

### 3.5. Properties

In order to analyze real-world systems, we must be able to relate describe the properties and the relation between the properties of systems. In this section we examine how properties are presented and how they're related for a few important classes of materials. First, however, we must define a few terms:

- The phase of a substance has homogeneous chemical composition and physical structure. The three phases of matter are solid, liquid, and vapor.
- A pure substance is one that is uniform and invariable in chemical composition. A pure substance can exist in more than one phase, but all of the phases must have identical chemical compositions.
- A simple, compressible system is one in which electrical, magnetic, surface tension, gravitational, and motion effects are negligible. Systems consisting of pure water or uniform mixtures of non-reacting gases are examples of simple, compressible systems. The state principle states that any two independent intensive thermodynamic properties will uniquely define the system's state. If additional effects are significant, e.g., gravitational forces and accelerations, then additional properties are required, e.g., elevation and velocity.

#### *p-v-T Diagrams*

For simple, compressible systems, we can show the relationship between pressure, specific volume, and temperature in (3D)  $p-v-T$  diagrams (Figures 3.12 and 3.13). It's most convenient, however, to show (2D) projections of these diagrams onto the  $p-T$ ,  $p-v$ , and  $T-v$  planes. Please refer to the following two figures in the following discussion of the important features of these diagrams.

- Three single-phase regions can be identified on the plots: solid, liquid, and vapor.
  - The state in these regions is fixed by two independent properties, e.g.,  $(p, v)$ ,  $(p, T)$ , or  $(v, T)$ .
  - Above a critical pressure, temperature, and specific volume, known as the critical point, the difference between liquid and vapor is no longer discernible. The properties at the critical point are referred to as critical properties. Values for the critical pressure and temperature may often be found in the back of textbooks (e.g., Table A-1 in Moran et al., 8th ed.) or online.
- Three two-phase regions in which two phases exist in equilibrium can also be identified in the plots. These regions correspond to solid-vapor, solid-liquid, and liquid-vapor.
  - These regions correspond to situations involving melting or freezing (transition from solid to liquid or liquid to solid, respectively); vaporization (boiling – turning to vapor by increasing the temperature while holding pressure constant, or cavitation – turning to vapor by decreasing the pressure while holding the temperature constant) or condensation (transition from vapor to a liquid); and sublimation (transition from a solid to a vapor).
  - In these two-phase regions, the pressure and temperature are not independent. Hence, to define a state we need the specific volume,  $v$ , and either the pressure or the temperature,  $(p$  or  $T)$ .
- A single three-phase line, along which solid, liquid, and vapor exist in equilibrium. This line is referred to as the triple line.
- The state at which a phase change begins is known as a saturation state.
  - The two-phase (liquid-vapor) dome-shaped region is known as the vapor dome.
  - The lines bordering the vapor dome are known as the saturated liquid and saturated vapor lines.
  - The point at the top of the dome is known as the critical point, which is at the critical temperature,  $T_c$ , and critical pressure,  $p_c$ , and critical specific volume,  $v_c$ . The critical temperature is the maximum temperature at which liquid and vapor phases can co-exist in equilibrium.
- The projection of the  $p-v-T$  diagram onto the  $p-T$  plane is known as a phase diagram.
  - The two-phase regions are projected as lines in this view.
  - The saturation temperature is the temperature at which a phase change takes place at a given pressure. The corresponding pressure is known as the saturation pressure. For each saturation pressure there is a unique corresponding saturation temperature, hence,  $T$  and  $p$  are not independent during a change of phase.
  - The triple line projects to the triple point in a phase diagram.

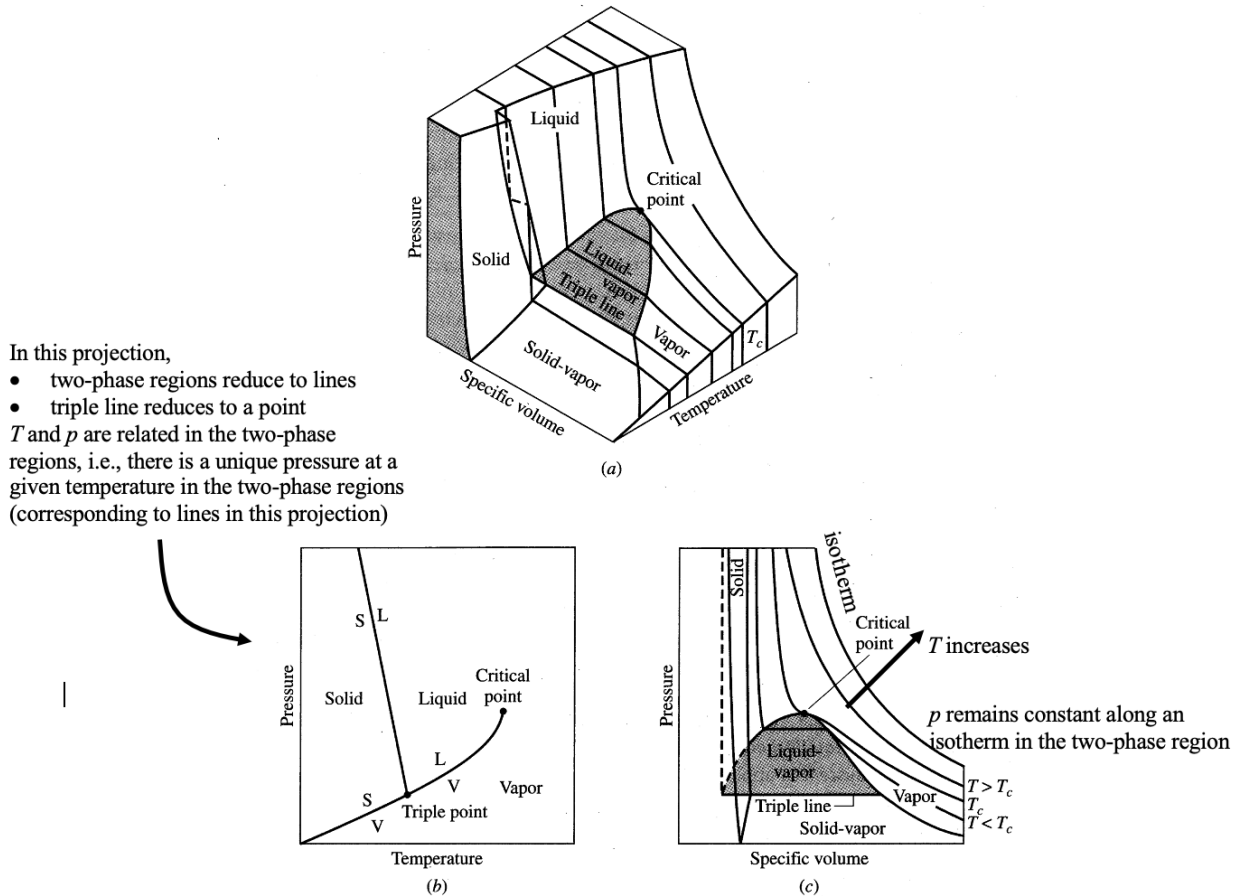


FIGURE 3.12.  $p$ - $v$ - $T$  surface and projections for a substance that expands on freezing. (a) 3D view, (b) phase diagram, (c)  $p$ - $v$  diagram. Figure 3.2 from Moran et al., 7th ed.

- The triple point of water occurs at 273.16 K and 0.6113 kPa (abs).
- For a substance that expands on freezing, e.g., water, the solid-liquid phase line tilts toward the left. For a substance that contracts upon freezing, the solid-liquid saturation line tilts toward the right.
- The projection of the  $p$ - $v$ - $T$  diagram onto the  $p$ - $v$  plane is also useful (Figure 3.14).
  - Lines of constant temperature are referred to as isotherms.
  - For  $T < T_c$ , the pressure remains constant in the two-phase regions along an isotherm. In the single-phase regions along an isotherm, the pressure decreases with increasing  $v$ .
  - The isotherm is at an inflection point when passing through the critical point.
- The projection of the  $p$ - $v$ - $T$  diagram onto the  $T$ - $v$  plane is frequently used (Figure 3.15).
  - Lines of constant pressure are known as isobars.
  - The temperature remains constant with pressure along an isobar in the two-phase region.
  - In the single-phase regions, the temperature increases with increasing specific volume along an isobar.
  - For pressures greater than the critical pressure, the temperature increases continuously with increasing specific volume along an isobar.

