As shown in the figure, a gas contained within a piston-cylinder assembly, initially at a volume of 0.1 m^3 , undergoes a constant-pressure expansion at 2 bar (abs) to a final volume of 0.12 m^3 , while being slowly heated through the base. The change in internal energy of the gas is 0.25 kJ. The piston and cylinder walls are fabricated from heat-resistant material, and the piston moves smoothly in the cylinder. The local atmospheric pressure is 1 bar (abs).

- a. Evaluate the work done by the gas and the heat added to the gas.
- b. What is the work done on the piston due to the surrounding pressures?
- c. Evaluate the change in potential energy of the piston.



SOLUTION:

Apply the First Law to a system consisting of the gas (indicated by the red dashed line in the figure).



The work done by the gas on the surroundings is,

$$\frac{W_{by\ sys} = \int_{V_1}^{V_2} p dV = p \int_{V_1}^{V_2} dV = p(V_2 - V_1) = \left(2\ bar \cdot \frac{100 \cdot 10^3 P a}{1\ bar}\right) (0.12\ m^3 - 0.1\ m^3) = 4000\ J \tag{1}$$

$$\frac{W_{by\ sys} = 4\ kJ}{W_{by\ sys} = 4\ kJ}$$

Use the 1st Law of Thermodynamics to determine the heat added to the system,

 $\Delta E_{sys} = Q_{into sys} - W_{by sys}$ where $\Delta E_{sys} = \Delta U_{sys} = 0.25 \ kJ \quad \text{(changes to KE and PE are negligible)}$ (3)

Thus, using the previous result for the work,

 $Q_{into sys} = 4.25 \text{ kJ}$

Now make the piston the system (indicated by the blue dashed line in the figure). The work done on the piston due to the surrounding pressures is,

$$W_{on\,sys} = \int_{1}^{2} \mathbf{F} \cdot d\mathbf{s} = \int_{1}^{2} (p_{gas,abs}A - p_{atm,abs}A) \hat{\mathbf{k}} \cdot dz \hat{\mathbf{k}} = (p_{gas,abs} - p_{atm,abs}) A \int_{0}^{\Delta h} dz, \tag{4}$$

$$W_{on\,sys} = (n_{on,s} - n_{on,s}) A \Delta h \tag{5}$$

$$\frac{W_{on sys} - (p_{gas,abs} - p_{atm,abs})}{W_{on sys} = (p_{gas,abs} - p_{atm,abs})\Delta V.}$$
(6)

Using,

 $p_{gas,abs} = 2 \text{ bar} = 200*10^3 \text{ Pa},$ $p_{atm,abs} = 1 \text{ bar} = 100*10^3 \text{ Pa},$

 $\Delta V = V_2 - V_1 = 0.12 \text{ m}^3 - 0.1 \text{ m}^3 = 0.02 \text{ m}^3$, Thus,

 $W_{on sys} = 2 \text{ kJ}.$

Applying the 1st Law to the system, i.e, the piston,

 $\Delta E_{sys} = Q_{into\ sys} - W_{by\ sys} \tag{7}$

where

 $\Delta E_{sys} = \Delta P E_{sys} \quad \text{(changes to KE and IE are negligible)} \tag{8}$

 $Q_{into sys} = 0$ (the piston is heat resistant so it doesn't absorb much heat) (9)

Thus, using the previous result for the work,

$$\Delta P E_{sys} = -W_{by\,sys} = W_{on\,sys},\tag{10}$$

so that, $\Delta P E_{sys} = 2 \text{ kJ}$