FIGURE 3.24. Schematic showing how to linearly interpolate a property value.

3.5.2. Quality

Recall that Figure 3.19 presented two tables for saturated water properties (aka, water in two-phase, liquid-vapor equilibrium along the saturated liquid and the saturated vapor lines). The pressure and temperature are uniquely related in this two-phase region (and along the saturated liquid and vapor lines). The top table (Table A-2) presents the property data organized according to temperature while the bottom table (Table A-3) presents the same data organized according to pressure. The subscripts "f" and "g" in the table refer to "fluid" and "gas", which is a historical notation. It is better to refer to the properties as being either at the saturated liquid state (subscript "f" in the table) or in the saturated vapor state (subscript "g" in the table). Similar tables exist for two-phase solid-liquid and solid-vapor.

Within the two-phase liquid-vapor region (i.e, the vapor dome), the fraction of mass that is vapor is given by the quality, x, which is defined as,

$$\boxed{x := \frac{m_v}{m_l + m_v}} \tag{3.51}$$

where m_v and m_l are the masses of vapor and liquid, respectively. Note that the fraction of mass that is liquid is,

$$\frac{m_l}{m_l + m_v} = \frac{m_l + m_v - m_v}{m_l + m_v} = \frac{m_l + m_v}{m_l + m_v} - \frac{m_v}{m_l + m_v} = 1 - x \tag{3.52}$$

Hence, a quality of zero corresponds to a saturated liquid (all liquid, $m_v = 0$) while a quality of one corresponds to a saturated vapor (all vapor, $m_l = 0$). The quality can be used to determine the value of properties within the two-phase region, given the saturated liquid and saturated vapor properties. For example, the specific volume of a mixture (subscript "m") of liquid (subscript "l") and vapor (subscript "v") in equilibrium (i.e., in the vapor dome), assuming the quality x is known, is,

$$V_m = V_l + V_v \tag{3.53}$$

$$v_m = \frac{V_m}{m_m} = \frac{V_l + V_v}{m_m} = \frac{V_l}{m_m} + \frac{V_v}{m_m} \tag{3.54}$$

where V_m is the total volume of the mixture. The quantity m_m is the total mass of the mixture, i.e., $m_m = m_l + m_v$. Hence,

$$v_m = \frac{V_l}{m_l + m_v} + \frac{V_v}{m_l + m_v} = \frac{m_l v_l}{m_l + m_v} + \frac{m_v + v_v}{m_l + m_v} = \left(\frac{m_l}{m_l + m_v}\right) v_l + \left(\frac{m_v}{m_l + m_v}\right) v_v \tag{3.55}$$

where the volume is related to the specific volume via $V = mv$. Making use of Eqs. (3.51) and (3.52),

$$\boxed{v_m = (1 - x)\, v_l + x v_v} \tag{3.56}$$

Thus, the specific volume of a mixture of liquid and vapor can be thought of as the specific volume of the saturated liquid multiplied by its mass fraction $((1 - x)v_l)$ plus the specific volume of the saturated vapor

multiplied by its mass fraction (xv_v). Equation (3.56) may also be re-arranged to give,

$$v_m = v_l + x \underbrace{(v_v - v_l)}_{=v_{lv}} \tag{3.57}$$

where v_{lv} is the change in the specific volume during vaporization (liquid turns to vapor). Hence, the specific volume of the liquid-vapor mixture is the specific volume of the liquid (v_l) plus the mass fraction that has turned to vapor multiplied by the change in specific volume during vaporization $(x(v_v - v_l))$.

A similar approach may be used to find other properties in the two-phase liquid-vapor region, such as specific internal energy, e.g.,

$$\boxed{u_m = (1-x)\, u_l + x u_v = u_l + x\, (u_v - u_l)} \tag{3.58}$$

What is the quality of water at a pressure of 1.00 bar (abs) and specific volume of 0.01 m³/kg?

