By injecting liquid water into superheated vapor, the de-superheater shown in the figure has a saturated vapor stream at its exit. Steady-state operating data are shown in the figure. Ignoring stray heat transfer and kinetic and potential energy effects, determine:

- a. the mass flow rate of the superheated vapor stream, in kg/min, and
- b. the rate of entropy production within the de-superheater, in kW/K.
- c. Sketch the process on a *T*-*s* diagram.





https://www.enggcyclopedia.com/2011/07/steam-desuperheater/

(10)

SOLUTION:



First apply Conservation of Mass to the control volume shown above,

$\frac{dM}{dt} = \sum_{in}$	$\dot{m} - \sum_{out} \dot{m},$	(1)
where,		
$\frac{dM}{dt} = 0$	(steady state operation),	(2)

$$\sum_{in} \dot{m} - \sum_{out} \dot{m} = \dot{m}_1 + \dot{m}_2 - \dot{m}_3, \tag{3}$$

Substituting,

$$\dot{m}_1 + \dot{m}_2 - \dot{m}_3 = 0.$$

$$\dot{m}_3 = \dot{m}_1 + \dot{m}_2.$$
(4)
(5)

Now apply the 1st Law to the same control volume,

$$\frac{dE}{dt} = \sum_{in} \dot{m}(h + ke + pe) - \sum_{out} \dot{m}(h + ke + pe) + \dot{Q}_{in} - \dot{W}_{out},$$
(6)
where,

$$\frac{dE}{dt} = 0 \quad \text{(steady state operation)},\tag{7}$$

$$\sum_{in} \dot{m}(h + ke + pe) - \sum_{out} \dot{m}(h + ke + pe) = \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3,$$
(neglecting changes in KE and PE between the inlets and outlets),
(8)

 $\dot{Q}_{in} = 0$ (assuming the device is well insulated), (9)

$$\dot{W}_{out} = 0$$
 (the device is passive).

Substituting and solving for the mass flow rate at inlet 2, making use of Eq. (3),

 $\dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3 = 0, \tag{11}$ $\dot{m}_1 h_1 + \dot{m}_2 h_2 - (\dot{m}_1 + \dot{m}_2) h_2 = 0. \tag{12}$

$$\dot{m}_1 n_1 + m_2 n_2 - (m_1 + m_2) n_3 = 0,$$

$$\dot{m}_1 (h_1 - h_3) + \dot{m}_2 (h_2 - h_3) = 0,$$
(12)

$$\dot{m}_2 = \dot{m}_1 \left(\frac{h_3 - h_1}{h_2 - h_3} \right). \tag{14}$$

Determine the properties at each inlet/outlet using the property tables for water.

State 1 (compressed liquid): $T_1 = 20 \text{ °C},$ $p_1 = 0.3 \text{ MPa (abs)} = 3 \text{ bar (abs)} > p_{\text{sat } @ 20 \text{ °C}} = 0.023393 \text{ bar (abs)},$ $v_{CL}(T_1, p_1) \approx v_f(T_1) = 0.0010018 \text{ m}^3/\text{kg},$ $h_{CL}(T_1, p_1) \approx h_f(T_1) + [p_1 - p_{sat}(T_1)]v_f(T_1) =$ $= (83.914 \text{ kJ/kg}) + (3 \text{ bar} - 0.023393 \text{ bar})^*(0.0010018 \text{ m}^3/\text{kg})^*(100 \text{ kPa/bar}) = 84.2122 \text{ kJ/kg},$ $s_{CL}(T_1, p_1) \approx s_f(T_1) = 0.296480 \text{ kJ/(kg.K)}.$

State 2 (superheated vapor):

 $T_2 = 200 \text{ °C},$ $p_2 = 0.3 \text{ MPa (abs)} = 3 \text{ bar (abs)},$ $h(T_2, p_2) = 2865.9 \text{ kJ/kg},$ $s(T_2, p_2) = 7.313 \text{ kJ/(kg.K)}.$

State 3 (saturated vapor):

 $p_1 = 0.3 \text{ MPa (abs)} = 3 \text{ bar (abs)},$ $T_{\text{sat (@ 3 bar}} = 133.52 \text{ °C},$ $h(T_3, p_3) = 2724.9 \text{ kJ/kg},$ $s(T_3, p_3) = 6.9916 \text{ kJ/(kg.K)}.$

Substituting these values along with $\dot{m}_1 = 6.37$ kg/min (given) into Eq. (14), $\dot{m}_2 = 119$ kg/min \Rightarrow (using Eq. (5)) $\dot{m}_3 = 126$ kg/min.

Now apply the Entropy Equation to the same control volume,

 $\frac{ds}{dt} = \sum_{in} \dot{m}s - \sum_{out} \dot{m}s + \int_b \frac{\dot{q}_{in}}{T} + \dot{\sigma},$ (15) where,

 $\frac{ds}{dt} = 0 \quad (\text{steady state operation}), \tag{16}$ $\sum_{in} \dot{m}s - \sum_{out} \dot{m}s = \dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3, \tag{17}$ $\int_b \frac{\dot{Q}_{in}}{T} = 0 \quad (\text{assuming the device is well insulated}), \tag{18}$ $\dot{\sigma} = ? \tag{19}$

Substitute and solve for the rate of entropy production,

$$\dot{\sigma} = \dot{m}_3 s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2. \tag{20}$$

Using the previously evaluated parameters,

 $\dot{\sigma} = 0.0718 \text{ kW/K}$. (Note that 1 kg/min = (1/60) kg/s.) There is positive entropy production, as expected.

