\int_{CV} e\rho\, dV + \int_{CS}\left(h + \tfrac{1}{2}V^2 + gz\right)\left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = \dot{Q}_{\substack{\text{into}\\ \text{CV}}} + \dot{W}_{\substack{\text{other,}\\ \text{on CV}}} \quad (2)$$

where,

$$\frac{d}{dt}\int_{CV} e\rho dV = 0 \quad \text{(steady flow)} \quad (3)$$

$$\int_{CS}\left(h + \tfrac{1}{2}V^2 + gz\right)\left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = \dot{m}_2\left(h_2 + \tfrac{1}{2}V_2^2\right) - \dot{m}_1\left(h_1 + \tfrac{1}{2}V_1^2\right) \quad (4)$$

$$\dot{Q}_{\substack{\text{into}\\ \text{CV}}} = 0 \quad \text{(Flow through diffusers is usually assumed to occur adiabatically.)} \quad (5)$$

$$\dot{W}_{\substack{\text{other,}\\ \text{on CV}}} = 0 \quad (6)$$

Note that from conservation of mass for the same control volume,

$$\dot{m}_2 = \dot{m}_1 = \dot{m} \quad (7)$$

Substitute into the First Law and simplify,

$$\dot{m}\left(h_2 + \tfrac{1}{2}V_2^2\right) - \dot{m}\left(h_1 + \tfrac{1}{2}V_1^2\right) = 0 \quad \text{(Note: } V_2 << V_1\text{.)} \quad (8)$$

$$\boxed{h_2 = h_1 + \tfrac{1}{2}V_1^2} \quad (9)$$

From a thermodynamics table for air, h_1 = 283.14 kJ/kg @ 283 K giving,

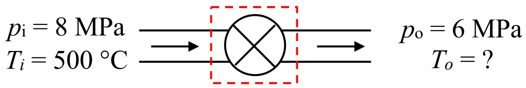
h_2 = 303.14 kJ/kg $\Rightarrow$ $\boxed{T_2 = 302.9 \text{ K} \approx 30 \text{ °C}}$ (from a thermodynamics table for air)

Alternately, if air is assumed to behave as a perfect gas so that $\Delta h = c_p \Delta T$ with c_p = 1000 J/(kg·K), then,

$$\boxed{T_2 = T_1 + \frac{V_1^2}{2c_p}} \Rightarrow T_2 = 303 \text{ K} \quad \text{(Same as before!)} \quad (10)$$

A well-insulated valve is used to throttle steam from 8 MPa (abs) and 500 °C to 6 MPa (abs). Determine the final temperature of the steam.

SOLUTION:



Apply the First Law to the CV shown assuming 1D, steady flow, no heat transfer (the valve is well-insulated), and no "other" work done on the CV besides pressure work.

$$\frac{d}{dt}\int_{CV} e\rho\, dV + \int_{CS}\left(h+\tfrac{1}{2}V^2+gz\right)\left(\rho\mathbf{u}_{\text{rel}}\cdot d\mathbf{A}\right) = \dot{Q}_{\substack{\text{into}\\ \text{CV}}} + \dot{W}_{\substack{\text{other,}\\ \text{on CV}}} \tag{1}$$

where,

$$\frac{d}{dt}\int_{CV} e\rho dV = 0 \quad \text{(steady flow)} \tag{2}$$

$$\int_{CS}\left(h+\tfrac{1}{2}V^2+gz\right)\left(\rho\mathbf{u}_{\text{rel}}\cdot d\mathbf{A}\right) = \dot{m}\left(h_o - h_i\right) \tag{3}$$

(The changes in kinetic and potential energies are assumed negligible and, since the flow is steady and 1D, the mass flow rate is the same at the inlet and outlet.)

$$\dot{Q}_{\substack{\text{into}\\ \text{CV}}} = 0 \quad \text{(The valve is well-insulated.)} \tag{4}$$

$$\dot{W}_{\substack{\text{other,}\\ \text{on CV}}} = 0 \quad \text{(No work is performed other than pressure work.)} \tag{5}$$

Substitute and simplify,

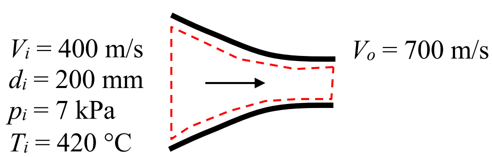
$$\dot{m}\left(h_o - h_i\right) = 0 \;\Rightarrow\; \boxed{h_o = h_i} \tag{6}$$

From steam tables at p_i = 8 MPa = 800 bars, T_i = 500 °C, h_i = 3398.05 kJ/kg. Again, using the steam tables for h_o = 3398.05 kJ/kg and p_o = 6 MPa = 600 bars, $\boxed{T_o = 490\ ^\circ\text{C}}$. Hence, the temperature drops by 10 °C across the valve.

Air flows through a nozzle with an inlet diameter of 200 mm, velocity of 400 m/s, pressure of 7 kPa (abs), and temperature of 420 °C. The nozzle exit diameter is adjusted such that the exiting velocity is 700 m/s. Determine:

a. the exit temperature, and
b. the mass flow rate through the nozzle

SOLUTION:



Apply the First Law to the CV shown assuming 1D, steady flow, no heat transfer, and no "other" work done on the CV besides pressure work,

$$\frac{d}{dt}\int_{CV} e\rho\, dV + \int_{CS}\left(h + \tfrac{1}{2}V^2 + gz\right)\left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = \dot{Q}_{\substack{\text{into}\\ \text{CV}}} + \dot{W}_{\substack{\text{other,}\\ \text{on CV}}} \tag{1}$$

where,

$$\frac{d}{dt}\int_{CV} e\rho dV = 0 \quad \text{(steady state)} \tag{2}$$

$$\int_{CS}\left(h + \tfrac{1}{2}V^2 + gz\right)\left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = \dot{m}\left(h_o + \tfrac{1}{2}V_o^2 - h_i - \tfrac{1}{2}V_i^2\right) \tag{3}$$

(The change in potential energies is assumed negligible and, since the flow is steady and 1D, the mass flow rate is the same at the inlet and outlet.)

$$\dot{Q}_{\substack{\text{into}\\ \text{CV}}} = 0 \quad \text{(Assume little heat transfer occurs over the nozzle surface area.)} \tag{4}$$

$$\dot{W}_{\substack{\text{other,}\\ \text{on CV}}} = 0 \quad \text{(No work is performed other than pressure work.)} \tag{5}$$