SOLUTION:
The specific volume of a saturated substance is,
$$v = xv_v + (1-x)v_l . \tag{1}$$
Re-arrange to solve for the quality,
$$x = \frac{v - v_l}{v_v - v_l} . \tag{2}$$
For water at 1.00 bar (abs) (using Table A.3),
    v_v = 1.694 m³/kg,
    v_l = 1.0432*10⁻³ m³/kg.
Solving Eq. (2) when v = 0.01 m³/kg,
    $x = 0.0053.$

TABLE A-3

Properties of Saturated Water (Liquid–Vapor): Pressure Table

Pressure Conversions:
1 bar = 0.1 MPa
= 10² kPa

Press. bar	Temp. °C	Specific Volume m³/kg Sat. Liquid $v_f \times 10^3$	Specific Volume m³/kg Sat. Vapor v_g	Internal Energy kJ/kg Sat. Liquid u_f	Internal Energy kJ/kg Sat. Vapor u_g	Enthalpy kJ/kg Sat. Liquid h_f	Enthalpy kJ/kg Evap. h_{fg}	Enthalpy kJ/kg Sat. Vapor h_g	Entropy kJ/kg·K Sat. Liquid s_f	Entropy kJ/kg·K Sat. Vapor s_g	Press. bar
0.04	28.96	1.0040	34.800	121.45	2415.2	121.46	2432.9	2554.4	0.4226	8.4746	0.04
0.06	36.16	1.0064	23.739	151.53	2425.0	151.53	2415.9	2567.4	0.5210	8.3304	0.06
0.08	41.51	1.0084	18.103	173.87	2432.2	173.88	2403.1	2577.0	0.5926	8.2287	0.08
0.10	45.81	1.0102	14.674	191.82	2437.9	191.83	2392.8	2584.7	0.6493	8.1502	0.10
0.20	60.06	1.0172	7.649	251.38	2456.7	251.40	2358.3	2609.7	0.8320	7.9085	0.20
0.30	69.10	1.0223	5.229	289.20	2468.4	289.23	2336.1	2625.3	0.9439	7.7686	0.30
0.40	75.87	1.0265	3.993	317.53	2477.0	317.58	2319.2	2636.8	1.0259	7.6700	0.40
0.50	81.33	1.0300	3.240	340.44	2483.9	340.49	2305.4	2645.9	1.0910	7.5939	0.50
0.60	85.94	1.0331	2.732	359.79	2489.6	359.86	2293.6	2653.5	1.1453	7.5320	0.60
0.70	89.95	1.0360	2.365	376.63	2494.5	376.70	2283.3	2660.0	1.1919	7.4797	0.70
0.80	93.50	1.0380	2.087	391.58	2498.8	391.66	2274.1	2665.8	1.2329	7.4346	0.80
0.90	96.71	1.0410	1.869	405.06	2502.6	405.15	2265.7	2670.9	1.2695	7.3949	0.90
1.00	99.63	1.0432	1.694	417.36	2506.1	417.46	2258.0	2675.5	1.3026	7.3594	1.00
1.50	111.4	1.0528	1.159	466.94	2519.7	467.11	2226.5	2693.6	1.4336	7.2233	1.50
2.00	120.2	1.0605	0.8857	504.49	2529.5	504.70	2201.9	2706.7	1.5301	7.1271	2.00

A closed, rigid tank fitted with a fine-wire electric resistor is filled with Refrigerant 22, initially at -10 °C, a quality of 80%, and a volume of 0.01 m³. A 12 V battery provides a 5 A current to the resistor for 5 min. If the final temperature of the refrigerant is 40 °C, determine the heat transfer, in kJ, from the refrigerant.
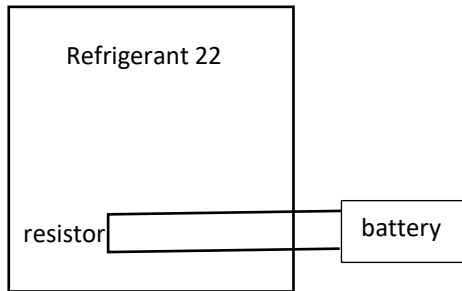
SOLUTION:
The heat transferred from the refrigerant to the surroundings may be found using the 1ˢᵗ Law applied to the refrigerant (our system),
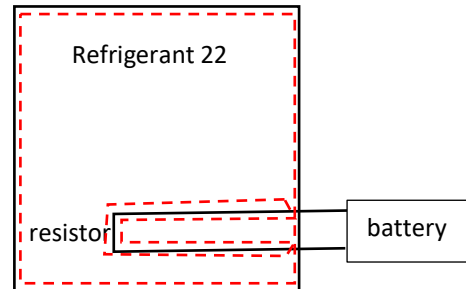
$$\Delta E_{R22} = Q_{\underset{R22}{into}} + W_{\underset{R22}{on}} \quad \Rightarrow \quad Q_{\underset{R22}{into}} = \Delta E_{R22} - W_{\underset{R22}{on}} , \tag{1}$$

where,

$$\Delta E_{R22} = \Delta U_{R22} = U_2 - U_1 = m(u_2 - u_1), \tag{2}$$

assuming that other forms of energy change, e.g., kinetic and potential, are negligible. Note that since the container is closed, the initial and final refrigerant masses will be the same. Furthermore, the resistor wire is not considered to be part of the system.

