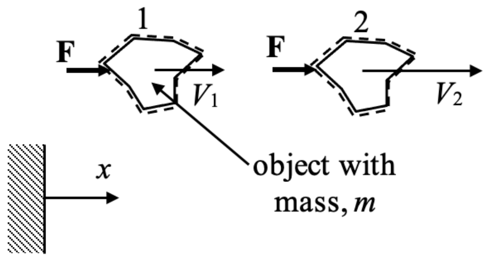


FIGURE 3.3. Illustration of the work due to acceleration.

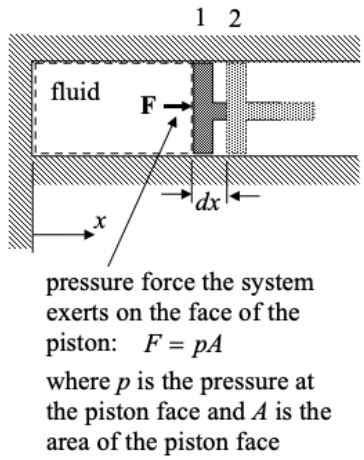


FIGURE 3.4. Illustration of the work due to a pressure force.

3.2.2.3. Pressure Work

Consider the work done by the expansion of a fluid (a gas or liquid) in a piston (Figure 3.4),

$$W_{\text{on surr},12} = \int_1^2 \mathbf{F}_{\text{on surr}} \cdot d\mathbf{s}, \tag{3.16}$$

$$= \int_{x_1}^{x_2} (pA\hat{\mathbf{e}}_x) \cdot (dx\hat{\mathbf{e}}_x), \tag{3.17}$$

$$= \int_{x_1}^{x_2} pAdx, \tag{3.18}$$

$$\boxed{W_{\text{on surr},12} = \int_{V_1}^{V_2} pdV}. \tag{3.19}$$

Note that $dV = Adx$. Note also that in this example, the work on the surroundings has been calculated instead of the work acting on the system. To get the work done on the system, we simply have, $W_{\text{on sys}} = -W_{\text{on surr}}$. If we plot how the pressure changes with volume, we get a p-V diagram, as shown in Figure 3.5. Note that different paths from state 1 to state 2 will give different work values, as shown in Figure 3.6. One example of a particular pressure-volume relationship is known as a polytropic process in which the pressure and volume are related by,

$$pV^n = c, \tag{3.20}$$

where n and c are constants.

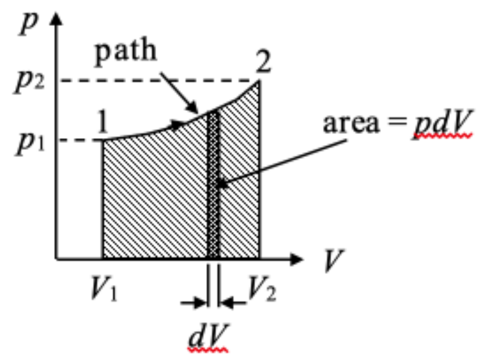


FIGURE 3.5. Illustration of an example path on a $p - V$ plot. The area under the curve is equal to the work done by the fluid on the surroundings in going from volume 1 to volume 2.

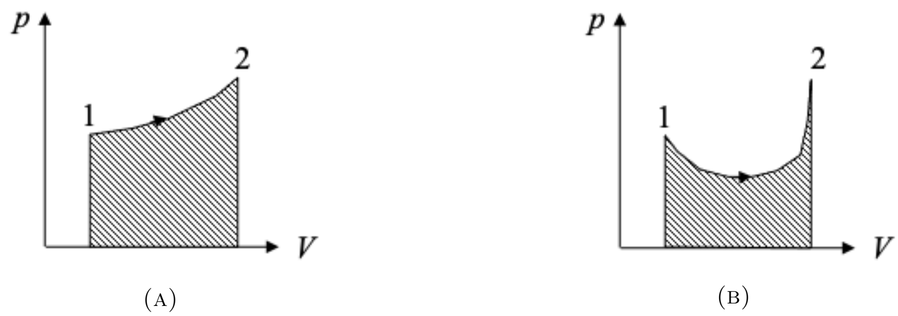


FIGURE 3.6. Different paths result in different values for the work. Here, $W_{12,A} > W_{12,B}$.