Substitute and simplify,

$$\dot{m}\left(h_o + \tfrac{1}{2}V_o^2 - h_i - \tfrac{1}{2}V_i^2\right) = 0 \;\Rightarrow\; \boxed{h_o = h_i + \tfrac{1}{2}\left(V_i^2 - V_o^2\right)} \tag{6}$$

For the given conditions,

h_i = 705.75 kJ/kg (from thermodynamics tables for air, assumed to be an ideal gas, at T_i = 420 °C)
V_i = 400 m/s
V_o = 700 m/s
$\Rightarrow$ h_o = 540.75 kJ/kg $\Rightarrow$ $\boxed{T_o = 264\text{ °C}}$ (from thermo tables assuming air is an ideal gas)

The mass flow rate is,

$$\boxed{\dot{m} = \rho_i V_i A_i = \frac{p_i}{RT_i} V_i \tfrac{\pi}{4} d_i^2} \tag{7}$$

Using the given parameters:

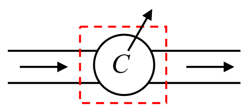
p_i = 7 kPa
R = 287 J/(kg·K)
T_i = 420 °C = 673 K
V_i = 400 m/s
d_i = 0.2 m
$\Rightarrow$ $\boxed{\dot{m} = 0.44\text{ kg/s}}$

Air at 100 kPa and 280 K is compressed steadily to 600 kPa and 400 K. The mass flow rate of the air is 0.02 kg/s and a heat loss per unit of flowing mass of 16 kJ/kg occurs during the process. Assuming the changes in kinetic and potential energies are negligible, determine the necessary power input to the compressor.

SOLUTION:

16 kJ/kg

p_i = 100 kPa
T_i = 280 K
$\dot{m}$ = 0.02 kg/s



p_o = 600 kPa
T_i = 400 K

Apply the First Law to a control volume (CV) surrounding the compressor,

$$\frac{d}{dt}\int_{\mathrm{CV}} e\rho\, dV + \int_{\mathrm{CS}} \left(h + \tfrac{1}{2}V^2 + gz\right)\left(\rho \mathbf{u}_{\mathrm{rel}} \cdot d\mathbf{A}\right) = \dot{Q}_{\substack{\text{into}\\ \text{CV}}} + \dot{W}_{\substack{\text{other,}\\ \text{on CV}}} \tag{1}$$

where,

$$\frac{d}{dt}\int_{\mathrm{CV}} e\rho dV = 0 \quad \text{(steady state)} \tag{2}$$

$$\int_{\mathrm{CS}} \left(h + \tfrac{1}{2}V^2 + gz\right)\left(\rho \mathbf{u}_{\mathrm{rel}} \cdot d\mathbf{A}\right) = \dot{m}\left(h_o - h_i\right) \tag{3}$$

(Changes in KE and PE are negligible; the mass flow rate at the inlet and outlet are the same since the flow is steady and 1D.)

$$\dot{Q}_{\substack{\text{into}\\ \text{CV}}} \Big/ \dot{m} = -16 \ \text{kJ/kg} \quad \text{(conduction through the sides of the can)} \tag{4}$$

$$\dot{W}_{\substack{\text{other,}\\ \text{on CV}}} = \dot{W}_{\substack{\text{shaft,}\\ \text{on CV}}} \tag{5}$$

Substitute and simplify,

$$\dot{m}\left(h_o - h_i\right) = \dot{Q}_{\substack{\text{into}\\ \text{CV}}} + \dot{W}_{\substack{\text{shaft,}\\ \text{on CV}}} \tag{6}$$

$$\boxed{\dot{W}_{\substack{\text{shaft,}\\ \text{on CV}}} = \dot{m}\left(h_o - h_i\right) - \dot{Q}_{\substack{\text{into}\\ \text{CV}}}} \tag{7}$$

For the given conditions,

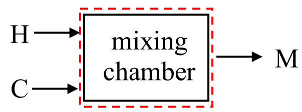
h_o = 400.98 kJ/kg (thermodynamic tables for air at p_o = 100 kPa, T_o = 280 K)
h_i = 280.13 kJ/kg (thermodynamic tables for air at p_i = 600 kPa, T_i = 400 K)
$\dot{Q}_{\substack{\text{into}\\ \text{CV}}} \Big/ \dot{m}$ = -16 kJ/kg
$\dot{m}$ = 0.02 kg/s
$\Rightarrow \boxed{\dot{W}_{\substack{\text{shaft,}\\ \text{on CV}}} = 2.74 \text{ kW}}$

Thus, 2.74 kW must be supplied to the air. The power supplied to the compressor would, in fact, be larger than this due to inefficiencies in the compressor.

Consider an ordinary shower where hot water at 140 °F is mixed with cold water at 50 °F. If it is desired that a steady stream of warm water at 110 °F be supplied, determine the ratio of the mass flow rates of the hot to cold water. Assume the heat losses from the mixing chamber to be negligible and the mixing to take place at a pressure of 20 psia.

SOLUTION:

Apply the First Law to the control volume shown below,



$$\frac{d}{dt}\int_{\text{CV}} e\rho\, dV + \int_{\text{CS}} \left(h + \tfrac{1}{2}V^2 + gz\right)\left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = \dot{Q}_{\substack{\text{into}\\ \text{CV}}} + \dot{W}_{\substack{\text{on}\\ \text{CV}}} \tag{1}$$

where,

$$\frac{d}{dt}\int_{\text{CV}} e\rho dV = 0 \quad \text{(steady flow)}$$

$$\int_{\text{CS}} \left(h + \tfrac{1}{2}V^2 + gz\right)\left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = -\dot{m}_H h_H - \dot{m}_C h_C + \dot{m}_M h_M$$

(Neglect changes in potential and kinetic energies since they won't be significant for flow through a shower head.)

$$\dot{Q}_{\substack{\text{into}\\ \text{CV}}} = 0 \quad \text{(negligible heat transfer to the surroundings} \Rightarrow \text{assume adiabatic)}$$

$$\dot{W}_{\substack{\text{on}\\ \text{CV}}} = 0 \quad \text{(no work besides pressure work is being done on the CV)}$$

Substitute,

$$-\dot{m}_H h_H - \dot{m}_C h_C + \dot{m}_M h_M = 0 \tag{2}$$

Apply conservation of mass to the same control volume to find,

$$\dot{m}_M = \dot{m}_H + \dot{m}_C \tag{3}$$

Combine Eqs. (2) and (3) and simplify,

$$-\dot{m}_H h_H - \dot{m}_C h_C + \left(\dot{m}_H + \dot{m}_C\right) h_M = 0$$

$$-\frac{\dot{m}_H}{\dot{m}_C} h_H - h_C + \left(\frac{\dot{m}_H}{\dot{m}_C} + 1\right) h_M = 0$$

$$\frac{\dot{m}_H}{\dot{m}_C}\left(h_M - h_H\right) = h_C - h_M$$

$$\boxed{\therefore \frac{\dot{m}_H}{\dot{m}_C} = \frac{h_C - h_M}{h_M - h_H}} \tag{4}$$