The specific internal energy at state 1 is also found using the thermodynamic property tables,

$$u_1 = xu_v + (1-x)u_l, \tag{3}$$

where, at -10 °C in the saturated liquid-vapor phase,

x = 0.80,
u_v = 223.02 kJ/kg,
u_l = 33.27 kJ/kg,
$\Rightarrow u_1$ = 185.07 kJ/kg.

The specific volume at state 1 may be found in a similar manner,

$$v_1 = xv_v + (1-x)v_l, \tag{4}$$

where,

x = 0.80,
v_v = 0.0652 m³/kg,
v_l = 0.7606*10⁻³ m³/kg,
$\Rightarrow v_1$ = 0.0523 m³/kg.

The mass of the refrigerant may be found from the initial state,

$$m = \frac{V}{v_1}, \quad \text{(The electrical wire volume is assumed negligible compared to the tank volume.)} \tag{5}$$

where,

V = 0.01 m³,
$\Rightarrow m$ = 0.191 kg.

The specific internal energy at state 2 (after the 5 min) is found using the thermodynamic property tables for Refrigerant 22 at a temperature of 40 °C and a specific volume of,

$v_2 = v_1$ (since the container volume and refrigerant mass remain constant). (6)

Using the two-phase liquid-vapor thermodynamic table, observe that at the final temperature of $T_2 = 40$ °C, the saturated vapor specific volume is 0.0151 m^3/kg, which is smaller than the specific volume at state 2, $v_2 = 0.0523$ m^3/kg. Hence, the refrigerant must be in a superheated vapor phase. Interpolating from the superheated vapor table using T_2 and v_2,

$u_2 = 250.33$ kJ/kg.

Combining m, u_2, and u_1, Eq. (2) becomes,

$\Delta U = 12.46$ kJ/kg.

There is no work acting on the refrigerant since the container volume remains constant and because the electrical work goes into the wire, which is not part of the system,

$W_{\substack{\text{on} \\ \text{R22}}} = 0$. (7)

There is, however, heat that is transferred from the wire into the system. This heat may be found by applying the 1st Law to the wire. Assuming steady conditions so that the change in total energy of the wire is zero, the total heat from the wire will equal the total (electrical) work done on the wire,

$\underbrace{\Delta E_{\text{wire}}}_{=0 \text{ (steady)}} = -Q_{\substack{\text{from} \\ \text{wire}}} + W_{\substack{\text{on} \\ \text{wire}}} \Rightarrow Q_{\substack{\text{from} \\ \text{wire}}} = W_{\substack{\text{on} \\ \text{wire}}}$, (8)

where the total work done on the wire is,

$W_{\substack{\text{on} \\ \text{wire}}} = VI\Delta t$ (assuming that neither the voltage nor current change over time Δt), (9)

with,

V = 12 V,
I = 5 A,
Δt = 5 min = 300 s,
$\Rightarrow W_{\text{on wire}} = 18$ kJ $\Rightarrow Q_{\text{from wire}} = 18$ kJ.

Break the heat into the refrigerant into two heat components, one from the wire and one from the remainder of the surroundings,

$Q_{\text{into R22}} = Q_{\substack{\text{into R22,} \\ \text{from wire}}} + Q_{\substack{\text{into R22,} \\ \text{from elsewhere}}}$. (10)

Substituting the expressions for heat, work, and energy into Eq. (1),

$Q_{\substack{\text{into R22,} \\ \text{from elsewhere}}} = \Delta U - Q_{\substack{\text{into R22,} \\ \text{from wire}}}$, (11)

$\Rightarrow Q_{\substack{\text{into R22,} \\ \text{from elsewhere}}} = $ -5.54 kJ.

Since we're interested in the heat from the refrigerant,

$\boxed{Q_{\substack{\text{from R22,} \\ \text{into elsewhere}}} = -Q_{\substack{\text{into R22,} \\ \text{from elsewhere}}} = 5.54 \text{ kJ.}}$ (12)

The process and states are shown schematically in the following T-v plot.