The  $p$ - $v$ - $T$  sketches shown in the previous figures are distorted from what they actual plots look like. Figure 3.16 shows an example of a  $p$ - $v$  plot for water drawn to scale. Note the use of logarithmic axes.

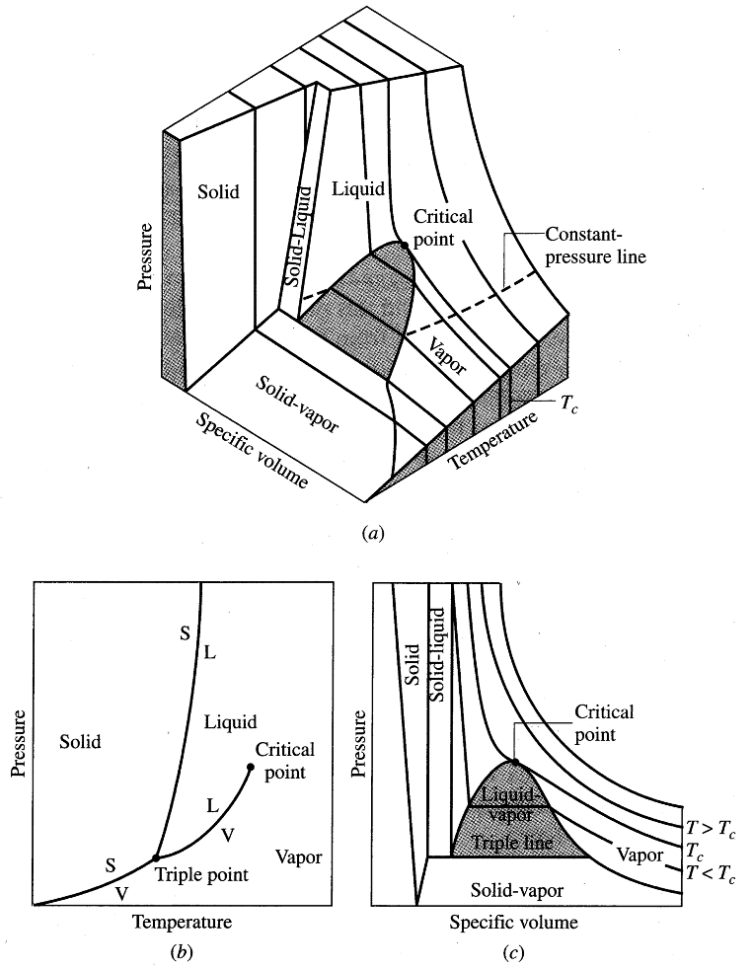


FIGURE 3.13.  $p$ - $v$ - $T$  surface and projections for a substance that contracts on freezing. (a) 3D view, (b) phase diagram, (c)  $p$ - $v$  diagram. Figure 3.2 from Moran et al., 7th ed.

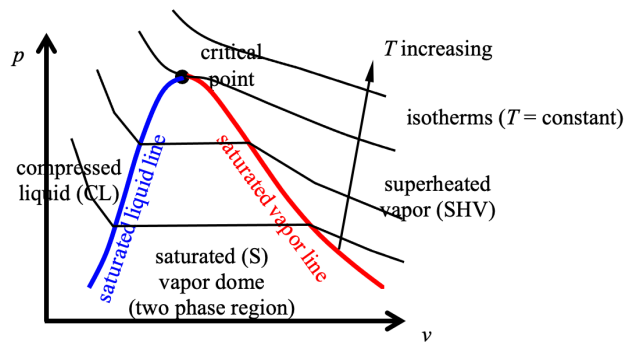
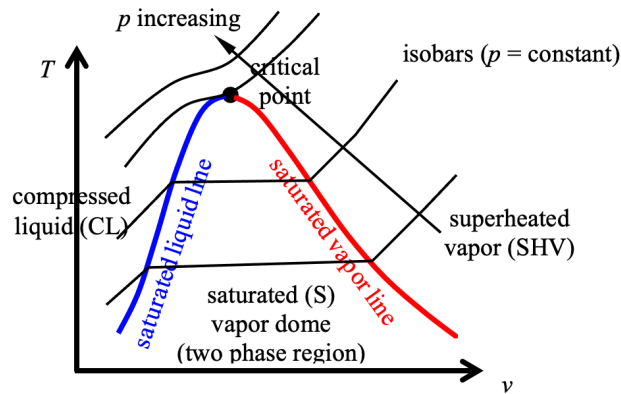
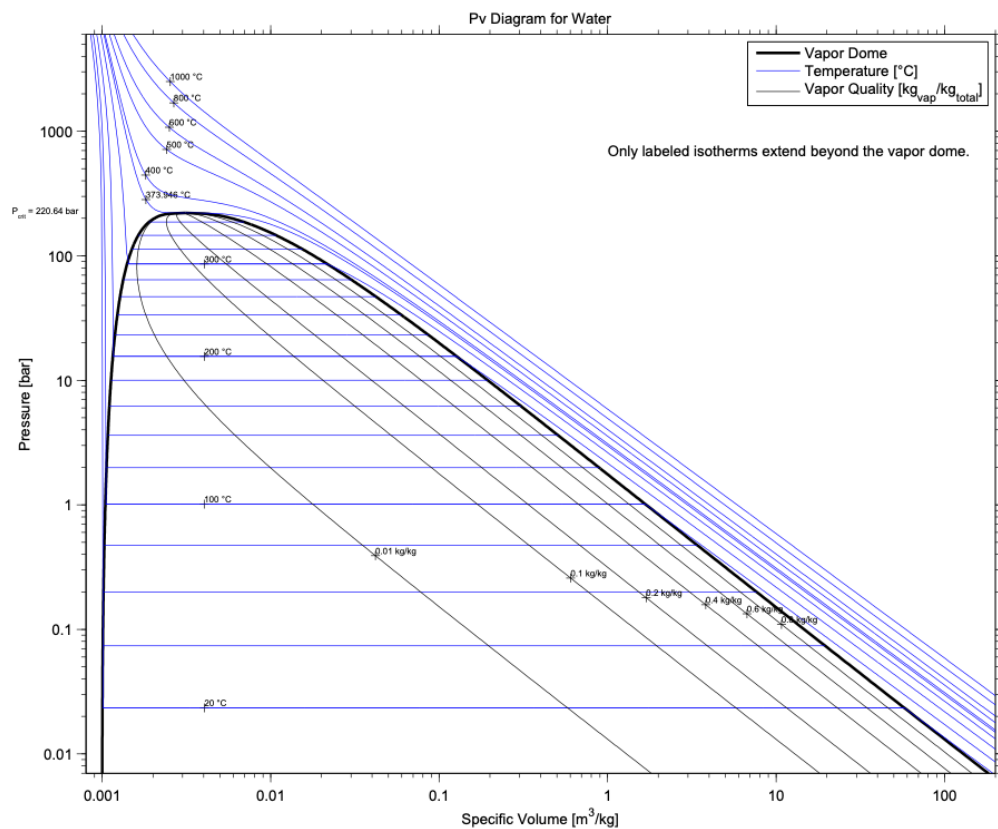


FIGURE 3.14. Sketch of a  $p$ - $v$  plot.


 FIGURE 3.15. Sketch of a  $T-v$  plot.

 FIGURE 3.16. A  $p-v$  plot for water. Note the logarithmic scales.

Now consider the  $T-v$  projection more closely and, specifically, the region near the vapor dome as shown in Figure 3.17.

- The phase of a substance to the left of the vapor dome is known as a liquid, subcooled liquid, or compressed liquid (CL).
  - Point “1” in the figure is in this liquid region.
  - The term “subcooled” refers to the fact that along an isobar, the temperature is too low for the substance to be a vapor.