Look up the specific enthalpies for water in a thermodynamics reference (note that since the water is a pure liquid, the mixing pressure is irrelevant):

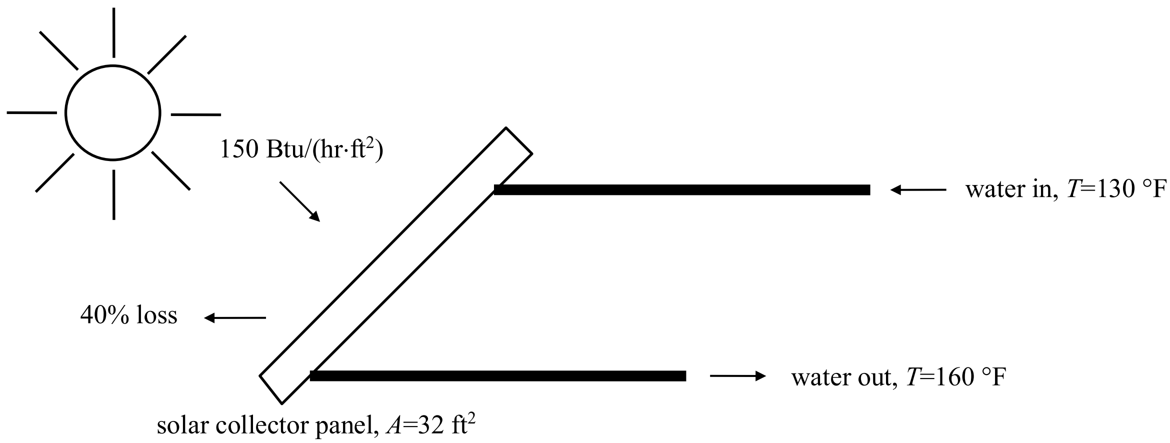
$h_C = h_{50\,°\text{F}} = 18.06 \text{ Btu/lb}_\text{m}$

$h_H = h_{140\,°\text{F}} = 107.96 \text{ Btu/lb}_\text{m}$

$h_M = h_{110\,°\text{F}} = 78.02 \text{ Btu/lb}_\text{m}$

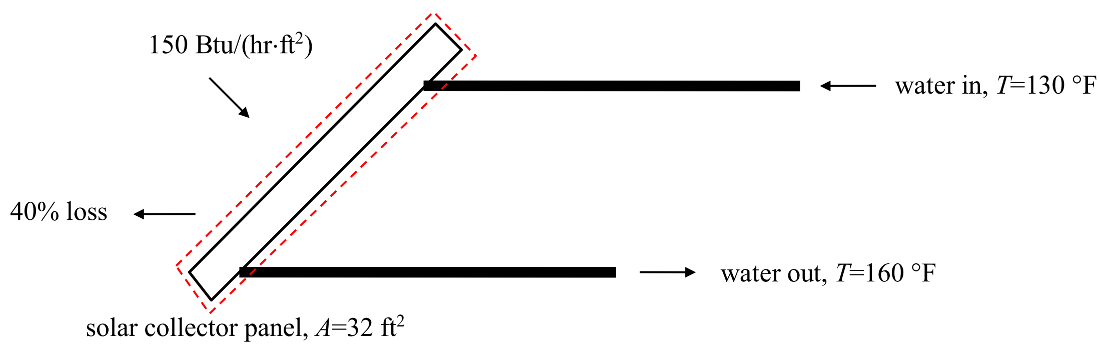
$$\boxed{\therefore \frac{\dot{m}_H}{\dot{m}_C} = 2.0}$$

The figure below shows a solar collector panel with a surface area of 32 ft^2. The panel receives energy from the sun at a rate of 150 Btu/hr per ft^2 of collector surface. Forty percent of the incoming energy is lost to the surroundings. The remainder is used to warm liquid water from 130 to 160 °F. The water passes through the solar collector with a negligible pressure drop. Neglecting kinetic and potential energy effects, determine at steady state the mass flow rate of the water in lb$_m$/min. How many solar collectors would be needed to provide a total of 40 gal of 160 °F water in 30 min?



SOLUTION:

Apply the First Law to the control volume shown below.



$$\frac{d}{dt}\int_{CV} e\rho \, dV + \int_{CS}\left(h + \tfrac{1}{2}V^2 + gz\right)\left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = \dot{Q}_{\substack{\text{into}\\ \text{CV}}} + \dot{W}_{\substack{\text{on}\\ \text{CV}}}$$

where,

$$\frac{d}{dt}\int_{CV} e\rho dV = 0 \quad \text{(steady state)}$$

$$\int_{CS}\left(h + \tfrac{1}{2}V^2 + gz\right)\left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = -\left(\dot{m}h\right)_{\text{in}} + \left(\dot{m}h\right)_{\text{out}} \quad \text{(Neglect changes in kinetic and potential energies.)}$$

$$\dot{Q}_{\substack{\text{into}\\ \text{CV}}} = \dot{Q}_{\substack{\text{in from}\\ \text{sun}}} - \dot{Q}_{\substack{\text{out from}\\ \text{panel}}} = \left(1-\alpha\right)\dot{Q}_{\substack{\text{in from}\\ \text{sun}}} \quad \text{(where } \alpha \text{ is the fraction of heat lost from the panel)}$$

$$\dot{W}_{\substack{\text{on}\\ \text{CV}}} = 0$$

Substitute and simplify,

$$-\left(\dot{m}h\right)_{\text{in}}+\left(\dot{m}h\right)_{\text{out}}=\left(1-\alpha\right)\dot{Q}_{\substack{\text{in from}\\\text{sun}}} \tag{1}$$

Applying conservation of mass to the same control volume gives,

$$\dot{m}_{\text{in}}=\dot{m}_{\text{out}} \tag{2}$$

Combine Eqs. (1) and (2) and simplify,

$$\boxed{\dot{m}=\frac{\left(1-\alpha\right)\dot{Q}_{\substack{\text{in from}\\\text{sun}}}}{h_{\text{out}}-h_{\text{in}}}} \tag{3}$$

Using the given data:

α = 0.40

$\dot{Q}_{\substack{\text{in from}\\\text{sun}}}$ = [150 Btu/(hr·ft^2)][32 ft^2] = 4800 Btu/hr

h_{out} = 127.96 Btu/lb$_m$ (water @ 160 °F, from thermodynamics tables)

h_{in} = 97.98 Btu/lb$_m$ (water @ 130 °F, from thermodynamics tables)

$\Rightarrow$ $\boxed{\dot{m} = 96.1\ \text{lb}_m/\text{hr} = 1.60\ \text{lb}_m/\text{min}}$

To get 40 gal of 160 °F water in 30 min,

$$\boxed{\text{Vol}=nQ\Delta t} \tag{4}$$

where Vol is the total volume, n is the number of solar collectors, Q is the volumetric flow rate, and Δt is the duration. Using the given data:

Vol = 40 gal = 5.35 ft^3

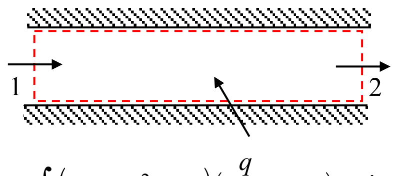
Q = (1.60 lb$_m$/min)/(62.4 lb$_m$/ft^3) = 2.56*10^{-2} ft^3/min

Δt = 30 min

$\Rightarrow$ n = 6.95 $\Rightarrow$ $\boxed{\text{Seven solar collectors would be required.}}$

Carbon dioxide flows through a constant area duct. At the inlet to the duct, the velocity is 120 m/s and the temperature and pressure are 200 °C and 700 kPa (abs), respectively. Heat is added to the flow in the duct and at the exit of the duct the velocity is 240 m/s and the temperature is 450 °C. Find the amount of heat being added to the carbon dioxide per unit mass of gas and the mass flow rate through the duct per unit cross-sectional area of the duct. Assume that the specific heat ratio for carbon dioxide is 1.3 and the gas constant is 189 J/(kg·K).

SOLUTION:
Apply the First Law to the control volume shown below,



$$\frac{d}{dt}\int_{CV} e\rho\, dV + \int_{CS}\left(h+\tfrac{1}{2}V^2+gz\right)\left(\rho\mathbf{u}_{\text{rel}}\cdot d\mathbf{A}\right) = \dot{Q}_{\substack{\text{into}\\ \text{CV}}} + \dot{W}_{\substack{\text{other,}\\ \text{on CV}}} \tag{1}$$

where,

$$\frac{d}{dt}\int_{CV} e\rho dV = 0 \quad \text{(steady flow)} \tag{2}$$

$$\int_{CS}\left(h+\tfrac{1}{2}V^2+gz\right)\left(\rho\mathbf{u}_{\text{rel}}\cdot d\mathbf{A}\right) = \dot{m}_2\left(h_2+\tfrac{1}{2}V_2^2\right) - \dot{m}_1\left(h_1+\tfrac{1}{2}V_1^2\right) \tag{3}$$

$$\dot{Q}_{\substack{\text{into}\\ \text{CV}}} = ? \tag{4}$$

$$\dot{W}_{\substack{\text{other,}\\ \text{on CV}}} = 0 \tag{5}$$

Note that from conservation of mass for the same control volume,

$$\dot{m}_2 = \dot{m}_1 = \dot{m} \tag{6}$$

Substitute into the First Law and simplify,

$$\dot{m}\left(h_2+\tfrac{1}{2}V_2^2\right) - \dot{m}\left(h_1+\tfrac{1}{2}V_1^2\right) = \dot{Q}_{\substack{\text{into}\\ \text{CV}}} \tag{7}$$

$$q_{\substack{\text{into}\\ \text{CV}}} = \left(h_2-h_1\right)+\tfrac{1}{2}\left(V_2^2-V_1^2\right) \tag{8}$$

Assume that CO_2 behaves as a perfect gas so that $\Delta h = c_p\Delta T$ and Eq. (8) becomes,

$$\boxed{q_{\substack{\text{into}\\ \text{CV}}} = c_p\left(T_2-T_1\right)+\tfrac{1}{2}\left(V_2^2-V_1^2\right)} \tag{9}$$

Note that a more accurate solution would not use the perfect gas assumption, but would instead evaluate the specific enthalpies directly using thermodynamic property tables.

Using the given data:

γ = 1.3 $\qquad R$ = 189 J/(kg·K) $\qquad c_p = \dfrac{\gamma R}{\gamma - 1}$ = 819 J/(kg·K)

T_2 = 450 ºC = 723 K $\qquad T_1$ = 200 ºC = 473 K
V_2 = 240 m/s $\qquad V_1$ = 120 m/s
$\Rightarrow \boxed{q_{\text{into CV}} = 226 \text{ kJ/kg}}$

The mass flow rate per unit area is simply,

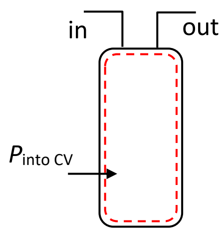
$$\dot{m} = \rho_1 V_1 A \;\Rightarrow\; \frac{\dot{m}}{A} = \rho_1 V_1 = \left(\frac{p_1}{RT_1}\right)V_1 \;\Rightarrow\; \boxed{\frac{\dot{m}}{A} = 940 \text{ kg/s/m}^2} \tag{10}$$

where p_1 = 700 kPa (abs).

If the water in a well-insulated, 50 gal electric water heater initially has the same temperature as the inlet water temperature (55 °F), determine how long it will take for the water at the outlet to reach a comfortable shower temperature of 105 °F if the water flows continuously in the shower at a rate of 2 gal/min (a typical flow rate for a shower). What will be the steady state temperature in the water heater for these conditions? This particular water heater can provide 4500 W of power to the heating element. The inlet supply line has a pressure of 50 psi and the pressure in the tank is 70 psi.

