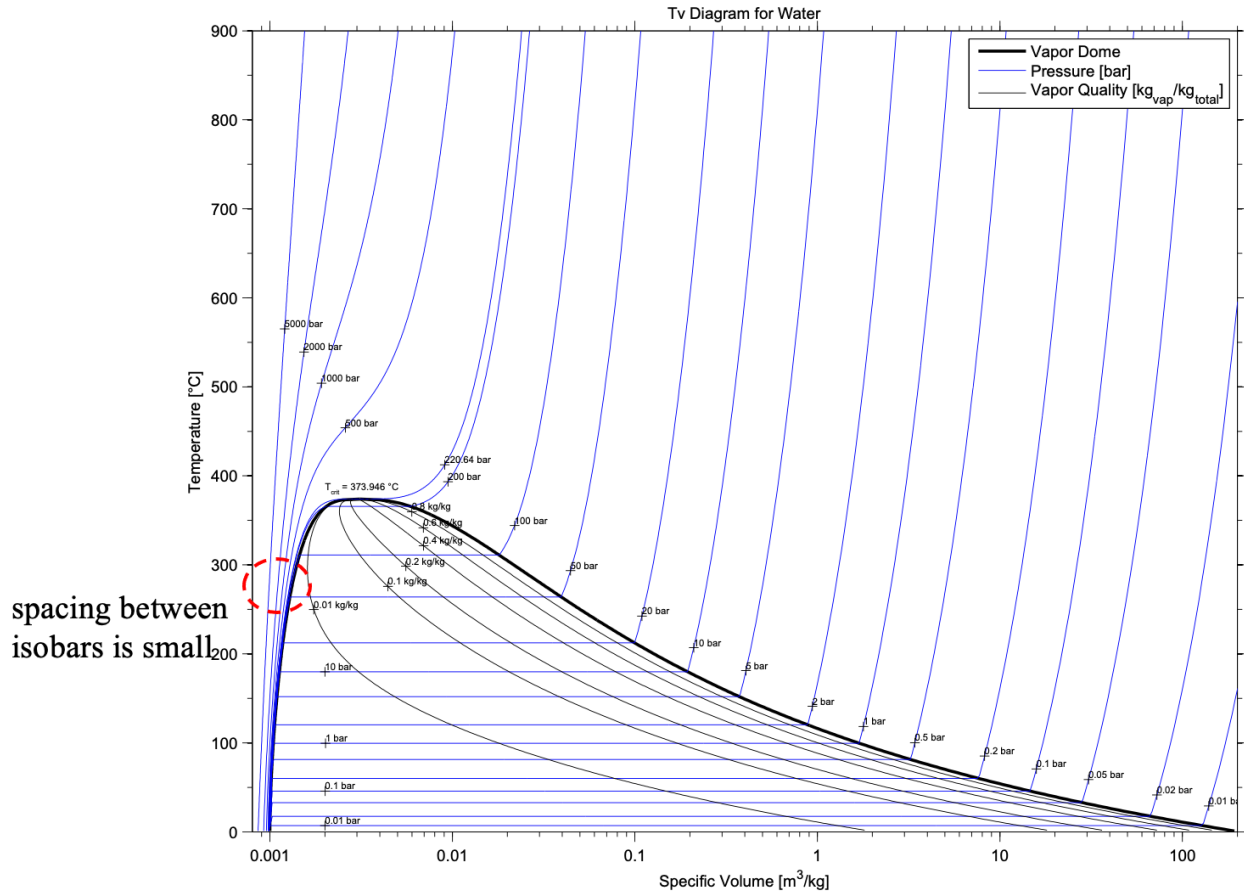FIGURE 3.25. A T-v plot for water highlighting the close spacing of isobars in the compressed liquid region.

3.5.4. Specific Heat

Recall that internal energy, sensible energy in particular, is related to temperature. Furthermore, we know from experience that some materials heat up at different rates than others. For example, 4.5 kJ of energy added to a 1 kg mass of iron will raise the iron's temperature from 20 °C to 30 °C. To raise 1 kg of water from 20 °C to 30 °C, however, requires 41.8 kJ; about nine times the amount of energy than is required to raise the iron's temperature an equivalent amount.

The property that quantifies the energy storage capability of matter is called the specific heat (aka specific heat capacity). The specific heat of a substance is the energy required to raise the temperature of a unit mass of a substance by one degree. In general, the energy required to raise the temperature of a substance will depend on the process path. Two particular processes of interest are where the system's volume is held constant while energy is added and where the pressure in the system is held constant while energy is added.