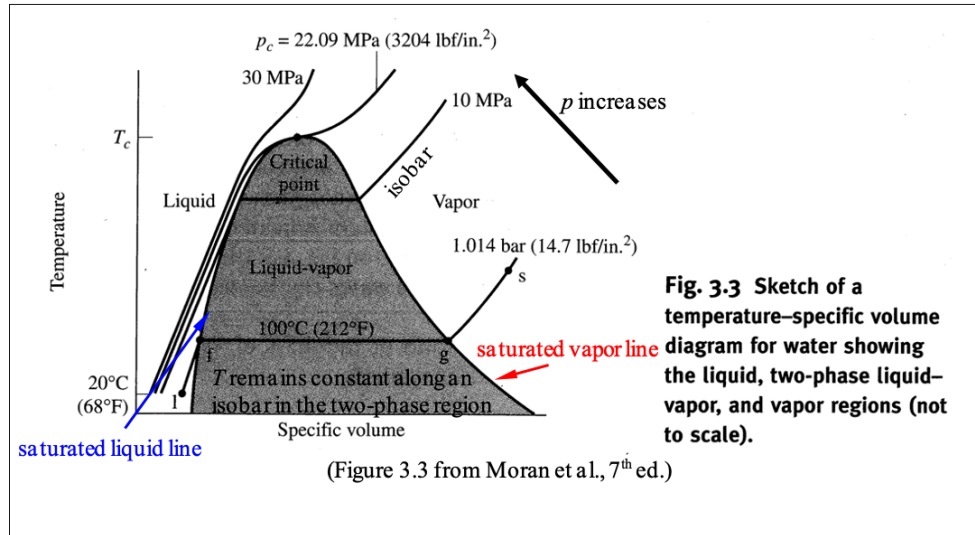


FIGURE 3.17. Sketch of a  $T$ - $v$  diagram. Figure 3.3 from Moran et al., 7th ed.

- The term “compressed” refers to the fact that at a given temperature, the pressure is larger than the pressure required to reach the saturation state. The pressure increases moving upward and toward the left across the isobars.
- The phase of a substance to the right of the vapor dome is known as a superheated vapor (SHV).
  - The term “superheated” refers the fact that the temperature is larger than what would be required to reach a liquid-vapor saturation state along a given isobar.
  - Point “s” is in the superheated vapor region.
- Within the vapor dome (the two-phase, or saturated (S) region), both liquid and vapor can exist in equilibrium. In order to specify how much of the substance is in liquid form versus vapor form, we define the quality of the mixture,  $x$ , which is the mass fraction of vapor at a given state, i.e., how much of the total mass is vapor. We’ll come back to this term a little later. For now, it’s sufficient to know that, from the definition of quality, a quality of zero corresponds to a saturated liquid (no vapor) while a quality of one corresponds to a saturated vapor (all vapor). Similar parameters can be defined for two-phase regions consisting of solid-vapor and solid-liquid.
- There are three similar sounding terms used frequently in the two-phase region, but each of these terms represents a different thing:
  - saturated liquid: In this phase, the state is on the saturated liquid line, i.e., at the left edge of the saturated phase, which means it’s 100% liquid, i.e., it has a quality of zero.
  - saturated vapor: In this phase, the state is on the saturated vapor line, i.e., at the right edge of the saturated phase, which means it’s 100% vapor, i.e., it has a quality of one.
  - saturated: In this phase, the substance contains both liquid and vapor. The state is within the vapor dome. The quality for a saturated substance is between zero and one.

Although the property plots are helpful for qualitatively understanding the relationship between properties and phases of a substance, they’re not particularly useful for quantitative analysis. Fortunately, tables (and computer databases – see for example <http://webbook.nist.gov/chemistry/fluid/>) have been prepared that provide quantitative values for the relationship between properties.

Figure 3.18 highlights the saturation (liquid-vapor) region in a  $T$ - $v$  diagram. The properties corresponding to this region for water are given in Figure 3.19 as two tables. Recall that the pressure and temperature are related in this two-phase region. Thus, there is a unique temperature, known as the saturation temperature, at each pressure, known as the saturation pressure. The difference between the two tables is that one is

ordered according to convenient temperature increments (top table) while the other is ordered by convenient pressure increments (bottom table).

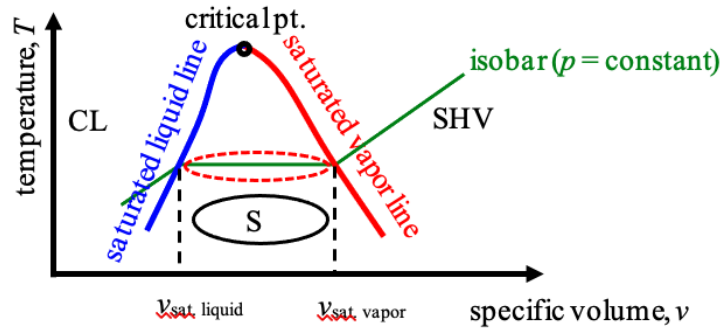


FIGURE 3.18. A sketch of a  $T-v$  diagram highlighting the region under the vapor dome corresponding to the saturation properties.

TABLE A-2

Properties of Saturated Water (Liquid-Vapor): Temperature Table

Pressure Conversions:  
1 bar = 0.1 MPa  
= 10<sup>5</sup> kPa

Temp. °C	Press. bar	Specific Volume m <sup>3</sup> /kg		Internal Energy kJ/kg		Enthalpy kJ/kg			Entropy kJ/kg · K		Temp. °C
		Sat. Liquid $v_f \times 10^3$	Sat. Vapor $v_g$	Sat. Liquid $u_f$	Sat. Vapor $u_g$	Sat. Liquid $h_f$	Evap. $h_{fg}$	Sat. Vapor $h_g$	Sat. Liquid $s_f$	Sat. Vapor $s_g$	
.01	0.00611	1.0002	206.136	0.00	2375.3	0.01	2501.3	2501.4	0.0000	9.1562	.01
4	0.00813	1.0001	157.232	16.77	2380.9	16.78	2491.9	2508.7	0.0610	9.0514	4
5	0.00872	1.0001	147.120	20.97	2382.3	20.98	2489.6	2510.6	0.0761	9.0257	5
6	0.00935	1.0001	137.734	25.19	2383.6	25.20	2487.2	2512.4	0.0912	9.0003	6
8	0.01072	1.0002	120.917	33.59	2386.4	33.60	2482.5	2516.1	0.1212	8.9501	8
10	0.01228	1.0004	106.379	42.00	2389.2	42.01	2477.7	2519.8	0.1510	8.9008	10
11	0.01312	1.0004	99.857	46.20	2390.5	46.20	2475.4	2521.6	0.1658	8.8765	11

TABLE A-3

Properties of Saturated Water (Liquid-Vapor): Pressure Table

Pressure Conversions:  
1 bar = 0.1 MPa  
= 10<sup>5</sup> kPa

Press. bar	Temp. °C	Specific Volume m <sup>3</sup> /kg		Internal Energy kJ/kg		Enthalpy kJ/kg			Entropy kJ/kg · K		Press. bar
		Sat. Liquid $v_f \times 10^3$	Sat. Vapor $v_g$	Sat. Liquid $u_f$	Sat. Vapor $u_g$	Sat. Liquid $h_f$	Evap. $h_{fg}$	Sat. Vapor $h_g$	Sat. Liquid $s_f$	Sat. Vapor $s_g$	
0.04	28.96	1.0040	34.800	121.45	2415.2	121.46	2432.9	2554.4	0.4226	8.4746	0.04
0.06	36.16	1.0064	23.739	151.53	2425.0	151.53	2415.9	2567.4	0.5210	8.3304	0.06
0.08	41.51	1.0084	18.103	173.87	2432.2	173.88	2403.1	2577.0	0.5926	8.2287	0.08
0.10	45.81	1.0102	14.674	191.82	2437.9	191.83	2392.8	2584.7	0.6493	8.1502	0.10
0.20	60.06	1.0172	7.649	251.38	2456.7	251.40	2358.3	2609.7	0.8320	7.9085	0.20
0.30	69.10	1.0223	5.229	289.20	2468.4	289.23	2336.1	2625.3	0.9439	7.7686	0.30

$p$  is an absolute pressure  
 $v$  is specific volume  
 (Note that  $v$  entries are multiplied by 1000, e.g.,  
 $v(2.5 \text{ MPa}, 20 \text{ }^\circ\text{C}) = 1.0006 \times 10^{-3} \text{ m}^3/\text{kg}$ .  
 $u$  is specific internal energy  
 $h$  is specific enthalpy  
 $s$  is specific entropy

FIGURE 3.19. Example table for the saturation properties for water organized by temperature (top) and pressure (bottom). These tables are from Moran et al., 7th ed.

At each saturation temperature and pressure, the tables provide the specific volume and other properties (specific internal energy, specific enthalpy, and specific entropy) at saturated liquid and saturated vapor conditions. The actual properties of the water will lie somewhere between or equal to the saturated liquid and saturated vapor conditions when the water is in a saturated state, e.g.,  $v_{\text{sat. liquid}} \leq v \leq v_{\text{sat. vapor}}$ .