SOLUTION:

Apply the First Law to a control volume surrounding the water heater tank.



$$\frac{d}{dt}\int_{\text{CV}} e\rho\, dV + \int_{\text{CS}} \left(h + \tfrac{1}{2}V^2 + gz\right)\left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = \dot{Q}_{\substack{\text{into}\\\text{CV}}} + \dot{W}_{\substack{\text{on}\\\text{CV}}} \tag{1}$$

where,

$$\frac{d}{dt}\int_{\text{CV}} e\rho dV = \frac{d}{dt}\left(u_{CV}\rho V_{CV}\right) = \rho V_{CV}\frac{du_{CV}}{dt} \quad \text{(only the internal energy changes in the CV)} \tag{2}$$

$$\int_{\text{CS}} \left(h + \tfrac{1}{2}V^2 + gz\right)\left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = \dot{m}\left(h_{CV} - h_{\text{in}}\right) \quad \text{(same velocity in and out, same elevation)} \tag{3}$$

Note that the water leaving the control volume is assumed to have the same properties as the water in the CV.

$$\dot{Q}_{\substack{\text{into}\\\text{CV}}} = 0 \quad \text{(the tank is well insulated)} \tag{4}$$

$$\dot{W}_{\substack{\text{on}\\\text{CV}}} = P \quad \text{(the input electrical power)} \tag{5}$$

Substitute and simplify,

$$\rho V_{CV}\frac{du_{CV}}{dt} + \dot{m}\left(h_{CV} - h_{\text{in}}\right) = P \tag{6}$$

Model water as an incompressible substance with constant specific heat,

$$\frac{du_{CV}}{dt} = c\frac{dT_{CV}}{dt} \quad \text{and} \quad h_{CV} - h_{\text{in}} = c\left(T_{CV} - T_{\text{in}}\right) + \frac{1}{\rho}\left(p_{CV} - p_{\text{in}}\right) \tag{7}$$

Substitute into Eq. (6),

$$\rho V_{CV} c\frac{dT_{CV}}{dt} + \dot{m}c\left(T_{CV} - T_{\text{in}}\right) + \frac{\dot{m}}{\rho}\left(p_{CV} - p_{\text{in}}\right) = P \tag{8}$$

$$\rho V_{CV} c\frac{dT_{CV}}{dt} + \dot{m}cT_{CV} - \dot{m}cT_{\text{in}} + \frac{\dot{m}}{\rho}\left(p_{CV} - p_{\text{in}}\right) = P \tag{9}$$

$$\frac{dT_{CV}}{dt}+\left(\frac{\dot{m}}{\rho V_{CV}}\right)T_{CV}=\frac{\dot{m}cT_{\text{in}}+\frac{\dot{m}}{\rho}\left(p_{\text{in}}-p_{CV}\right)+P}{\rho V_{CV}c} \tag{10}$$

$$\frac{dT_{CV}}{dt}+\alpha T_{CV}=\beta \tag{11}$$

where,

$$\alpha=\frac{\dot{m}}{\rho V_{CV}} \quad \text{and} \quad \beta=\frac{\dot{m}}{\rho V_{CV}}T_{\text{in}}+\frac{\dot{m}\left(p_{\text{in}}-p_{CV}\right)}{\rho^2 cV_{CV}}+\frac{P}{\rho cV_{CV}}=\frac{1}{\rho cV_{CV}}\left[\dot{m}cT_{\text{in}}+\frac{\dot{m}\left(p_{\text{in}}-p_{CV}\right)}{\rho}+P\right] \tag{12}$$

and,

$$T_{CV}\left(t=0\right)=T_0 \tag{13}$$

where T_0 is the initial water temperature in the tank.

Also note that,

$$\frac{\beta}{\alpha}=T_{\text{in}}+\frac{\left(p_{\text{in}}-p_{CV}\right)}{\rho c}+\frac{P}{\dot{m}c} \tag{14}$$

The solution to the ODE given in Eq. (11) subject to the initial condition given in Eq. (13) is,

$$\int_{T_0}^{T}\frac{dT_{CV}}{\beta-\alpha T_{CV}}=\int_0^t dt \tag{15}$$

$$-\frac{1}{\alpha}\ln\left(\frac{\beta-\alpha T}{\beta-\alpha T_0}\right)=t \tag{16}$$

$$\boxed{T=\frac{\beta}{\alpha}-\left(\frac{\beta}{\alpha}-T_0\right)\exp\left(-\alpha t\right)} \tag{17}$$

The time to reach a particular temperature may be found by re-arranging Eq. (17),

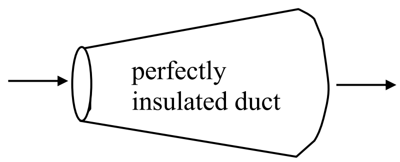
$$\boxed{t=-\frac{1}{\alpha}\ln\left[\frac{\left(\frac{\beta}{\alpha}-T\right)}{\left(\frac{\beta}{\alpha}-T_0\right)}\right]} \tag{18}$$

The steady state temperature is found by letting $t\to\infty$,

$$\boxed{T_{t\to\infty}=\frac{\beta}{\alpha}} \tag{19}$$

Using the given parameters, it is not possible to reach the desired shower temperature. In fact, the steady state temperature is only 70 °F. In order to reach the desired temperature, one would need to decrease the flow rate. A flow rate of approximately 0.6 gpm will give a steady state temperature of 105 °F, but it will take a long time to get there and you'll waste a lot of water!

A steady flow of viscous air at 20 °C and 1 atm enters a perfectly insulated, horizontal, circular duct at a velocity of 3 m/s. The duct diameter increases in the direction of flow as shown in the figure. There are no electrical lines or rotating shafts within the duct.



As the air flows through the duct, the temperature

A. increases

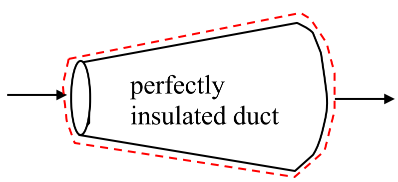
B. decreases

C. remains the same

D. there is insufficient information to determine the trend

SOLUTION:

Apply the First Law to the control volume shown below.



$$\frac{d}{dt}\int_{CV} e\rho\, dV + \int_{CS}\left(h+\tfrac{1}{2}V^2+gz\right)\left(\rho\mathbf{u}_{\text{rel}}\cdot d\mathbf{A}\right) = \dot{Q}_{\substack{\text{into}\\ \text{CV}}} + \dot{W}_{\substack{\text{other}\\ \text{on CV}}} \tag{1}$$

where

$$\frac{d}{dt}\int_{CV} e\rho dV = 0 \quad \text{(steady flow)} \tag{2}$$

$$\int_{CS}\left(h+\tfrac{1}{2}V^2+gz\right)\left(\rho\mathbf{u}_{\text{rel}}\cdot d\mathbf{A}\right) = \dot{m}\left[\Delta h+\tfrac{1}{2}\Delta\left(V^2\right)\right] \quad \text{(no elevation changes)} \tag{3}$$

$$\dot{Q}_{\substack{\text{into}\\ \text{CV}}} = 0 \quad \text{(perfectly insulated duct)} \tag{4}$$

$$\dot{W}_{\substack{\text{other}\\ \text{on CV}}} = 0 \quad \text{(the only work is pressure work)} \tag{5}$$