The specific heat at constant volume, c_v, is the energy required to raise the temperature of a unit mass by one degree during a constant volume process. For a pure, simple compressible substance with $u(T, v)$, the specific heat at constant volume is defined as,

$$c_v := \left.\frac{\partial u}{\partial T}\right|_v, \tag{3.67}$$

where the subscript "v" indicates that the volume is held constant during differentiation.

The specific heat at constant pressure, c_p, is the energy required to raise the temperature of a unit mass by one degree during a constant pressure process. For a pure, simple compressible substance with $h(T, p)$, the specific heat at constant pressure is defined as,

$$c_p := \left. \frac{\partial h}{\partial T} \right|_p, \tag{3.68}$$

where the subscript "p" indicates that the pressure is held constant during differentiation.

Notes:

(1) Note that c_p is greater than c_v since at constant pressure, the system is allowed to expand and the energy for this expansion work must also be supplied to the system. The exception is for incompressible substances where the system does not expand and thus $c_p = c_v$.

(2) The ratio of the specific heats frequently appears in thermo-fluid analyses and is defined as the specific heat ratio, k,

$$k := \frac{c_p}{c_v}. \tag{3.69}$$

Since $c_p \geq c_v$, $k \geq 1$.

(3) In general, c_v and c_p are functions of temperature and pressure; however, for certain classes of substances, such as incompressible materials and ideal gases, the pressure dependence is negligible and only the temperature dependence is considered.

(4) The specific heat at constant volume (c_v) can be used in processes where the volume varies. Likewise, the specific heat at constant pressure (c_p) can be used in processes where the pressure varies. The terms "at constant volume" and "at constant pressure" are used only in the definitions of the specific heats. For example, consider the constant pressure process path shown in Figure 3.26. Although the process is at constant pressure, we can still evaluate c_v at different points along the path since at each point one can imagine a local constant volume process.
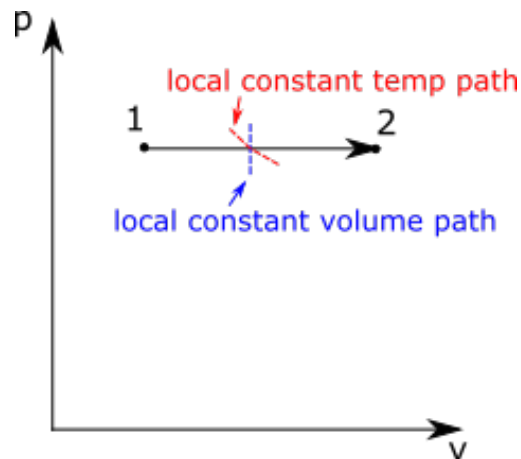

FIGURE 3.26. Illustration of a constant pressure process path. At each point along the path, one can locally evaluate the specific heat at constant volume, $c_v = \left. \frac{\partial u}{\partial T} \right|_v$, even though the process is at constant pressure.

3.5.5. Incompressible Substances

Liquids and solids are often approximated in engineering applications as being incompressible, meaning that their specific volumes (or densities) remain constant, i.e., $v = $ constant. For example, referring to Figure 3.25, large changes in pressure produce very small changes in specific volume in the compressed liquid region. Similarly, since the isobars are nearly vertical in the compressed liquid region, large changes in temperature at a given pressure also only change the specific volume only slightly.

In addition to the specific volume assumption, it's a reasonable approximation to assume that the specific internal energy is a function only of temperature, i.e., $u(T)$, for incompressible substances. Recall that when using the compressed liquid tables we often use the approximation that $u_{CL}(T,p) \approx u_l(T)$, i.e., for compressed liquids the pressure plays little role in determining the specific internal energy. From Eq. (3.67),

$$c_v = \frac{du}{dT}, \tag{3.70}$$

where $u(T)$ and, thus, $c_v = c_v(T)$. In addition,

$$h(T,p) = u(T) + pv \implies c_p = \left.\frac{\partial h}{\partial T}\right|_p = \frac{du}{dT} = c_v. \tag{3.71}$$

Thus,

$$\boxed{c_p(T) = c_v(T) = c(T)}, \tag{3.72}$$

for an incompressible substance. Note that $c = c(T)$, in general.

Notes:

(1) Recall that the properties of liquids can be approximated using the corresponding saturated liquid properties (Eqs. (3.61), (3.62), and (3.66)). The saturated liquid specific internal energy is only a function of temperature, identical to what is noted in Eq. (3.70).