What the actual property values are depends on the quality of the water (the mass fraction of vapor in the two-phase mixture), a topic to be discussed later.

Figure 3.20 presents super-heated vapor phase region in a  $T-v$  plot. Example properties for water in this region are given in Figure 3.21. Recall that the pressure and temperature are independent in this single-phase region and, thus, the table entries are organized based on a given pressure and range of temperatures. For each pressure, the saturation temperature (the temperature at which the vapor touches the vapor dome along the saturated vapor line) is also reported.

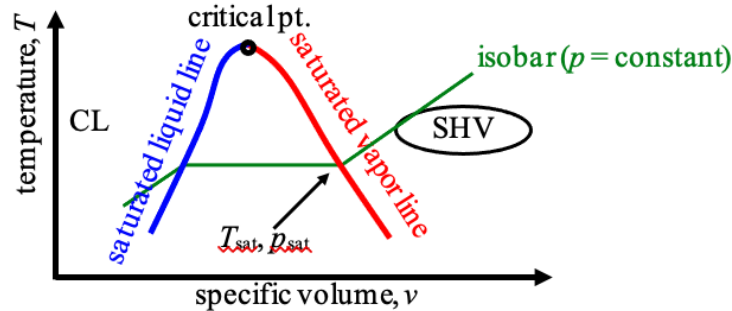


FIGURE 3.20. A sketch of a  $T-v$  diagram highlighting the super-heated vapor region.

TABLE A-4 Properties of Superheated Water Vapor									
$T$ °C	$v$ m <sup>3</sup> /kg	$u$ kJ/kg	$h$ kJ/kg	$s$ kJ/kg · K	$v$ m <sup>3</sup> /kg	$u$ kJ/kg	$h$ kJ/kg	$s$ kJ/kg · K	
$p = 0.06 \text{ bar} = 0.006 \text{ MPa}$ ( $T_{\text{sat}} = 36.16^\circ\text{C}$ )					$p = 0.35 \text{ bar} = 0.035 \text{ MPa}$ ( $T_{\text{sat}} = 72.69^\circ\text{C}$ )				
Sat.	23.739	2425.0	2567.4	8.3304	4.526	2473.0	2631.4	7.7158	
80	27.132	2487.3	2650.1	8.5804	4.625	2483.7	2645.6	7.7564	
120	30.219	2544.7	2726.0	8.7840	5.163	2542.4	2723.1	7.9644	
160	33.302	2602.7	2802.5	8.9693	5.696	2601.2	2800.6	8.1519	
200	36.383	2661.4	2879.7	9.1398	6.228	2660.4	2878.4	8.3237	
240	39.462	2721.0	2957.8	9.2982	6.758	2720.3	2956.8	8.4828	
280	42.540	2781.5	3036.8	9.4464	7.287	2780.9	3036.0	8.6314	
320	45.618	2843.0	3116.7	9.5859	7.815	2842.5	3116.1	8.7712	
360	48.696	2905.5	3197.7	9.7180	8.344	2905.1	3197.1	8.9034	
400	51.774	2969.0	3279.6	9.8435	8.872	2968.6	3279.2	9.0291	
440	54.851	3033.5	3362.6	9.9633	9.400	3033.2	3362.2	9.1490	
500	59.467	3132.3	3489.1	10.1336	10.192	3132.1	3488.8	9.3194	

Pressure Conversion:  
 1 bar = 0.1 MPa  
 = 10<sup>5</sup> kPa

H<sub>2</sub>O

$p$  is an absolute pressure  
 $v$  is specific volume  
 $u$  is specific internal energy  
 $h$  is specific enthalpy  
 $s$  is specific entropy

FIGURE 3.21. Example table for the superheated vapor (SHV) properties for water. This table is from Moran et al., 7th ed.

Figure 3.22 presents the compressed liquid phase region in a  $T-v$  plot. Example properties for water in this region are given in Figure 3.23. Recall that the pressure and temperature are independent in this single-phase region and, thus, the table entries are organized based on a given pressure and range of temperatures. For each pressure, the saturation temperature (the temperature at which the liquid touches the vapor dome along the saturated liquid line) is also reported.

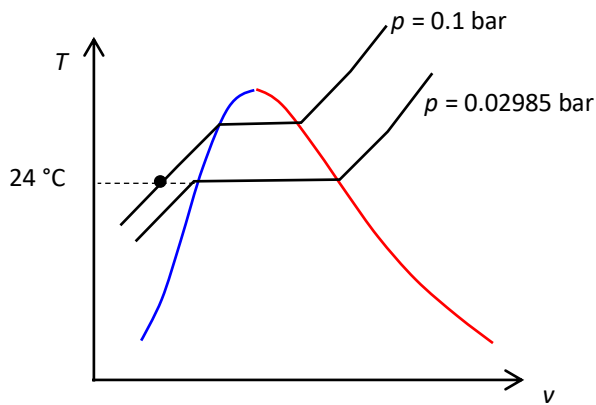
In what phase (liquid, saturated, vapor, etc.) is water for the following conditions?

- $T = 24\text{ }^\circ\text{C}$ ,  $p = 0.1\text{ bar (abs)}$
- $p = 4\text{ bar (abs)}$ ,  $T = 180\text{ }^\circ\text{C}$
- $p = 20\text{ bar (abs)}$ ,  $v = 0.01\text{ m}^3/\text{kg}$
- $T = 30\text{ }^\circ\text{C}$ ,  $p = 0.04246\text{ bar (abs)}$
- $T = 30\text{ }^\circ\text{C}$ ,  $v = 1.0043 \cdot 10^{-3}\text{ m}^3/\text{kg}$
- $p = 25\text{ bar (abs)}$ ,  $v = 0.07998\text{ m}^3/\text{kg}$

SOLUTION:

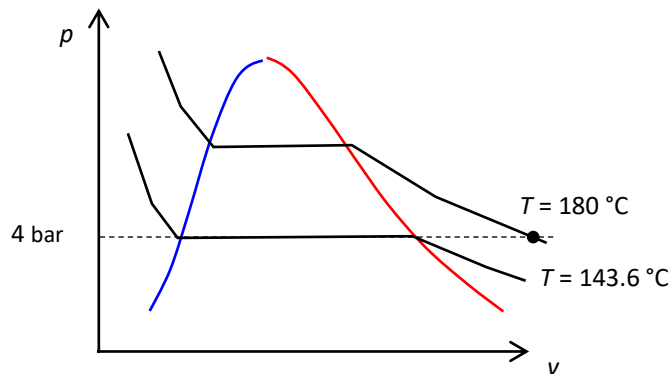
- $T = 24\text{ }^\circ\text{C}$ ,  $p = 0.1\text{ bar (abs)}$

Using Table A.2 (attached to the end of this example), at  $T = 24\text{ }^\circ\text{C}$ ,  $p_{\text{sat}} = 0.02985\text{ bar (abs)}$ . Since  $p > p_{\text{sat}}$ , the water will be in a compressed liquid phase.



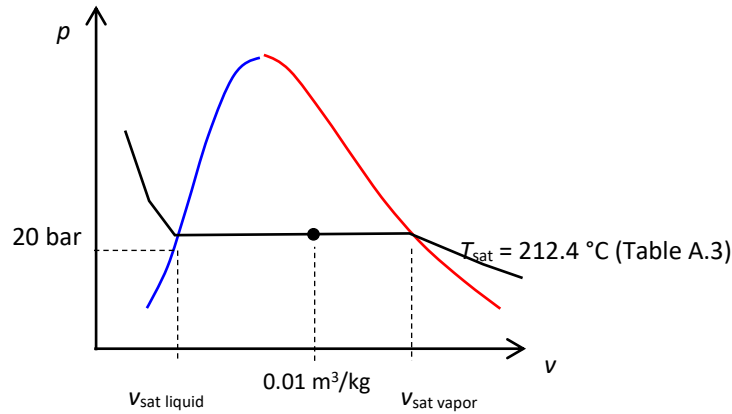
- $p = 4\text{ bar (abs)}$ ,  $T = 180\text{ }^\circ\text{C}$

Using Table A.3 (attached to the end of this example), at  $p = 4\text{ bar (abs)}$ ,  $T_{\text{sat}} = 143.6\text{ }^\circ\text{C}$ . Since  $T > T_{\text{sat}}$ , the water will be in a superheated vapor phase.