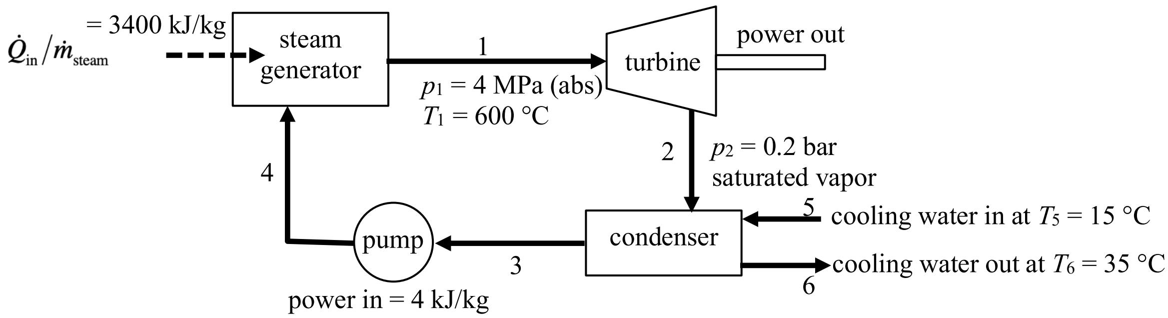
Substitute and simplify.

$$\dot{m}\left[\Delta h+\tfrac{1}{2}\Delta\left(V^2\right)\right] = 0 \;\Rightarrow\; \Delta h = -\tfrac{1}{2}\Delta\left(V^2\right) \tag{6}$$

Since $\Delta(V^2) < 0$ (from conservation of mass on the same CV), $\Delta h > 0$. Treating air as a perfect gas, $\Delta h = c_p \Delta T$ (note that if the air is assumed incompressible, then $\Delta h = c\Delta T$). $\boxed{\text{Hence, the temperature must be increasing}}$.

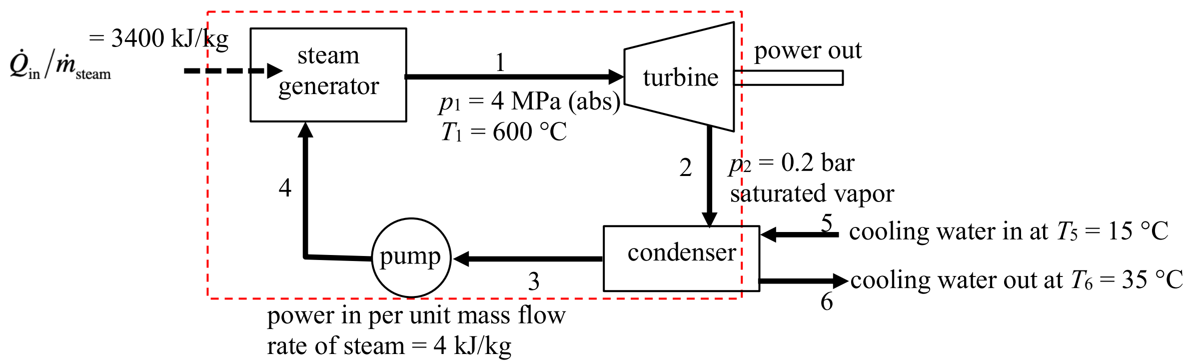
Steady-state operating data for a simple steam power plant are provided in the figure. Stray heat transfer and kinetic and potential energy effects can be ignored. Determine the:

a. thermal efficiency and
b. the ratio of the cooling water mass flow rate to the steam mass flow rate.



SOLUTION:

Consider the following control volume.



The efficiency of the system is the ratio of the rate at which work is produced by the system divided by the rate at which heat is put into the system,

$$\eta = \frac{\dot{W}_{\text{by system}}}{\dot{Q}_{\text{into system}}}, \tag{1}$$

or, since the rate at which heat enters the system is given in terms of heat rate per mass flow rate of steam,

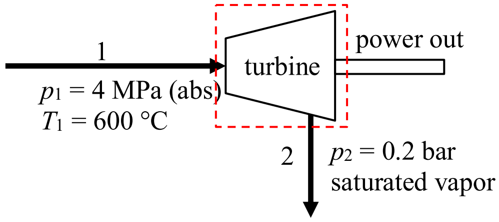
$$\eta = \frac{\dot{W}_{\text{by system}}/\dot{m}_{\text{steam}}}{\dot{Q}_{\text{into system}}/\dot{m}_{\text{steam}}}. \tag{2}$$

Note that the steam mass flow rates along connections 1 – 4 are the same since the system operates at steady conditions,

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \dot{m}_4 = \dot{m}. \tag{3}$$

The rate at which work is done by the system is the power produced by the turbine minus the power entering the system into the pump,

$$\dot{W}_{\text{by system}} = \dot{W}_{\text{by turbine}} - \dot{W}_{\text{on pump}}. \tag{4}$$



The power produced by the turbine may be found by applying the 1st Law to a control volume surrounding just the turbine,

$$\frac{d}{dt}\int_{CV} e\rho\, dV + \int_{CS}\left(h + \tfrac{1}{2}V^2 + gz\right)\left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = \dot{Q}_{\substack{\text{into}\\\text{CV}}} - \dot{W}_{\substack{\text{by}\\\text{CV}}}, \tag{5}$$

where,

$$\frac{d}{dt}\int_{CV} e\rho\, dV = 0 \quad \text{(steady operation)}, \tag{6}$$

$$\int_{CS}\left(h + \tfrac{1}{2}V^2 + gz\right)\left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = \dot{m}_{\text{steam}}\left(h_2 - h_1\right), \tag{7}$$

(kinetic and potential energy changes are negligible; mass flow rate is the same at 1 and 2 due to conservation of mass),

$$\dot{Q}_{\substack{\text{into}\\\text{CV}}} = 0 \quad \text{(adiabatic operation assumed)}. \tag{8}$$

Substitute and simplify,

$$\dot{m}_{\text{steam}}\left(h_2 - h_1\right) = -\dot{W}_{\text{by CV}} \Rightarrow \frac{\dot{W}_{\text{by CV}}}{\dot{m}_{\text{steam}}} = h_1 - h_2. \tag{9}$$

The specific enthalpies at the inlet may be found using the thermodynamic property tables for water (e.g., Table A-4 in Moran et al., 7th ed.),

h_1 = 3674.4 kJ/kg (superheated vapor, e.g., Table A-4 in Moran et al., 7th ed.)
h_2 = 2609.7 kJ/kg (saturated vapor, e.g., Table A-3 in Moran et al., 7th ed.)

