

## Diesel Cycle & Four Processes Of Diesel Cycle

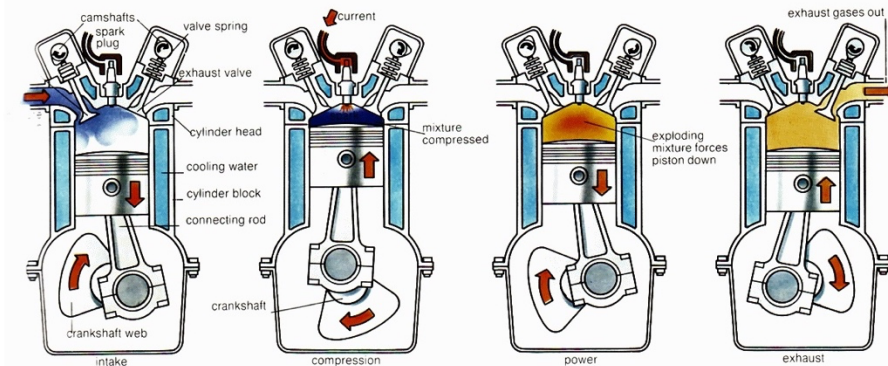


Image: <https://engineeringinsider.org/diesel-cycle-four-processes/>

As an idealization, it is assumed that air in a diesel engine cylinder, initially with a volume of 1 L, is compressed isentropically from 1 bar (abs) and 300 K to 10 bar (abs) in order to compress and heat the air prior to fuel injection. Assume that the molecular weight of air is 28.9 kg/kmol.

- Determine the final temperature and volume assuming the compression process is isentropic and the specific heats vary with temperature.
- Determine the final temperature and volume if the process is isentropic but the specific heats are assumed constant. For this case assume the specific heat ratio for air is 1.4.
- The work required to compress the air for case (a).
- Plot the process on a  $T$ - $s$  diagram.
- Is it harder or easier to compress the air if the air starts at a higher temperature? Assume that the air mass is the same and the pressures at the initial and final states are still 1 bar (abs) and 10 bar (abs), respectively. Support your answer.

SOLUTION:

For an isentropic process involving an ideal gas,

$$\Delta s = 0 = s^0(T_2) - s^0(T_1) - R \ln \left( \frac{p_2}{p_1} \right), \quad (1)$$

$$s^0(T_2) = s^0(T_1) + R \ln \left( \frac{p_2}{p_1} \right). \quad (2)$$

Using the given data and the ideal gas table for air,

$$T_1 = 300 \text{ K} \Rightarrow s^0(T_1) = 1.703 \text{ kJ}/(\text{kg}\cdot\text{K}),$$

$$p_1 = 1 \text{ bar (abs)},$$

$$p_2 = 10 \text{ bar (abs)},$$

$$R_{air} = \frac{\bar{R}_u}{MW_{air}} = \frac{8.314 \text{ kJ}/(\text{kmol}\cdot\text{K})}{28.9 \text{ kg}/\text{kmol}} = 0.2877 \text{ kJ}/(\text{kg}\cdot\text{K}),$$

$$\Rightarrow s^0(T_2) = 2.36541 \text{ kJ}/(\text{kg}\cdot\text{K}) \Rightarrow \boxed{T_2 = 575 \text{ K}}.$$

The final volume may be found using the ideal gas law,

$$\frac{p_2 V_2}{RT_2} = m = \frac{p_1 V_1}{RT_1} \Rightarrow V_2 = V_1 \left( \frac{p_1}{p_2} \right) \left( \frac{T_2}{T_1} \right). \quad (3)$$

Using the given and calculated data,

$$\boxed{V_2 = 0.192 \text{ L}}.$$

Note that the air mass is calculated to be:  $m = 0.0116 \text{ kg}$ .

An alternate method for working this part of the problem is to make use of the “relative pressure”,  $p_r$  (recall that  $p_r$  is not, in fact, a pressure),

$$\frac{p_{r2}}{p_{r1}} = \frac{p_2}{p_1} \Rightarrow p_{r2} = p_{r1} \left( \frac{p_2}{p_1} \right), \quad (4)$$

where,

$$p_{r1} = p_r(T_1 = 300 \text{ K}) = 1.3860 \quad (\text{from the Ideal Gas Table for air}),$$

$$p_2 = 10 \text{ bar (abs)} \quad \text{and} \quad p_1 = 1 \text{ bar (abs)},$$

$$\Rightarrow p_{r2} = 13.860 \Rightarrow T_2 = 574 \text{ K, which is the same answer found previously (to within numerical error).}$$

Recall that Eq. (4) can only be used for *ideal gases undergoing an isentropic process*.