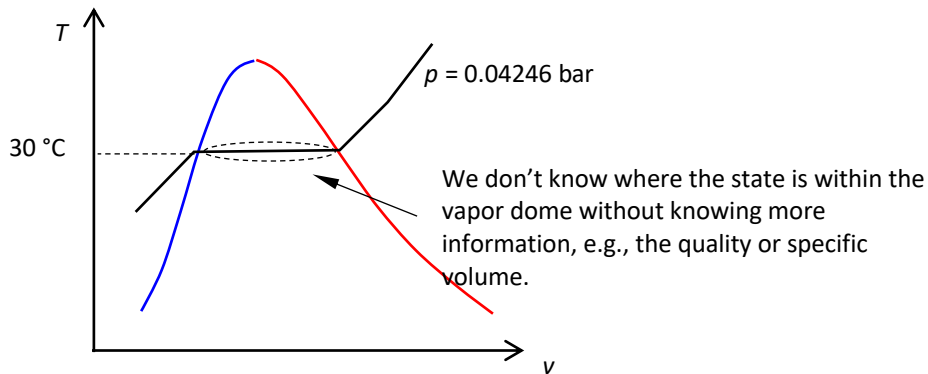
c.  $p = 20 \text{ bar (abs)}, v = 0.01 \text{ m}^3/\text{kg}$

Using Table A.3 (attached to the end of this example), at  $p = 20 \text{ bar (abs)}$ ,  $v_{\text{sat liquid}} = 1.1767 \cdot 10^{-3} \text{ m}^3/\text{kg}$  and  $v_{\text{sat vapor}} = 0.09963 \text{ m}^3/\text{kg}$ . Since  $v_{\text{sat liquid}} < v < v_{\text{sat vapor}}$ , the water will be in a saturated (i.e., two phase) phase.



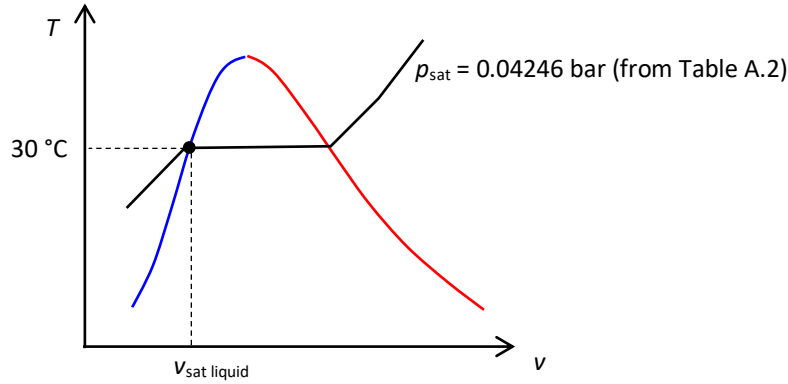
d.  $T = 30 \text{ }^\circ\text{C}, p = 0.04246 \text{ bar (abs)}$

Using Table A.2 (attached to the end of this example), at  $T = 30 \text{ }^\circ\text{C}$ ,  $p_{\text{sat}} = 0.04246 \text{ bar (abs)}$ . Since  $p = p_{\text{sat}}$ , the water will be in a saturated (two-phase) phase.



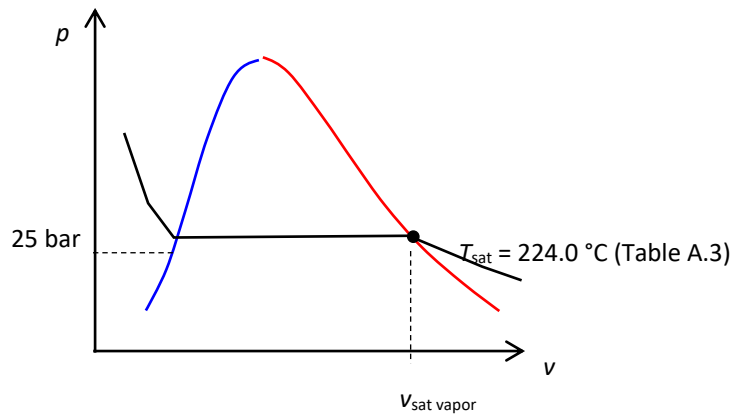
e.  $T = 30\text{ }^\circ\text{C}$ ,  $v = 1.0043 \cdot 10^{-3}\text{ m}^3/\text{kg}$

Using Table A.2 (attached to the end of this example), at  $T = 30\text{ }^\circ\text{C}$ ,  $v_{\text{sat liquid}} = 1.0043 \cdot 10^{-3}\text{ m}^3/\text{kg}$ . Since  $v = v_{\text{sat liquid}}$ , the water will be in saturated liquid phase.



f.  $p = 25\text{ bar (abs)}$ ,  $v = 0.07998\text{ m}^3/\text{kg}$

Using Table A.3 (attached to the end of this example), at  $p = 25\text{ bar (abs)}$ ,  $v_{\text{sat vapor}} = 0.07998\text{ m}^3/\text{kg}$ . Since  $v = v_{\text{sat vapor}}$ , the water will be in a saturated vapor phase.





**TABLE A-2**

**Properties of Saturated Water (Liquid-Vapor): Temperature Table**

Pressure Conversions:  
1 bar = 0.1 MPa  
= 10<sup>2</sup> kPa

Temp. °C	Press. bar	Specific Volume m <sup>3</sup> /kg		Internal Energy kJ/kg		Enthalpy kJ/kg			Entropy kJ/kg · K		Temp. °C
		Sat. Liquid <i>v<sub>f</sub></i> × 10 <sup>3</sup>	Sat. Vapor <i>v<sub>g</sub></i>	Sat. Liquid <i>u<sub>f</sub></i>	Sat. Vapor <i>u<sub>g</sub></i>	Sat. Liquid <i>h<sub>f</sub></i>	Evap. <i>h<sub>fg</sub></i>	Sat. Vapor <i>h<sub>g</sub></i>	Sat. Liquid <i>s<sub>f</sub></i>	Sat. Vapor <i>s<sub>g</sub></i>	
.01	0.00611	1.0002	206.136	0.00	2375.3	0.01	2501.3	2501.4	0.0000	9.1562	.01
4	0.00813	1.0001	157.232	16.77	2380.9	16.78	2491.9	2508.7	0.0610	9.0514	4
5	0.00872	1.0001	147.120	20.97	2382.3	20.98	2489.6	2510.6	0.0761	9.0257	5
6	0.00935	1.0001	137.734	25.19	2383.6	25.20	2487.2	2512.4	0.0912	9.0003	6
8	0.01072	1.0002	120.917	33.59	2386.4	33.60	2482.5	2516.1	0.1212	8.9501	8
10	0.01228	1.0004	106.379	42.00	2389.2	42.01	2477.7	2519.8	0.1510	8.9008	10
11	0.01312	1.0004	99.857	46.20	2390.5	46.20	2475.4	2521.6	0.1658	8.8765	11
12	0.01402	1.0005	93.784	50.41	2391.9	50.41	2473.0	2523.4	0.1806	8.8524	12
13	0.01497	1.0007	88.124	54.60	2393.3	54.60	2470.7	2525.3	0.1953	8.8285	13
14	0.01598	1.0008	82.848	58.79	2394.7	58.80	2468.3	2527.1	0.2099	8.8048	14
15	0.01705	1.0009	77.926	62.99	2396.1	62.99	2465.9	2528.9	0.2245	8.7814	15
16	0.01818	1.0011	73.333	67.18	2397.4	67.19	2463.6	2530.8	0.2390	8.7582	16
17	0.01938	1.0012	69.044	71.38	2398.8	71.38	2461.2	2532.6	0.2535	8.7351	17
18	0.02064	1.0014	65.038	75.57	2400.2	75.58	2458.8	2534.4	0.2679	8.7123	18
19	0.02198	1.0016	61.293	79.76	2401.6	79.77	2456.5	2536.2	0.2823	8.6897	19
20	0.02339	1.0018	57.791	83.95	2402.9	83.96	2454.1	2538.1	0.2966	8.6672	20
21	0.02487	1.0020	54.514	88.14	2404.3	88.14	2451.8	2539.9	0.3109	8.6450	21
22	0.02645	1.0022	51.447	92.32	2405.7	92.33	2449.4	2541.7	0.3251	8.6229	22
23	0.02810	1.0024	48.574	96.51	2407.0	96.52	2447.0	2543.5	0.3393	8.6011	23
24	0.02985	1.0027	45.883	100.70	2408.4	100.70	2444.7	2545.4	0.3534	8.5794	24
25	0.03169	1.0029	43.360	104.88	2409.8	104.89	2442.3	2547.2	0.3674	8.5580	25
26	0.03363	1.0032	40.994	109.06	2411.1	109.07	2439.9	2549.0	0.3814	8.5367	26
27	0.03567	1.0035	38.774	113.25	2412.5	113.25	2437.6	2550.8	0.3954	8.5156	27
28	0.03782	1.0037	36.690	117.42	2413.9	117.43	2435.2	2552.6	0.4093	8.4946	28
29	0.04008	1.0040	34.733	121.60	2415.2	121.61	2432.8	2554.5	0.4231	8.4739	29
30	0.04246	1.0043	32.894	125.78	2416.6	125.79	2430.5	2556.3	0.4369	8.4533	30
31	0.04496	1.0046	31.165	129.96	2418.0	129.97	2428.1	2558.1	0.4507	8.4329	31
32	0.04759	1.0050	29.540	134.14	2419.3	134.15	2425.7	2559.9	0.4644	8.4127	32
33	0.05034	1.0053	28.011	138.32	2420.7	138.33	2423.4	2561.7	0.4781	8.3927	33
34	0.05324	1.0056	26.571	142.50	2422.0	142.50	2421.0	2563.5	0.4917	8.3728	34
35	0.05628	1.0060	25.216	146.67	2423.4	146.68	2418.6	2565.3	0.5053	8.3531	35
36	0.05947	1.0063	23.940	150.85	2424.7	150.86	2416.2	2567.1	0.5188	8.3336	36
38	0.06632	1.0071	21.602	159.20	2427.4	159.21	2411.5	2570.7	0.5458	8.2950	38
40	0.07384	1.0078	19.523	167.56	2430.1	167.57	2406.7	2574.3	0.5725	8.2570	40
45	0.09593	1.0099	15.258	188.44	2436.8	188.45	2394.8	2583.2	0.6387	8.1648	45