(2) The change in specific internal energy may be found by integrating Eq. (3.70),

$$c = \frac{du}{dT} \implies du = cdT, \tag{3.73}$$

$$u_2 - u_1 = \int_{T_1}^{T_2} c(T)dT. \tag{3.74}$$

The specific enthalpy may be found using its definition and the previous relation,

$$h_2 - h_1 = u_2 - u_1 + (p_2 - p_1)v, \tag{3.75}$$

$$h_2 - h_1 = \int_{T_1}^{T_2} c(T)dT + (p_2 - p_1)v, \tag{3.76}$$

where $v = $ constant. In applications where the change in temperature is small ("small" depends on the substance, but as a rough guide assume less than a few hundreds of Kelvin), the specific heat may be reasonably assumed to be constant and thus Eqs. (3.74) and (3.76) may be written as,

$$u_2 - u_1 = c(T_2 - T_1), \tag{3.77}$$

$$h_2 - h_1 = c(T_2 - T_1) + (p_2 - p_1)v. \tag{3.78}$$

(3) Values for the specific heat for liquids and solids are often tabulated in the back of textbooks (e.g., Table A-1 in Moran et al., 8th ed.) or can be found online.

A passive solar house that is losing heat to the outdoors at an average rate of 50,000 kJ/hr is maintained at 22 °C at all times during a winter night for 10 hr. The house is to be heated by 50 glass containers each containing 20 L of water that is heated to 80 °C during the day by absorbing solar energy. A thermostat-controlled, 15 kW back-up electric resistance heater turns on whenever necessary to keep the house at 22 °C.

a.   How long will the electric heating system need to run during the night?
b.   How long would the electric heater run during the night if the house did not incorporate solar heating?
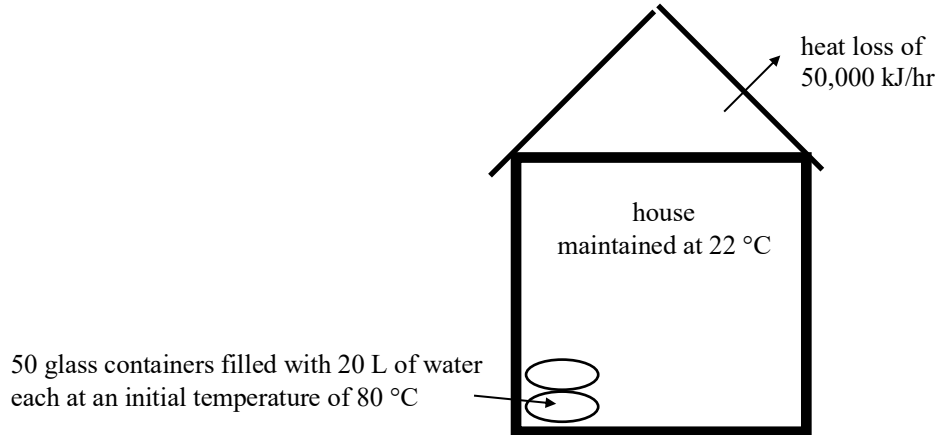
SOLUTION:

Apply the 1st Law to the house.
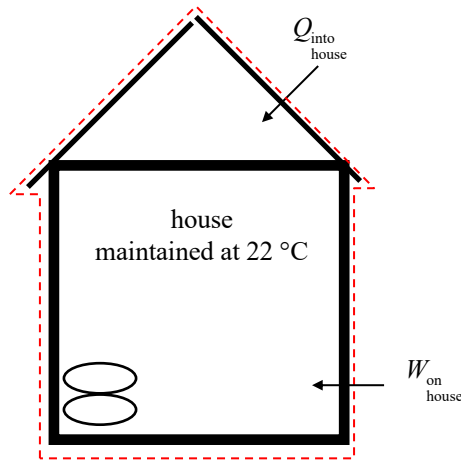
$$\Delta E_{system} = Q_{\substack{into \\ system}} + W_{\substack{on \\ system}} \tag{1}$$

The change in total energy of the house will consist of the change in the internal energy (potential and kinetic energy changes will be negligible). Furthermore, the total internal energy change will include the total energy change in the house structure, house air, and water tanks.

$$\Delta E_{system} = \Delta U_{system} = \Delta U_{\substack{house \\ structure}} + \Delta U_{air} + \Delta U_{water} \tag{2}$$

