By injecting liquid water into superheated vapor, the de-superheater shown in the figure has a saturated vapor stream at its exit. Steady-state operating data are shown in the figure. Ignoring stray heat transfer and kinetic and potential energy effects, determine:
a. the mass flow rate of the superheated vapor stream, in $\mathrm{kg} / \mathrm{min}$, and
b. the rate of entropy production within the de-superheater, in $\mathrm{kW} / \mathrm{K}$.
c. Sketch the process on a $T-S$ diagram.


## SOLUTION:



First apply Conservation of Mass to the control volume shown above,

$$
\begin{equation*}
\frac{d \dot{M}}{d t}=\sum_{i n} \dot{m}-\sum_{o u t} \dot{m} \tag{1}
\end{equation*}
$$

where,

$$
\begin{align*}
& \frac{d M}{d t}=0 \quad \text { (steady state operation) }  \tag{2}\\
& \sum_{\text {in }} \dot{m}-\sum_{\text {out }} \dot{m}=\dot{m}_{1}+\dot{m}_{2}-\dot{m}_{3} \tag{3}
\end{align*}
$$

Substituting,

$$
\begin{align*}
& \dot{m}_{1}+\dot{m}_{2}-\dot{m}_{3}=0 .  \tag{4}\\
& \dot{m}_{3}=\dot{m}_{1}+\dot{m}_{2} \tag{5}
\end{align*}
$$

Now apply the $1^{\text {st }}$ Law to the same control volume,

$$
\begin{equation*}
\frac{d E}{d t}=\sum_{i n} \dot{m}(h+k e+p e)-\sum_{\text {out }} \dot{m}(h+k e+p e)+\dot{Q}_{\text {in }}-\dot{W}_{o u t} \tag{6}
\end{equation*}
$$

where,

$$
\begin{align*}
& \frac{d E}{d t}=0 \quad \text { (steady state operation) },  \tag{7}\\
& \sum_{\text {in }} \dot{m}(h+k e+p e)-\sum_{\text {out }} \dot{m}(h+k e+p e)=\dot{m}_{1} h_{1}+\dot{m}_{2} h_{2}-\dot{m}_{3} h_{3}, \tag{8}
\end{align*}
$$

(neglecting changes in KE and PE between the inlets and outlets),
$\dot{Q}_{i n}=0$ (assuming the device is well insulated),
$\dot{W}_{\text {out }}=0$ (the device is passive).
Substituting and solving for the mass flow rate at inlet 2, making use of Eq. (3),

$$
\begin{align*}
& \dot{m}_{1} h_{1}+\dot{m}_{2} h_{2}-\dot{m}_{3} h_{3}=0  \tag{11}\\
& \dot{m}_{1} h_{1}+\dot{m}_{2} h_{2}-\left(\dot{m}_{1}+\dot{m}_{2}\right) h_{3}=0,  \tag{12}\\
& \dot{m}_{1}\left(h_{1}-h_{3}\right)+\dot{m}_{2}\left(h_{2}-h_{3}\right)=0,  \tag{13}\\
& \dot{m}_{2}=\dot{m}_{1}\left(\frac{h_{3}-h_{1}}{h_{2}-h_{3}}\right) . \tag{14}
\end{align*}
$$

Determine the properties at each inlet/outlet using the property tables for water.
State 1 (compressed liquid):