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**TABLE A-3**

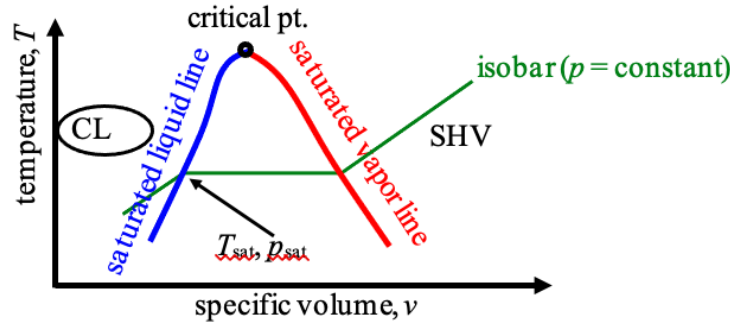
**Properties of Saturated Water (Liquid–Vapor): Pressure Table**

Pressure Conversions:  
1 bar = 0.1 MPa  
= 10<sup>5</sup> kPa

Press. bar	Temp. °C	Specific Volume m <sup>3</sup> /kg		Internal Energy kJ/kg		Enthalpy kJ/kg			Entropy kJ/kg · K		Press. bar
		Sat. Liquid v <sub>f</sub> × 10 <sup>3</sup>	Sat. Vapor v <sub>g</sub>	Sat. Liquid u <sub>f</sub>	Sat. Vapor u <sub>g</sub>	Sat. Liquid h <sub>f</sub>	Evap. h <sub>fg</sub>	Sat. Vapor h <sub>g</sub>	Sat. Liquid s <sub>f</sub>	Sat. Vapor s <sub>g</sub>	
0.04	28.96	1.0040	34.800	121.45	2415.2	121.46	2432.9	2554.4	0.4226	8.4746	0.04
0.06	36.16	1.0064	23.739	151.53	2425.0	151.53	2415.9	2567.4	0.5210	8.3304	0.06
0.08	41.51	1.0084	18.103	173.87	2432.2	173.88	2403.1	2577.0	0.5926	8.2287	0.08
0.10	45.81	1.0102	14.674	191.82	2437.9	191.83	2392.8	2584.7	0.6493	8.1502	0.10
0.20	60.06	1.0172	7.649	251.38	2456.7	251.40	2358.3	2609.7	0.8320	7.9085	0.20
0.30	69.10	1.0223	5.229	289.20	2468.4	289.23	2336.1	2625.3	0.9439	7.7686	0.30
0.40	75.87	1.0265	3.993	317.53	2477.0	317.58	2319.2	2636.8	1.0259	7.6700	0.40
0.50	81.33	1.0300	3.240	340.44	2483.9	340.49	2305.4	2645.9	1.0910	7.5939	0.50
0.60	85.94	1.0331	2.732	359.79	2489.6	359.86	2293.6	2653.5	1.1453	7.5320	0.60
0.70	89.95	1.0360	2.365	376.63	2494.5	376.70	2283.3	2660.0	1.1919	7.4797	0.70
0.80	93.50	1.0380	2.087	391.58	2498.8	391.66	2274.1	2665.8	1.2329	7.4346	0.80
0.90	96.71	1.0410	1.869	405.06	2502.6	405.15	2265.7	2670.9	1.2695	7.3949	0.90
1.00	99.63	1.0432	1.694	417.36	2506.1	417.46	2258.0	2675.5	1.3026	7.3594	1.00
1.50	111.4	1.0528	1.159	466.94	2519.7	467.11	2226.5	2693.6	1.4336	7.2233	1.50
2.00	120.2	1.0605	0.8857	504.49	2529.5	504.70	2201.9	2706.7	1.5301	7.1271	2.00
2.50	127.4	1.0672	0.7187	535.10	2537.2	535.37	2181.5	2716.9	1.6072	7.0527	2.50
3.00	133.6	1.0732	0.6058	561.15	2543.6	561.47	2163.8	2725.3	1.6718	6.9919	3.00
3.50	138.9	1.0786	0.5243	583.95	2546.9	584.33	2148.1	2732.4	1.7275	6.9405	3.50
4.00	143.6	1.0836	0.4625	604.31	2553.6	604.74	2133.8	2738.6	1.7766	6.8959	4.00
4.50	147.9	1.0882	0.4140	622.25	2557.6	623.25	2120.7	2743.9	1.8207	6.8565	4.50
5.00	151.9	1.0926	0.3749	639.68	2561.2	640.23	2108.5	2748.7	1.8607	6.8212	5.00
6.00	158.9	1.1006	0.3157	669.90	2567.4	670.56	2086.3	2756.8	1.9312	6.7600	6.00
7.00	165.0	1.1080	0.2729	696.44	2572.5	697.22	2066.3	2763.5	1.9922	6.7080	7.00
8.00	170.4	1.1148	0.2404	720.22	2576.8	721.11	2048.0	2769.1	2.0462	6.6628	8.00
9.00	175.4	1.1212	0.2150	741.83	2580.5	742.83	2031.1	2773.9	2.0946	6.6226	9.00
10.0	179.9	1.1273	0.1944	761.68	2583.6	762.81	2015.3	2778.1	2.1387	6.5863	10.0
15.0	198.3	1.1539	0.1318	843.16	2594.5	844.84	1947.3	2792.2	2.3150	6.4448	15.0
20.0	212.4	1.1767	0.09963	906.44	2600.3	908.79	1890.7	2799.5	2.4474	6.3409	20.0
25.0	224.0	1.1973	0.07998	959.11	2603.1	962.11	1841.0	2803.1	2.5547	6.2575	25.0
30.0	233.9	1.2165	0.06668	1004.8	2604.1	1008.4	1795.7	2804.2	2.6457	6.1869	30.0
35.0	242.6	1.2347	0.05707	1045.4	2603.7	1049.8	1753.7	2803.4	2.7253	6.1253	35.0
40.0	250.4	1.2522	0.04978	1082.3	2602.3	1087.3	1714.1	2801.4	2.7964	6.0701	40.0
45.0	257.5	1.2692	0.04406	1116.2	2600.1	1121.9	1676.4	2798.3	2.8610	6.0199	45.0
50.0	264.0	1.2859	0.03944	1147.8	2597.1	1154.2	1640.1	2794.3	2.9202	5.9734	50.0
60.0	275.6	1.3187	0.03244	1205.4	2589.7	1213.4	1571.0	2784.3	3.0267	5.8892	60.0
70.0	285.9	1.3513	0.02737	1257.6	2580.5	1267.0	1505.1	2772.1	3.1211	5.8133	70.0
80.0	295.1	1.3842	0.02352	1305.6	2569.8	1316.6	1441.3	2758.0	3.2068	5.7432	80.0
90.0	303.4	1.4178	0.02048	1350.5	2557.8	1363.3	1378.9	2742.1	3.2858	5.6772	90.0
100.	311.1	1.4524	0.01803	1393.0	2544.4	1407.6	1317.1	2724.7	3.3596	5.6141	100.
110.	318.2	1.4886	0.01599	1433.7	2529.8	1450.1	1255.5	2705.6	3.4295	5.5527	110.