Similarly, the volume at State 2 for an *ideal gas undergoing an isentropic process* can be found using a “relative volume” (not really a volume, but that’s what it’s called),

$$\frac{v_{r2}}{v_{r1}} = \frac{v_2}{v_1} \Rightarrow \frac{v_{r2}}{v_{r1}} = \frac{V_2}{V_1} \Rightarrow V_2 = V_1 \left( \frac{v_{r2}}{v_{r1}} \right), \quad (4)$$

where,

$$V_1 = 1 \text{ L,}$$

$$v_{r1} = v_r(T_1 = 300 \text{ K}) = 621.2 \quad (\text{from the Ideal Gas Table for air}),$$

$$v_{r2} = v_r(T_2 = 574 \text{ K}) = 119.0 \quad (\text{from the Ideal Gas Table for air}),$$

$$\Rightarrow V_2 = 0.192 \text{ L, which is the same answer found previously.}$$

For part (b), we are to assume the air behaves as a perfect gas, i.e., an ideal gas with constant specific heats. For a *perfect gas undergoing an isentropic process*,

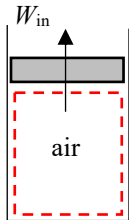
$$\frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{k-1}{k}} \Rightarrow T_2 = T_1 \left( \frac{p_2}{p_1} \right)^{\frac{k-1}{k}} \quad (5)$$

Using the given data and  $k = 1.4$ ,  $T_2 = 579 \text{ K}$ . This value is less than 1% larger than the more accurate value calculated previously (where the specific heats can vary with temperature).

The volume can be found using the following expression, which is also for a *perfect gas undergoing an isentropic process*,

$$\frac{T_2}{T_1} = \left( \frac{v_1}{v_2} \right)^{k-1} = \left( \frac{V_1}{V_2} \right)^{k-1} \Rightarrow \text{using Eq. (5)} \Rightarrow \left( \frac{p_2}{p_1} \right)^{\frac{k-1}{k}} = \left( \frac{V_1}{V_2} \right)^{k-1} \Rightarrow V_2 = V_1 \left( \frac{p_2}{p_1} \right)^{-\frac{1}{k}}. \quad (6)$$

Substituting the given and calculated values,  $V_2 = 0.193 \text{ L}$ , which is less than 1% larger than the more accurate value found previously. As can be observed in this problem, the perfect gas assumption gives accurate values over this range in temperature values (300 K to 575 K).



For part (c), apply the 1<sup>st</sup> Law using a control volume surrounding the air in the cylinder,

$$\Delta E_{CV} = \sum_{in} m(h + ke + pe) - \sum_{out} m(h + ke + pe) + Q_{in} + W_{in}, \quad (7)$$

where,

$$\Delta E_{CV} = \Delta U + \Delta KE + \Delta PE = \Delta U = m(u_2 - u_1) \quad (\text{neglecting changes in KE and PE}), \quad (8)$$

$$\sum_{in} m(h + ke + pe) - \sum_{out} m(h + ke + pe) = 0 \quad (\text{no inlets or outlets}), \quad (9)$$

$$Q_{in} = 0 \quad (\text{assuming adiabatic operation}), \quad (10)$$

$$W_{in} = ? \quad (11)$$

Substitute and solve for the work,

$$W_{in} = m(u_2 - u_1), \quad (12)$$

Using the temperatures of  $T_1 = 300 \text{ K}$  and  $T_2 = 575 \text{ K}$ ,

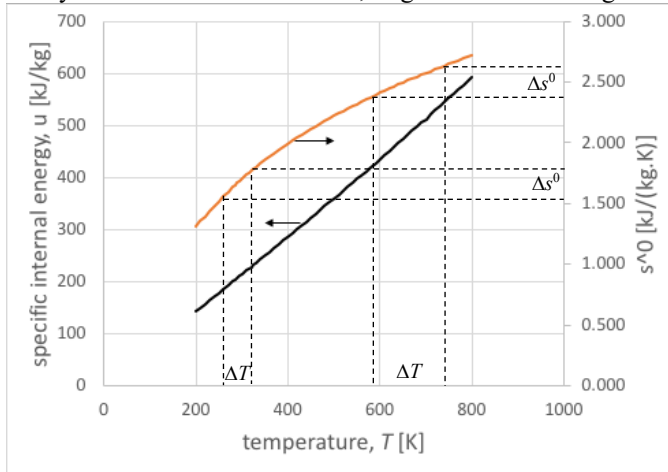
$$u_1 = u(T_1) = 214.1 \text{ kJ/kg} \quad \text{and} \quad u_2 = u(T_2) = 415.8 \text{ kJ/kg,}$$

$$\Rightarrow W_{in} = 2.34 \text{ kJ.}$$

Now consider how the work changes as the initial temperature increases, but the air mass and pressures remain the same. From Eq. (12) we observe that the work is proportional to the change in internal energy, i.e.,

$$W_{in} \propto \Delta u. \tag{13}$$

For an ideal gas,  $u = u(T)$  and from the figure below where  $u$  is plotted as a function of  $T$ , we observe that  $u$  is a nearly linear function of  $T$ . Thus, larger  $\Delta T$  values will give larger  $\Delta u$  values.



The  $\Delta T$  is found indirectly from Eq. (2),

$$s^0(T_2) = s^0(T_1) + R \ln \left( \frac{p_2}{p_1} \right) \Rightarrow s^0(T_2) - s^0(T_1) = R \ln \left( \frac{p_2}{p_1} \right) \Rightarrow \Delta s^0 = \text{constant}, \tag{14}$$

since  $p_2/p_1$  and  $R$  are constants. Now examine the plot of  $s^0$  as a function of  $T$  in the figure above. Here the curve has negative curvature, meaning that for the same  $\Delta s^0$ , the  $\Delta T$  will be larger at larger  $T$  values. Thus, as  $T$  increases,  $\Delta s^0$  increases  $\Rightarrow \Delta T$  increases  $\Rightarrow \Delta u$  increases  $\Rightarrow W_{in}$  increases.