$$
\begin{aligned}
& T_{1}=20^{\circ} \mathrm{C}, \\
& p_{1}=0.3 \mathrm{MPa}(\mathrm{abs})=3 \mathrm{bar}(\mathrm{abs})>p_{\text {sat }} @ 20{ }^{\circ} \mathrm{C}=0.023393 \mathrm{bar}(\mathrm{abs}), \\
& v_{C L}\left(T_{1}, p_{1}\right) \approx v_{f}\left(T_{1}\right)=0.0010018 \mathrm{~m}^{3} / \mathrm{kg}, \\
& h_{C L}\left(T_{1}, p_{1}\right) \approx h_{f}\left(T_{1}\right)+\left[p_{1}-p_{\text {sat }}\left(T_{1}\right)\right] v_{f}\left(T_{1}\right)= \\
& \quad=(83.914 \mathrm{~kJ} / \mathrm{kg})+(3 \mathrm{bar}-0.023393 \mathrm{bar})^{*}\left(0.0010018 \mathrm{~m}^{3} / \mathrm{kg}\right)^{*}(100 \mathrm{kPa} / \mathrm{bar})=84.2122 \mathrm{~kJ} / \mathrm{kg}, \\
& s_{C L}\left(T_{1}, p_{1}\right) \approx s_{f}\left(T_{1}\right)=0.296480 \mathrm{~kJ} /(\mathrm{kg} . \mathrm{K}) .
\end{aligned}
$$

State 2 (superheated vapor):
$T_{2}=200^{\circ} \mathrm{C}$,
$p_{2}=0.3 \mathrm{MPa}(\mathrm{abs})=3 \mathrm{bar}(\mathrm{abs})$,
$h\left(T_{2}, p_{2}\right)=2865.9 \mathrm{~kJ} / \mathrm{kg}$,
$s\left(T_{2}, p_{2}\right)=7.313 \mathrm{~kJ} /(\mathrm{kg} . \mathrm{K})$.
State 3 (saturated vapor):

$$
\begin{aligned}
& p_{1}=0.3 \mathrm{MPa}(\mathrm{abs})=3 \mathrm{bar}(\mathrm{abs}), \\
& T_{\text {sat }} @ 3 \text { bar }=133.52^{\circ} \mathrm{C}, \\
& h\left(T_{3}, p_{3}\right)=2724.9 \mathrm{~kJ} / \mathrm{kg} \\
& s\left(T_{3}, p_{3}\right)=6.9916 \mathrm{~kJ} /(\mathrm{kg} . \mathrm{K}) .
\end{aligned}
$$

Substituting these values along with $\dot{m}_{1}=6.37 \mathrm{~kg} / \mathrm{min}$ (given) into Eq. (14), $\dot{m}_{2}=119 \mathrm{~kg} / \mathrm{min} \Rightarrow$ (using Eq. (5)) $\dot{m}_{3}=126 \mathrm{~kg} / \mathrm{min}$.

Now apply the Entropy Equation to the same control volume,

$$
\begin{equation*}
\frac{d s}{d t}=\sum_{i n} \dot{m} s-\sum_{o u t} \dot{m} s+\int_{b} \frac{\dot{Q}_{i n}}{T}+\dot{\sigma}, \tag{15}
\end{equation*}
$$

where,

$$
\begin{align*}
& \frac{d S}{d t}=0 \quad \text { (steady state operation) },  \tag{16}\\
& \sum_{i n} \dot{m} s-\sum_{o u t} \dot{m} s=\dot{m}_{1} s_{1}+\dot{m}_{2} s_{2}-\dot{m}_{3} s_{3},  \tag{17}\\
& \int_{b} \frac{\dot{Q}_{\text {in }}}{T}=0 \quad \text { (assuming the device is well insulated) },  \tag{18}\\
& \dot{\sigma}=? \tag{19}
\end{align*}
$$

Substitute and solve for the rate of entropy production,

$$
\begin{equation*}
\dot{\sigma}=\dot{m}_{3} s_{3}-\dot{m}_{1} s_{1}-\dot{m}_{2} s_{2} . \tag{20}
\end{equation*}
$$

Using the previously evaluated parameters, $\dot{\sigma}=0.0718 \mathrm{~kW} / \mathrm{K}$. (Note that $1 \mathrm{~kg} / \mathrm{min}=(1 / 60) \mathrm{kg} / \mathrm{s}$.
There is positive entropy production, as expected.