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 FIGURE 3.22. A sketch of a  $T$ - $v$  diagram highlighting the compressed liquid region.

**TABLE A-5**  
**Properties of Compressed Liquid Water**

$T$ °C	$v \times 10^3$ m <sup>3</sup> /kg	$u$ kJ/kg	$h$ kJ/kg	$s$ kJ/kg · K	$v \times 10^3$ m <sup>3</sup> /kg	$u$ kJ/kg	$h$ kJ/kg	$s$ kJ/kg · K
$p = 2.5 \text{ bar} = 2.5 \text{ MPa}$ ( $T_{\text{sat}} = 223.99^\circ\text{C}$ )				$p = 5.0 \text{ bar} = 5.0 \text{ MPa}$ ( $T_{\text{sat}} = 263.99^\circ\text{C}$ )				
20	1.0006	83.80	86.30	.2961	.9995	83.65	88.65	.2956
40	1.0067	167.25	169.77	.5715	1.0056	166.95	171.97	.5705
80	1.0280	334.29	336.86	1.0737	1.0268	333.72	338.85	1.0720
100	1.0423	418.24	420.85	1.3050	1.0410	417.52	422.72	1.3030
140	1.0784	587.82	590.52	1.7369	1.0768	586.76	592.15	1.7343
180	1.1261	761.16	763.97	2.1375	1.1240	759.63	765.25	2.1341
200	1.1555	849.9	852.8	2.3294	1.1530	848.1	853.9	2.3255
220	1.1898	940.7	943.7	2.5174	1.1866	938.4	944.4	2.5128
Sat.	1.1973	959.1	962.1	2.5546	1.2859	1147.8	1154.2	2.9202

Pressure Conversions:  
1 bar = 0.1 MPa  
= 10<sup>2</sup> kPa

H<sub>2</sub>O

$p$  is an absolute pressure  
 $v$  is specific volume  
(Note that  $v$  entries are multiplied by 1000, e.g.,  
 $v(2.5 \text{ MPa}, 20^\circ\text{C}) = 1.0006 \times 10^{-3} \text{ m}^3/\text{kg}$ .  
 $u$  is specific internal energy  
 $h$  is specific enthalpy  
 $s$  is specific entropy

 (Table from Moran et al., 7<sup>th</sup> ed.)

 FIGURE 3.23. Example table for the compressed liquid (CL) properties for water. This table is from Moran et al., 7<sup>th</sup> ed.

### 3.5.1. Linear Interpolation

In order to save space, the properties in the tables are listed in coarse increments, e.g., increments of 2.5 MPa and 20 or 40 °C. To approximate property values between the ones stated in the tables, we can use linear interpolation. Linear interpolation is the process of estimating the values of a property assuming a linear relationship between neighboring data points. Hence, to estimate the value of a property  $P$  at a value  $V$  given the property's values at neighboring points  $V_S$  and  $V_L$ , corresponding to points in the table just smaller than and larger than the value  $V$  of interest, we can use the equation of a line (Figure 3.24),

$$P - P_S = \left( \frac{P_L - P_S}{V_L - V_S} \right) (V - V_S) \quad (3.50)$$

where  $P_S$  and  $P_L$  are the property values at  $V_S$  and  $V_L$ , respectively.

What is the specific volume of compressed liquid water at 5.0 MPa (abs) and 60 °C?

SOLUTION:

Since there is no specific volume data at 60 °C at 5.0 MPa in Table A-5 shown below, we can approximate the specific volume at 60 °C using linear interpolation,

$$v_{60\text{ }^\circ\text{C}} - v_{40\text{ }^\circ\text{C}} = \left( \frac{v_{80\text{ }^\circ\text{C}} - v_{40\text{ }^\circ\text{C}}}{T_{80\text{ }^\circ\text{C}} - T_{40\text{ }^\circ\text{C}}} \right) (T_{60\text{ }^\circ\text{C}} - T_{40\text{ }^\circ\text{C}}), \tag{1}$$

where

$$\begin{aligned} v_{40\text{ }^\circ\text{C}} &= 1.0056 \cdot 10^{-3} \text{ m}^3/\text{kg} \\ v_{80\text{ }^\circ\text{C}} &= 1.0268 \cdot 10^{-3} \text{ m}^3/\text{kg} \\ T_{40\text{ }^\circ\text{C}} &= 40\text{ }^\circ\text{C} \\ T_{60\text{ }^\circ\text{C}} &= 60\text{ }^\circ\text{C} \\ T_{80\text{ }^\circ\text{C}} &= 80\text{ }^\circ\text{C} \\ \Rightarrow v_{60\text{ }^\circ\text{C}} &= 1.0162 \cdot 10^{-3} \text{ m}^3/\text{kg} \end{aligned}$$

**TABLE A-5**

**Properties of Compressed Liquid Water**

<i>T</i> °C	<i>v</i> × 10 <sup>3</sup> m <sup>3</sup> /kg	<i>u</i> kJ/kg	<i>h</i> kJ/kg	<i>s</i> kJ/kg · K	<i>v</i> × 10 <sup>3</sup> m <sup>3</sup> /kg	<i>u</i> kJ/kg	<i>h</i> kJ/kg	<i>s</i> kJ/kg · K
<i>p</i> = 25 bar = 2.5 MPa ( <i>T</i> <sub>sat</sub> = 223.99°C)								
20	1.0006	83.80	86.30	.2961	.9995	83.65	88.65	.2956
40	1.0067	167.25	169.77	.5715	1.0056	166.95	171.97	.5705
80	1.0280	334.29	336.86	1.0737	1.0268	333.72	338.85	1.0720
100	1.0423	418.24	420.85	1.3050	1.0410	417.52	422.72	1.3030
140	1.0784	587.82	590.52	1.7369	1.0768	586.76	592.15	1.7343
180	1.1261	761.16	763.97	2.1375	1.1240	759.63	765.25	2.1341
200	1.1555	849.9	852.8	2.3294	1.1530	848.1	853.9	2.3255
220	1.1898	940.7	943.7	2.5174	1.1866	938.4	944.4	2.5128
Sat.	1.1973	959.1	962.1	2.5546	1.2859	1147.8	1154.2	2.9202
<i>p</i> = 50 bar = 5.0 MPa ( <i>T</i> <sub>sat</sub> = 263.99°C)								

Pressure Conversions:  
1 bar = 0.1 MPa  
= 10<sup>2</sup> kPa

H<sub>2</sub>O

(Table from Moran et al., 7<sup>th</sup> ed.)

What is the specific internal energy of compressed liquid water at 3.0 MPa and 60 °C?