Since the house structure and air are maintained at a constant temperature, $\Delta U_{house} = \Delta U_{air} = 0$. Hence, Eqn. (1) can be re-written as:

$$\Delta U_{water} = Q_{\substack{into \\ system}} + W_{\substack{on \\ system}} \tag{3}$$

The total change in the internal energy of the water (assuming an incompressible fluid) is given by:
$$\Delta U_{\text{water}} = m_{\text{water}} c_{\text{water}} \left(T_{f,\text{water}} - T_{i,\text{water}} \right) \tag{4}$$

The total heat added to the house is:
$$Q_{\underset{\text{system}}{\text{into}}} = \left(-50,000 \text{ kJ/hr} \right) \left(10 \text{ hr} \right) = -500,000 \text{ kJ} \tag{5}$$
and the total work done on the house by the electric heater is:
$$W_{\underset{\text{system}}{\text{on}}} = \left(15 \text{ kW} \right) \Delta t \tag{6}$$
where Δt is the time over which the heater operates.

Substitute Eqns. (4)-(6) into Eqn. (3).
$$m_{\text{water}} c_{\text{water}} \left(T_{f,\text{water}} - T_{i,\text{water}} \right) = -500,000 \text{ kJ} + \left(15 \text{ kW} \right) \Delta t \tag{7}$$

Using the given parameters in Eqn. (7).
|		
m_{water}	=	50(20 L)(0.001 m³/L)(1000 kg/m³) = 1000 kg
c_{water}	=	4.179 kJ/(kg·K) (from a thermodynamics table)
$T_{f,\text{water}}$	=	22 °C
$T_{i,\text{water}}$	=	80 °C
$\Rightarrow$ Δt	=	4.8 hrs Hence, the heater must be on for 4.8 hrs at night with the water tanks.

If the water containers were not present, then the left-hand side of Eqn (7) would be zero ($\Delta U_{\text{water}} = 0$) and:
$$\Rightarrow \quad \Delta t \quad = \quad 9.3 \text{ hrs} \quad \text{Hence, the heater must be on for 9.3 hrs at night without the water tanks.}$$

A steel rivet of mass 2 lb_m, initially at 1000 °F, is placed in a large tank containing 5 ft³ of liquid water initially at 70 °F. Eventually, the rivet and water cool back to 70 °F as a result of heat transfer to the surroundings. Taking the rivet and water as the system, determine the heat transfer, in Btu, to the surroundings. The specific heat for steel is 0.11 Btu/(lb_m·°R).

SOLUTION:

Apply the 1st Law to the rivet/water system.
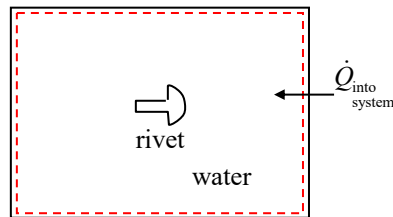
$$\Delta E_{\substack{system}} = Q_{\substack{into \\ system}} + W_{\substack{on \\ system}} \tag{1}$$

Assuming that the tank is rigid, $W_{on\ system} = 0$. Furthermore, the change in the total energy of the system will be due solely to changes in the internal energy, *i.e.* $\Delta E_{system} = \Delta U_{system}$ where:

$$\Delta U_{\substack{system}} = \Delta U_{\substack{rivet}} + \Delta U_{\substack{water}}$$

$$= m_{rivet} c_{rivet} \left(T_{f,rivet} - T_{i,rivet} \right) + m_{water} c_{water} \left(T_{f,water} - T_{i,water} \right) \tag{2}$$

Substitute Eqn. (2) into Eqn. (1) and simplify.

$$Q_{\substack{into \\ system}} = m_{rivet} c_{rivet} \left(T_{f,rivet} - T_{i,rivet} \right) + m_{water} c_{water} \left(T_{f,water} - T_{i,water} \right) \tag{3}$$

Since the final water temperature is the same as the initial water temperature, *i.e.* $T_{f,water} = T_{i,water}$, the change in the water internal energy will be zero. Hence,

$$Q_{\substack{into \\ system}} = m_{rivet} c_{rivet} \left(T_{f,rivet} - T_{i,rivet} \right) \tag{4}$$

Using the given parameters.

$T_{f,rivet}$ = 70 °F
$T_{i,rivet}$ = 1000 °F
c_{rivet} = 0.11 Btu/(lb_m·°R)
m_{rivet} = 2 lb_m
$\Rightarrow$ $Q_{into\ system}$ = -205 Btu (205 Btu leave the system)