$$\Rightarrow \frac{\dot{W}_{\text{by CV}}}{\dot{m}_{\text{steam}}} = 1064.7 \text{ kJ/kg} \tag{10}$$

Substituting this result into Eqs. (4) and (2) gives,

$$\frac{\dot{W}_{\text{by system}}}{\dot{m}_{\text{steam}}} = 1060.7 \text{ kJ/kg}, \tag{11}$$

$$\boxed{\eta = 0.312} \tag{12}$$

where

$$\dot{Q}_{\text{into system}}/\dot{m}_{\text{steam}} = 3400 \text{ kJ/kg and } \dot{W}_{\text{on pump}}/\dot{m}_{\text{steam}} = 4 \text{ kJ/kg}.$$

To find the ratio of the cooling water mass flow rate to the steam mass flow rate, apply conservation of energy to a control volume surrounding the entire system.

$$\frac{d}{dt}\int_{CV} e\rho\, dV + \int_{CS}\left(h + \tfrac{1}{2}V^2 + gz\right)\left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = \dot{Q}_{\substack{\text{into}\\\text{CV}}} - \dot{W}_{\substack{\text{by}\\\text{CV}}}, \tag{13}$$

where,

$$\frac{d}{dt}\int_{CV} e\rho\, dV = 0 \quad \text{(steady operation)}, \tag{14}$$

$$\int_{CS}\left(h + \tfrac{1}{2}V^2 + gz\right)\left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = \dot{m}_{\text{cw}}\left(h_6 - h_5\right) \quad \text{(cooling water)}, \tag{15}$$

(kinetic and potential energy changes are negligible; mass flow rate is the same at 5 and 6 due to conservation of mass),

$$\frac{\dot{Q}_{\text{into CV}}}{\dot{m}_{\text{steam}}} = 3400 \text{ kJ/kg (given)}, \tag{16}$$

$$\frac{\dot{W}_{\text{by system}}}{\dot{m}_{\text{steam}}} = 1060.7 \text{ kJ/kg (given)}. \tag{17}$$

Substitute and simplify,

$$\dot{m}_{\text{cw}}\left(h_6 - h_5\right) = \dot{m}_{\text{steam}}\frac{\dot{Q}_{\text{into CV}}}{\dot{m}_{\text{steam}}} - \dot{m}_{\text{steam}}\frac{\dot{W}_{\text{by system}}}{\dot{m}_{\text{steam}}}, \tag{18}$$

$$\boxed{\frac{\dot{m}_{\text{cw}}}{\dot{m}_{\text{steam}}} = \frac{\frac{\dot{Q}_{\text{into CV}}}{\dot{m}_{\text{steam}}} - \frac{\dot{W}_{\text{by system}}}{\dot{m}_{\text{steam}}}}{\left(h_6 - h_5\right)}}. \tag{19}$$

Using the given and calculated parameters,

$$\frac{\dot{Q}_{\text{into CV}}}{\dot{m}_{\text{steam}}} = 3400 \text{ kJ/kg (given)},$$

$$\frac{\dot{W}_{\text{by system}}}{\dot{m}_{\text{steam}}} = 1060.7 \text{ kJ/kg (calculated, Eq. (10))}$$

$h_5 = 62.99$ kJ/kg (20)

(subcooled water at $T_5 = 15$ °C, use $h_5 \approx h_l(T_5)$, e.g., Table A.2 in Moran et al., 7th ed.)

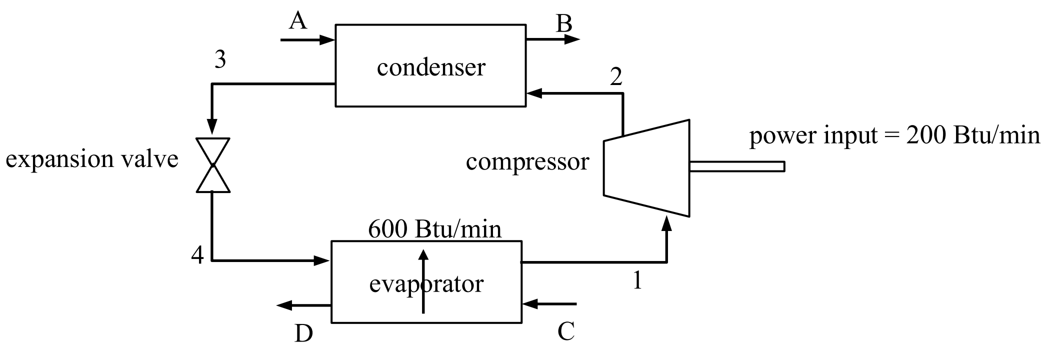
$h_6 = 146.68$ kJ/kg (21)

(subcooled water at $T_6 = 35$ °C, use $h_6 \approx h_l(T_6)$, e.g., Table A.2 in Moran et al., 7th ed.)

$$\Rightarrow \boxed{\frac{\dot{m}_{\text{cw}}}{\dot{m}_{\text{steam}}} = 28.0} \tag{22}$$

A residential air-conditioning system operates at steady state. Refrigerant 22 circulates through the components of the system. If the evaporator removes energy by heat transfer from the room air at a rate of 600 Btu/min, determine:

a. the rate of heat transfer between the compressor and the surroundings, in Btu/min, and
b. the coefficient of performance.

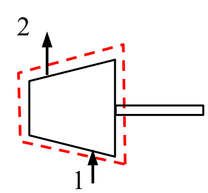


Location	Properties
A	outside air at 90 °F
B	air at a temperature greater than 90 °F
C	return room air at 75 °F
D	supply air to residence at a temperature less than 75 °F
1	Refrigerant 22, 120 psia, saturated vapor
2	Refrigerant 22, 225 psia, specific enthalpy of 130 Btu/lb$_m$
3	Refrigerant 22, 225 psia, 100 °F
4	Refrigerant 22, 62 °F

SOLUTION:

The rate of heat transfer between the compressor and the surroundings may be found by applied the 1st Law to a control volume surrounding the compressor,

$$\frac{dE_{\text{sys}}}{dt} = \sum_{\text{in}}\left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} - \sum_{\text{out}}\left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} + \dot{Q}_{\substack{\text{into}\\ \text{sys}}} + \dot{W}_{\substack{\text{other,}\\ \text{on sys}}}, \tag{1}$$



where,

$$\frac{dE_{\text{sys}}}{dt} = 0 \quad \text{(assuming steady conditions)}, \tag{2}$$

$$\sum_{\text{in}}\left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} - \sum_{\text{out}}\left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} = \left(h_1 - h_2\right)\dot{m}, \tag{3}$$