SOLUTION:

For this case, there is no table entry for either 3.0 MPa or 60 °C so we must linearly interpolate with respect to both temperature and pressure (called bilinear interpolation),

$$u_{2.5 \text{ MPa}, 60 \text{ °C}} - u_{2.5 \text{ MPa}, 40 \text{ °C}} = \left( \frac{u_{2.5 \text{ MPa}, 80 \text{ °C}} - u_{2.5 \text{ MPa}, 40 \text{ °C}}}{T_{2.5 \text{ MPa}, 80 \text{ °C}} - T_{2.5 \text{ MPa}, 40 \text{ °C}}} \right) (T_{2.5 \text{ MPa}, 60 \text{ °C}} - T_{2.5 \text{ MPa}, 40 \text{ °C}}), \tag{1}$$

$$u_{5.0 \text{ MPa}, 60 \text{ °C}} - u_{5.0 \text{ MPa}, 40 \text{ °C}} = \left( \frac{u_{5.0 \text{ MPa}, 80 \text{ °C}} - u_{5.0 \text{ MPa}, 40 \text{ °C}}}{T_{5.0 \text{ MPa}, 80 \text{ °C}} - T_{5.0 \text{ MPa}, 40 \text{ °C}}} \right) (T_{5.0 \text{ MPa}, 60 \text{ °C}} - T_{5.0 \text{ MPa}, 40 \text{ °C}}),$$

$$u_{3.0 \text{ MPa}, 60 \text{ °C}} - u_{2.5 \text{ MPa}, 60 \text{ °C}} = \left( \frac{u_{5.0 \text{ MPa}, 60 \text{ °C}} - u_{2.5 \text{ MPa}, 60 \text{ °C}}}{p_{5.0 \text{ MPa}, 60 \text{ °C}} - p_{2.5 \text{ MPa}, 60 \text{ °C}}} \right) (p_{3.0 \text{ MPa}, 60 \text{ °C}} - p_{2.5 \text{ MPa}, 60 \text{ °C}}),$$

where,

- $u_{2.5 \text{ MPa}, 40 \text{ °C}} = 167.25 \text{ kJ/kg}$
- $u_{2.5 \text{ MPa}, 80 \text{ °C}} = 334.29 \text{ kJ/kg}$
- $u_{5.0 \text{ MPa}, 40 \text{ °C}} = 166.95 \text{ kJ/kg}$
- $u_{5.0 \text{ MPa}, 80 \text{ °C}} = 333.72 \text{ kJ/kg}$
- $T_{2.5 \text{ MPa}, 40 \text{ °C}} = T_{5.0 \text{ MPa}, 40 \text{ °C}} = 40 \text{ °C}$
- $T_{2.5 \text{ MPa}, 60 \text{ °C}} = T_{5.0 \text{ MPa}, 60 \text{ °C}} = 60 \text{ °C}$
- $T_{2.5 \text{ MPa}, 80 \text{ °C}} = T_{5.0 \text{ MPa}, 80 \text{ °C}} = 80 \text{ °C}$
- $p_{2.5 \text{ MPa}, 60 \text{ °C}} = 2.5 \text{ MPa}$
- $p_{3.0 \text{ MPa}, 60 \text{ °C}} = 3.0 \text{ MPa}$
- $p_{5.0 \text{ MPa}, 60 \text{ °C}} = 5.0 \text{ MPa}$

$$\Rightarrow u_{2.5 \text{ MPa}, 60 \text{ °C}} = 250.77 \text{ kJ/kg}, u_{5.0 \text{ MPa}, 60 \text{ °C}} = 250.34 \text{ kJ/kg} \Rightarrow \boxed{u_{3.0 \text{ MPa}, 60 \text{ °C}} = 250.68 \text{ kJ/kg}}$$

Note that the same result would be achieved if we interpolated first with respect to pressure and then with respect to temperature.

**TABLE A-5**  
**Properties of Compressed Liquid Water**

$T$ °C	$v \times 10^3$ m <sup>3</sup> /kg	$u$ kJ/kg	$h$ kJ/kg	$s$ kJ/kg · K	$v \times 10^3$ m <sup>3</sup> /kg	$u$ kJ/kg	$h$ kJ/kg	$s$ kJ/kg · K
$p = 25 \text{ bar} = 2.5 \text{ MPa}$ ( $T_{\text{sat}} = 223.99\text{°C}$ )				$p = 50 \text{ bar} = 5.0 \text{ MPa}$ ( $T_{\text{sat}} = 263.99\text{°C}$ )				
20	1.0006	83.80	86.30	.2961	.9995	83.65	88.65	.2956
40	1.0067	167.25	169.77	.5715	1.0056	166.95	171.97	.5705
80	1.0280	334.29	336.86	1.0737	1.0268	333.72	338.85	1.0720
100	1.0423	418.24	420.85	1.3050	1.0410	417.52	422.72	1.3030
140	1.0784	587.82	590.52	1.7369	1.0768	586.76	592.15	1.7343
180	1.1261	761.16	763.97	2.1375	1.1240	759.63	765.25	2.1341
200	1.1555	849.9	852.8	2.3294	1.1530	848.1	853.9	2.3255
220	1.1898	940.7	943.7	2.5174	1.1866	938.4	944.4	2.5128
Sat.	1.1973	959.1	962.1	2.5546	1.2859	1147.8	1154.2	2.9202

Pressure Conversions:  
1 bar = 0.1 MPa  
= 10<sup>2</sup> kPa

H<sub>2</sub>O

(Table from Moran et al., 7<sup>th</sup> ed.)

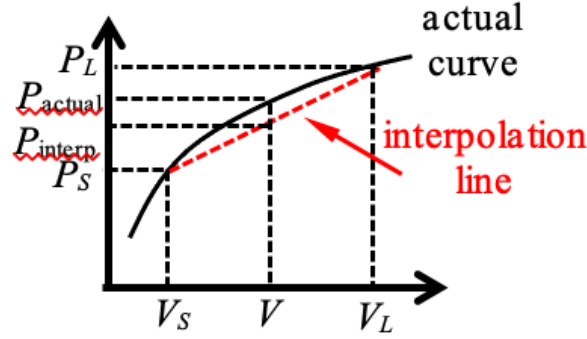


FIGURE 3.24. Schematic showing how to linearly interpolate a property value.

### 3.5.2. Quality

Recall that Figure 3.19 presented two tables for saturated water properties (aka, water in two-phase, liquid-vapor equilibrium along the saturated liquid and the saturated vapor lines). The pressure and temperature are uniquely related in this two-phase region (and along the saturated liquid and vapor lines). The top table (Table A-2) presents the property data organized according to temperature while the bottom table (Table A-3) presents the same data organized according to pressure. The subscripts “f” and “g” in the table refer to “fluid” and “gas”, which is a historical notation. It is better to refer to the properties as being either at the saturated liquid state (subscript “f” in the table) or in the saturated vapor state (subscript “g” in the table). Similar tables exist for two-phase solid-liquid and solid-vapor.

Within the two-phase liquid-vapor region (i.e, the vapor dome), the fraction of mass that is vapor is given by the quality,  $x$ , which is defined as,

$$x := \frac{m_v}{m_l + m_v} \quad (3.51)$$

where  $m_v$  and  $m_l$  are the masses of vapor and liquid, respectively. Note that the fraction of mass that is liquid is,

$$\frac{m_l}{m_l + m_v} = \frac{m_l + m_v - m_v}{m_l + m_v} = \frac{m_l + m_v}{m_l + m_v} - \frac{m_v}{m_l + m_v} = 1 - x \quad (3.52)$$

Hence, a quality of zero corresponds to a saturated liquid (all liquid,  $m_v = 0$ ) while a quality of one corresponds to a saturated vapor (all vapor,  $m_l = 0$ ). The quality can be used to determine the value of properties within the two-phase region, given the saturated liquid and saturated vapor properties. For example, the specific volume of a mixture (subscript “m”) of liquid (subscript “l”) and vapor (subscript “v”) in equilibrium (i.e., in the vapor dome), assuming the quality  $x$  is known, is,

$$V_m = V_l + V_v \quad (3.53)$$

$$v_m = \frac{V_m}{m_m} = \frac{V_l + V_v}{m_m} = \frac{V_l}{m_m} + \frac{V_v}{m_m} \quad (3.54)$$

where  $V_m$  is the total volume of the mixture. The quantity  $m_m$  is the total mass of the mixture, i.e.,  $m_m = m_l + m_v$ . Hence,

$$v_m = \frac{V_l}{m_l + m_v} + \frac{V_v}{m_l + m_v} = \frac{m_l v_l}{m_l + m_v} + \frac{m_v v_v}{m_l + m_v} = \left( \frac{m_l}{m_l + m_v} \right) v_l + \left( \frac{m_v}{m_l + m_v} \right) v_v \quad (3.55)$$

where the volume is related to the specific volume via  $V = mv$ . Making use of Eqs. (3.51) and (3.52),

$$v_m = (1 - x) v_l + x v_v \quad (3.56)$$

Thus, the specific volume of a mixture of liquid and vapor can be thought of as the specific volume of the saturated liquid multiplied by its mass fraction  $((1 - x)v_l)$  plus the specific volume of the saturated vapor