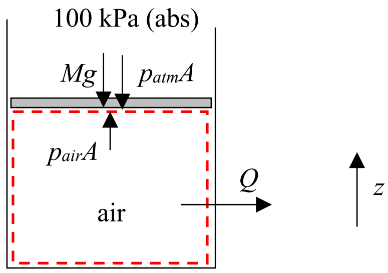
A vertical piston-cylinder assembly with a piston of mass 25 kg and having a face area of 0.005 m^2 contains air. The mass of air is 2.5 g and initially the air occupies a volume of 2.5 L. The atmosphere exerts a pressure of 100 kPa (abs) on the top of the piston. The volume of the air slowly decreases to 0.001 m^3 as energy with a magnitude of 1 kJ is slowly removed by heat transfer. Neglecting friction between the piston and the cylinder wall, determine the change in specific internal energy of the air.

SOLUTION:



Apply the First Law of Thermodynamics to a system consisting of the air inside the piston (indicated by the red, dashed line in the figure),

$$\Delta E_{sys} = Q_{into\ sys} + W_{on\ sys}, \quad (1)$$

where,

$$\Delta E_{sys} = \Delta U = m_{gas}\Delta u \quad (\Delta KE \text{ and } \Delta PE \text{ are negligible, } m_{gas} \text{ remains constant}), \quad (2)$$

$$Q_{into\ sys} = -1\ kJ \quad \text{(given)}, \quad (3)$$

$$W_{on\ sys} = \int_1^2 \boldsymbol{F}\cdot d\boldsymbol{s} = \int_1^2 \left(-Mg\hat{\boldsymbol{k}} - p_{atm}A\hat{\boldsymbol{k}}\right)\cdot dz\hat{\boldsymbol{k}} = -(Mg + p_{atm}A)\int_{z_1}^{z_2} dz = -M(g\Delta z + p_{atm}A\Delta z), \quad (4)$$

$$W_{on\ sys} = -(Mg\Delta z + p_{atm}\Delta V). \quad (5)$$

Note that,

$$\Delta z = \frac{\Delta V}{A}, \quad (6)$$

so that,

$$W_{on\ sys} = -\left(\frac{Mg}{A} + p_{atm}\right)\Delta V. \quad (7)$$

Substitute and solve for Δu,

$$m_{gas}\Delta u = Q_{into} - \left(\frac{Mg}{A} + p_{atm}\right)\Delta V, \quad (8)$$

$$\Delta u = \frac{Q_{into} - \left(\frac{Mg}{A} + p_{atm}\right)\Delta V}{m_{gas}}. \quad (9)$$

Using the given values,

Q_{into} = - 1 kJ,
M = 25 kg (mass of piston),
g = 9.81 m/s^2,
A = 0.005 m^2,
p_{atm} = 100 kPa (abs),
$\Delta V = V_2 - V_1$ = 0.001 m^3 – 2.5*10^{-3} m^3 = -1.5*10^{-3} m^3,
m_{gas} = 2.5*10^{-3} kg,

gives,

$\boxed{\Delta u = -311 \text{ kJ/kg}}$.

Note that the work on the system is,

$W_{on\ sys}$ = 224 J.

An alternate approach would be to calculate the work done by the system and use the following form of the First Law,

$$\Delta E_{sys} = Q_{into\ sys} - W_{by\ sys}, \quad (10)$$

where,

$$W_{by\ sys} = \int_{V_1}^{V_2} p_{air}\,dV = p_{air}\int_{V_1}^{V_2} dV = p_{air}(V_2 - V_1) \quad (\text{since } p_{air} = \text{constant}) \quad (11)$$

Note that the gas pressure is found by balancing forces on the piston,

$$p_{air}A = p_{atm}A + Mg, \quad (12)$$

$$p_{air} = p_{atm} + \frac{Mg}{A}, \quad (13)$$

Using the given values,

p_{air} = 149 kPa (abs),
$W_{by\ sys}$ = -224 J, which is the equal but opposite in sign to $W_{on\ sys}$, as expected.