3.2.2.4. Electric Work

Electrons moving across a system boundary can do work on a system since, in an electric field a force acts on an electron. When N Coulombs of electrons pass through a potential difference, V (the voltage), the electric work done on the system is,

$$W_{onsys} = NV. \tag{3.21}$$

The corresponding power is,

$$\boxed{\dot{W}_{\text{on sys}} = V\dot{N} = VI = I^2R = \frac{V^2}{R}}, \tag{3.22}$$

where I is the current and R is the resistance of the system. Note that Ohm's Law ($V = IR$) has been used in deriving the last two expressions on the right hand side of Eq. (3.22).

3.2.2.5. Spring Work

Now let's examine the work required to compress a spring with stiffness, k (Figure 3.7),

$$W_{\text{on sys},12} = \int_1^2 \mathbf{F}_{\text{on sys}} \cdot d\mathbf{s}, \tag{3.23}$$

$$= \int_{x_1}^{x_2} (-kx\hat{\mathbf{e}}_x) \cdot (dx\hat{\mathbf{e}}_x), \tag{3.24}$$

$$\boxed{W_{\text{on sys},12} = \frac{1}{2}k\left(x_1^2 - x_2^2\right)}. \tag{3.25}$$

Note that k is assumed constant in this equation.

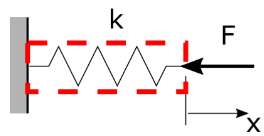


Figure 3.7. An illustration of spring work.

3.2.2.6. Shaft Work

Another method of transferring energy between a system and the surroundings is through shaft work (Figure 3.8). Shaft work is most often associated with rotating fluid machines such as compressors, pumps, turbines, fans, propellers, and windmills. The power acting on a system due to a rotating shaft is given by,

$$\boxed{\dot{W}_{\text{on sys}} = \mathbf{T}_{\text{on sys}} \cdot \boldsymbol{\omega}}, \tag{3.26}$$

where $\mathbf{T}_{\text{on sys}}$ is the torque acting on the system (assumed constant here) and $\boldsymbol{\omega}$ is the angular velocity of the shaft.

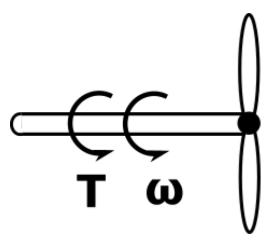
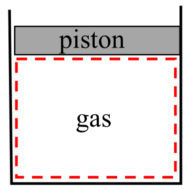


Figure 3.8. An illustration of shaft work.

A gas in a piston assembly undergoes a polytropic expansion from an initial volume, V_i=0.1 m^3, and initial pressure, p_i = 2 bar (abs) (1 bar = 1*10^5 Pa), to a final volume of V_f = 0.5 m^3. Determine the work the gas does on the piston for n = 1.5 and n = 1 (where pV^n = constant).

SOLUTION:



The work the gas performs on the piston is given by:

$$W_{i\to f} = \int_{V=0.1\text{ m}^3}^{V=0.5\text{ m}^3} p dV \tag{1}$$

where, for a polytropic expansion,

$$pV^n = \text{constant}=c \tag{2}$$

where n is a constant. Substitute Eq. (2) into Eq. (1).

$$W_{i\to f} = \int_{V=0.1\text{ m}^3}^{V=0.5\text{ m}^3} cV^{-n} dV = \begin{cases} \left.\dfrac{c}{1-n}V^{1-n}\right|_{0.1\text{ m}^3}^{0.5\text{ m}^3} & n \neq 1 \\ \left. c\ln V\right|_{0.1\text{ m}^3}^{0.5\text{ m}^3} & n = 1 \end{cases} \tag{3}$$

