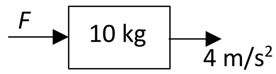
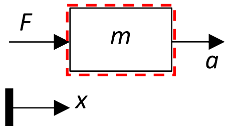
An object of mass 10 kg, initially at rest, experiences a constant horizontal acceleration of 4 m/s^2 due to the action of a resultant force applied for 20 s. Determine the total amount of energy transfer by work in kJ.



SOLUTION:



From the First Law of Thermodynamics, the work acting on the object (our system) will be related to the change in the object's kinetic energy, assuming there is no change in internal energy ($\Delta U = 0$) and there is no heat transfer ($Q_{\text{into sys}} = 0$),

$$\Delta E_{sys} = Q_{into\ sys} + W_{on\ sys}, \tag{1}$$

where,

$$\Delta E_{sys} = \Delta U_{sys} + \Delta KE_{sys} + \Delta PE_{sys} = \Delta KE_{sys} \text{ (no changes in internal or potential energy)}, \tag{2}$$

$$Q_{into\ sys} = 0 \text{ (no heat transfer)}. \tag{3}$$

Substituting into Eq. (1),

$$W_{on\ sys} = \Delta KE_{sys} = \tfrac{1}{2}m(V_2^2 - V_1^2) = \tfrac{1}{2}mV_2^2 \text{ (since } V_1 = 0\text{)}. \tag{4}$$

The speed after 20 s is find by integrating the acceleration,

$$V_2 = \int_{t=0}^{t=T} a\,dt = aT \text{ (}V(t = 0) = 0\text{ and the acceleration is constant)}. \tag{5}$$

Using the given values,

$a = 4\ \text{m/s}^2$,
$T = 20$ s,
=> $V_2 = 80$ m/s,
$m = 10$ kg,
=> $\boxed{W_{on\ sys} = 32\ \text{kJ}}$. This value is the work done <u>on</u> the system.

An alternate approach is to calculate the work directly from the applied force,

$$W_{on\ sys} = \int \boldsymbol{F}_{on\ sys} \cdot d\boldsymbol{s} = \int \underbrace{ma}_{=F}\,\hat{\boldsymbol{\imath}} \cdot ds\hat{\boldsymbol{\imath}} = ma\int_{s=0}^{s=L} ds = maL \text{ (the mass and acceleration are constants)}, \tag{6}$$

where L is the distance the mass moves during the process. To find this distance, integrate the object's acceleration twice in time,

$$a = \frac{d^2s}{dt^2} \Rightarrow \frac{ds}{dt} = at \text{ (}V(t = 0) = 0\text{, acceleration is a constant)}, \tag{7}$$

$$s = \tfrac{1}{2}at^2 \text{ (}s(t = 0) = 0\text{, acceleration is a constant)}. \tag{8}$$

Thus, for $a = 4\ \text{m/s}^2$ and $t = T = 20$ s, <u>$s = L = 800$ m</u>. Using Eq. (6), <u>$W_{on\ sys} = 32$ kJ</u>, which is the same answer found previously.