(assuming negligible differences in kinetic energy and potential energy across the compressor; the mass flow rate remains the same due to conservation of mass and the assumption of steady flow)

$$\dot{W}_{\substack{\text{other,}\\ \text{on sys}}} = \text{given (200 Btu/min)} \tag{4}$$

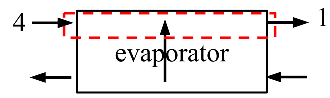
Substitute and simplify,

$$\underline{\dot{Q}_{\substack{\text{out of}\\ \text{sys}}} = \left(h_1 - h_2\right)\dot{m} + \dot{W}_{\substack{\text{other,}\\ \text{on sys}}}} \quad \text{(Note the change in the subscript for the heat transfer rate.)} \tag{5}$$

The specific enthalpy at state 2 is given in the problem statement (h_2 = 130 Btu/lb$_m$). The specific enthalpy at state 1 is found using a thermodynamic table (e.g., Table A.8E in Moran et al., 8th ed.) for Refrigerant 22 for saturated vapor at a pressure of 120 psia: h_1 = 109.88 Btu/lb$_m$.

The mass flow rate is not yet known, but can be determined by applying the 1st Law to the refrigerant in the evaporator where the energy transfer to the refrigerant via heat transfer is known,

$$\frac{dE_{\text{sys}}}{dt} = \sum_{\text{in}} \left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} - \sum_{\text{out}} \left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} + \dot{Q}_{\substack{\text{into}\\ \text{sys}}} + \dot{W}_{\substack{\text{other,}\\ \text{on sys}}}, \tag{6}$$



where,

$$\frac{dE_{\text{sys}}}{dt} = 0 \quad \text{(assuming steady conditions)}, \tag{7}$$

$$\sum_{\text{in}} \left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} - \sum_{\text{out}} \left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} = \left(h_4 - h_1\right)\dot{m}, \tag{8}$$

(assuming negligible differences in kinetic energy and potential energy across the evaporator; the mass flow rate remains the same due to conservation of mass and the assumption of steady flow)

$$\dot{Q}_{\text{into sys}} = \text{given (600 Btu/min)} \tag{9}$$

$$\dot{W}_{\substack{\text{other,}\\ \text{on sys}}} = 0 \quad \text{(no work other than pressure work is done on the evaporator)} \tag{10}$$

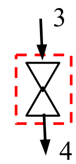
Substitute and simplify,

$$0 = \left(h_4 - h_1\right)\dot{m} + \dot{Q}_{\text{into sys}}, \tag{11}$$

$$\underline{\dot{m} = \frac{\dot{Q}_{\text{into sys}}}{h_1 - h_4}}. \tag{12}$$

The specific enthalpy of the Refrigerant 22 at state 1 was determined previously to be h_1 = 109.88 Btu/lb$_m$. The specific enthalpy at state 4 is not yet known, but it can be found by applying the 1st Law to a control volume surrounding the expansion valve,

$$\frac{dE_{\text{sys}}}{dt} = \sum_{\text{in}} \left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} - \sum_{\text{out}} \left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} + \dot{Q}_{\substack{\text{into}\\ \text{sys}}} + \dot{W}_{\substack{\text{other,}\\ \text{on sys}}}, \tag{13}$$



where,

$$\frac{dE_{\text{sys}}}{dt} = 0 \quad \text{(assuming steady conditions)}, \tag{14}$$

$$\sum_{\text{in}} \left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} - \sum_{\text{out}} \left(h + \tfrac{1}{2}V^2 + gz\right)\dot{m} = \left(h_3 - h_4\right)\dot{m}, \tag{15}$$

(assuming negligible differences in kinetic energy and potential energy across the valve; the mass flow rate remains the same due to conservation of mass and the assumption of steady flow)

$$\dot{Q}_{\text{into sys}} = 0 \quad \text{(assumed adiabatic)} \tag{16}$$

$$\dot{W}_{\substack{\text{other,}\\ \text{on sys}}} = 0 \quad \text{(no work other than pressure work)} \tag{17}$$

Substitute and simplify,

$$0 = \left(h_3 - h_4\right)\dot{m} \quad \Rightarrow \quad \underline{h_4 = h_3}. \tag{18}$$

The specific enthalpy at state 3 may be found using the thermodynamic tables given the pressure and temperature at that state. Using Table A-8E, we observe that at p_3 = 225 psia, $T_{3,\text{sat}}$ = 104.82 °F. Since T_3 = 100 °F < $T_{3,\text{sat}}$, the refrigerant must be in a compressed liquid phase at state 3. The specific enthalpy for a compressed liquid may be approximated as,

$$h_3(T_3, p_3) \approx h_l(T_3) + v(T_3)[p_3 - p_{\text{sat}}(T_3)], \tag{19}$$

where, from Table A-7E in Moran et al., 8th ed.,

$h_l(T_3)$ = 39.41 Btu/lb$_m$,

$v_l(T_3)$ = 0.01407 ft^3/lb$_m$,

$p_{\text{sat}}(T_3)$ = 210.69 psia,

$$\Rightarrow h_3(T_3, p_3) \approx 39.41 \text{ Btu/lb}_m + (0.01407 \text{ ft}^3/\text{lb}_m)[225 \text{ psia} - 210.69 \text{ psia}](1 \text{ Btu}/778.2 \text{ lb}_f.\text{ft})(144 \text{ in.}^2/\text{ft}^2), \tag{20}$$

$$\Rightarrow \underline{h_3(T_3, p_3) \approx 39.45 \text{ Btu/lb}_m}$$

$$\Rightarrow h_4 = h_3 = 39.45 \text{ Btu/lb}_m \text{ (from Eq. (18))} \tag{21}$$

Substitute values into Eq. (12),

$$\underline{\dot{m} = 8.52\ \text{lb}_\text{m}/\text{min}}. \tag{22}$$

Substitute values into Eq. (5),

$$\boxed{\dot{Q}_{\substack{\text{out of}\\ \text{compressor}}} = 28.58\ \text{Btu/min}}. \tag{23}$$

The coefficient of performance for this system, which is a type of refrigeration cycle since we're interested in removing energy via heat transfer from the room, is given by,

$$\text{COP}_\text{ref} = \frac{\dot{Q}_\text{into sys}}{\dot{W}_\text{on sys}} = \frac{600\ \text{Btu/min}}{200\ \text{Btu/min}} \quad \Rightarrow \quad \boxed{\text{COP}_\text{ref} = 3}. \tag{24}$$

where the system here is the refrigerant.