When n = 1.5, the constant is

$$c = \Big(\underbrace{2 * 10^5 \text{ Pa}}_{=p_i}\Big)\Big(\underbrace{0.1\text{ m}^3}_{=V_i}\Big)^{1.5} = 6.32 * 10^3 \text{ N}\cdot\text{m}^{2.5} \tag{4}$$

and the work performed by the gas, using Eq. (3), is:

$$W_{i\to f} = \frac{6.32*10^3 \text{ N}\cdot\text{m}^{2.5}}{-0.5}\left[(0.5\text{ m}^3)^{-0.5} - (0.1\text{ m}^3)^{-0.5}\right], \tag{5}$$

$$\boxed{W_{i\to f} = 2.2 * 10^4 \text{ N}\cdot\text{m}}. \tag{6}$$

When n = 1, the constant is:

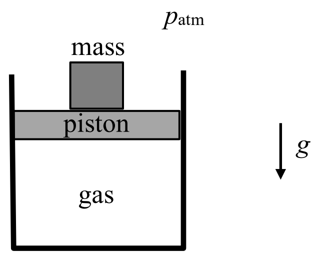
$$c = \Big(\underbrace{2 * 10^5 \text{ Pa}}_{=p_i}\Big)\Big(\underbrace{0.1\text{ m}^3}_{=V_i}\Big) = 2 * 10^4 \text{ N}\cdot\text{m} \tag{7}$$

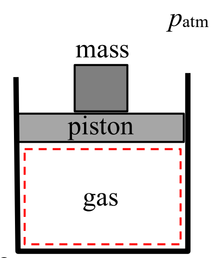
and the work performed by the gas, using Eq. (3), is:

$$W_{i\to f} = (2 * 10^4 \text{ N}\cdot\text{m})\ln\left(\frac{0.5\text{ m}^3}{0.1\text{ m}^3}\right), \tag{8}$$

$$\boxed{W_{i\to f} = 3.2 * 10^4 \text{ N}\cdot\text{m}}. \tag{9}$$

Determine the work done by the gas on the piston shown below as it expands quasi-statically from a volume of 0.02 m^3 to 0.04 m^3 given that the piston area is 0.01 m^2 and the mass resting on the piston is 100 kg (neglect the weight of the piston). Assume that atmospheric pressure is 101 kPa (abs).





SOLUTION:
The work done by the gas on the surroundings is,

$$W_{\text{by gas}} = \int_{V_1}^{V_2} p\,dV \text{ ,} \tag{1}$$

where,

$p = p_{atm} + mg/A = 101$ kPa (abs) + (100 kg)(9.81 m/s^2)/(0.01 m^2) = 1.99*10^5 Pa (2)

The pressure in the gas balances the atmospheric pressure plus the weight of the mass divided by the piston area. Note that this pressure is a constant throughout the process since we're always balancing the same mass and atmospheric pressure.

$V_1 = 0.02$ m^3
$V_2 = 0.04$ m^3

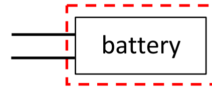
Since the pressure remains constant throughout the process, Eq. (1) may be written as,

$$W_{\text{by gas}} = \int_{V_1}^{V_2} p\,dV = p\int_{V_1}^{V_2} dV = p\left(V_2 - V_1\right). \tag{3}$$

Substituting the numbers given above,

$\boxed{W_{\text{by gas}} = 3.9 \text{ kJ}}$.

A 12 V automotive battery is charged with a constant current of 1.5 A for 3 hrs. Determine the work done on the battery.



SOLUTION:
The work done on the battery is,

$$W_{\text{on battery}} = \int_{t=0}^{t=T} \dot{W}\,dt = \int_{t=0}^{t=T} VI\,dt = VIT = \left(12\text{ V}\right)\left(1.5\text{ A}\right)\left(3\text{ hr}\cdot 3600\ {}^{\text{s}}\!/\!_{\text{hr}}\right),$$

$$\boxed{\therefore W_{\text{on battery}} = 0.2\text{ kJ}}.$$