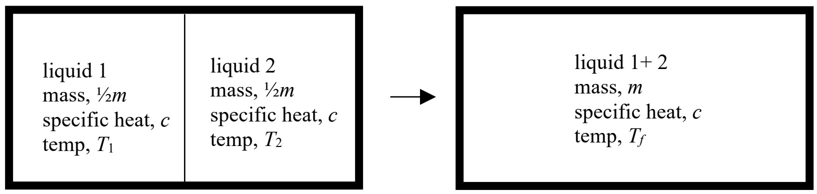
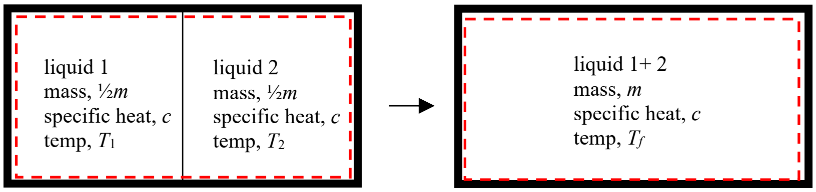
An isolated system of total mass m is formed by mixing two equal masses of the same liquid, assumed incompressible with the same specific heat c, initially at the absolute temperatures T_1 and T_2. Eventually the system attains an equilibrium state.

a. Determine the amount of entropy produced in terms of m, c, T_1, and T_2.

b. Demonstrate that σ must be positive if $T_1 \neq T_2$.



SOLUTION:



Apply the Entropy Equation to a control volume that includes the liquids,

$$\Delta S = \int_b \frac{\delta Q_{into\ CV}}{T} + \sigma + \sum_{in} ms - \sum_{out} ms, \quad (1)$$

where,

$$\Delta S = \Delta S_1 + \Delta S_2 = \tfrac{1}{2}mc\,ln\left(\frac{T_f}{T_1}\right) + \tfrac{1}{2}mc\,ln\left(\frac{T_f}{T_2}\right) \text{ (assuming incompressible and constant specific heats),} \quad (2)$$

$$\int_b \frac{\delta Q_{into\ CV}}{T} = 0, \text{ (the control volume is isolated),} \quad (3)$$

$$\sum_{in} ms - \sum_{out} ms = 0 \text{ (the control volume has no inlets or outlets).} \quad (4)$$

Substitute and solve for the entropy production,

$$\sigma = \tfrac{1}{2}mc\,ln\left(\frac{T_f}{T_1}\right) + \tfrac{1}{2}mc\,ln\left(\frac{T_f}{T_2}\right) = \tfrac{1}{2}mc\left[ln\left(\frac{T_f}{T_1}\right) + ln\left(\frac{T_f}{T_2}\right)\right], \quad (5)$$

$$\boxed{\sigma = \tfrac{1}{2}mc\,ln\left(\frac{T_f^2}{T_1T_2}\right)}, \quad (6)$$

Applying the 1st Law to the same CV,

$$\Delta E = \sum_{in} m(h + ke + pe) - \sum_{out} m(h + ke + pe) + Q_{into} - W_{by} \quad (7)$$

where,

$$\Delta E = \Delta U + \Delta KE + \Delta PE = \Delta U = \Delta U_1 + \Delta U_2 = \tfrac{1}{2}mc(T_f - T_1) + \tfrac{1}{2}mc(T_f - T_2), \quad (8)$$

$$\Delta E = \tfrac{1}{2}mc(2T_f - T_1 - T_2) \text{ (assuming incompressible and constant specific heats),}$$

$$\sum_{in} m(h + ke + pe) - \sum_{out} m(h + ke + pe) = 0 \text{ (the CV has no inlets or outlets),} \quad (9)$$

$$Q_{into} = 0 \text{ (the control volume is isolated),} \quad (10)$$

$$W_{by} = 0 \text{ (the control volume is isolated),} \quad (11)$$

Substitute and solve for T_f,

$$\tfrac{1}{2}mc(2T_f - T_1 - T_2) = 0, \quad (12)$$

$$T_f = \tfrac{1}{2}(T_1 + T_2). \quad (13)$$

Substitute Eq. (13) into Eq. (6) and simplify,

$$\sigma = \frac{1}{2}mc\ln\left[\frac{\frac{1}{4}(T_1+T_2)^2}{T_1T_2}\right], \quad (16)$$

$$\frac{\sigma}{\frac{1}{2}mc} = \ln\left[\frac{\frac{1}{4}(T_1+T_2)^2}{T_1T_2}\right], \quad (17)$$

Note that in order for $\sigma \geq 0$ (to satisfy the 2nd Law),

$$\frac{(T_1+T_2)^2}{4T_1T_2} \geq 1 \;\Rightarrow\; T_1^2 + 2T_1T_2 + T_2^2 \geq 4T_1T_2 \;\Rightarrow\; T_1^2 - 2T_1T_2 + T_2^2 \geq 0, \quad (18)$$

$$(T_1 - T_2)^2 \geq 0. \quad (19)$$

Equation (19) is equal to zero if $T_1 = T_2$, which results in $\sigma = 0$ and an internally reversible process. Equation (19) is greater than zero when either $T_1 > T_2$ or $T_2 > T_1$, which corresponds to $\sigma > 0$ and an internally irreversible process.