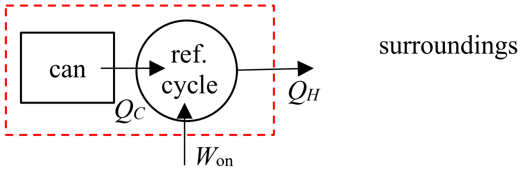
The temperature of a 12 oz (0.354 l) can of soft drink is reduced from 20 °C to 5 °C by a refrigeration cycle. The cycle receives energy by heat transfer from the soft drink and discharges energy by heat transfer at 20 °C to the surroundings. There are no other heat transfers. Determine the minimum theoretical work input required. You may ignore the aluminum can in your calculations.

SOLUTION:
A schematic of the situation is shown below.



Apply the First Law to a system consisting of the can and the refrigeration equipment,

$$\Delta E_{sys} = Q_{\underset{sys}{into}} + W_{\underset{sys}{on}}, \tag{1}$$

where,

$$\Delta E_{sys} = \Delta E_{can} + \Delta E_{ref.\ cycle}, \tag{2}$$

where changes in kinetic and potential energies are ignored for the can so $\Delta E_{can} = \Delta U_{can}$. Furthermore, since the refrigeration equipment operates over a cycle, $\Delta E_{ref.\ cycle} = 0$. Hence, Eq. (1) becomes,

$$\Delta U_{can} = -Q_H + W_{on} \quad \Rightarrow \quad W_{on} = \Delta U_{can} + Q_H . \tag{3}$$

The change in the can internal energy is,

$$\Delta U_{can} = mc(T_{can,f} - T_{can,i}), \tag{4}$$

where the soft drink is modeled as an incompressible substance since it's a liquid. The parameter m is the mass of the can and c is its specific heat.

The heat transferred out of the system may be found by applying the Entropy Equation,

$$\Delta S_{sys} = \int_{1,b}^{2} \frac{\delta Q_{into}}{T} + \sigma = \frac{-Q_H}{T_H} + \sigma \quad \Rightarrow \quad Q_H = T_H\sigma - T_H \Delta S_{sys}, \tag{5}$$

where,

$$\Delta S_{sys} = \Delta S_{can} + \Delta S_{ref.\ cycle}. \tag{6}$$

Since the refrigeration equipment operates on a cycle, $\Delta S_{ref.\ cycle} = 0$. The absolute temperature at the boundary of the system where the heat is transferred out of the system is T_H and σ is the entropy produced during the process due to irreversibilities.

Substituting Eq. (5) into Eq. (3) and simplifying gives,

$$W_{on} = \Delta U_{can} + T_H\sigma - T_H \Delta S_{sys} . \tag{7}$$

Since we're interested in the minimum amount of work required during the process, consider the case when $\sigma = 0$ (an internally reversible process). Recall that $\sigma > 0$ when irreversibilities are present. Since the soft drink is assumed to be an incompressible substance,

$$\Delta S_{can} = mc \ln\left(\frac{T_{can,f}}{T_{can,i}}\right). \tag{8}$$

Substituting Eqs. (4) and (8) into Eq. (7) (with $\sigma = 0$) gives,

$$W_{on,min} = mc\left(T_{can,f} - T_{can,i}\right) - T_H mc \ln\left(\frac{T_{can,f}}{T_{can,i}}\right), \tag{9}$$

$$\boxed{W_{on,min} = mc\left[\left(T_{can,f} - T_{can,i}\right) - T_H \ln\left(\frac{T_{can,f}}{T_{can,i}}\right)\right]}. \tag{10}$$

Using the following parameters,

m = (1000 kg/m^3)(0.354 l)(10^{-3} m^3/l) = 0.354 kg (assume the density of liquid water)
c = 4.2 kJ/(kg.K) (assume the specific heat of liquid water)
$T_{\text{can},f}$ = 5 °C = 278 K
$T_{\text{can},i}$ = 20 °C = 293 K
T_H = 20 °C = 293 K
⇒ $\boxed{W_{\text{in,min}} = 0.591 \text{ kJ}}$

If we assume the soft drink is a compressed liquid instead of being incompressible, then the change in entropy is,

$$\Delta S_{\text{soda}} = m\Delta s_{\text{soda}} \approx m\left[s_l\left(T_{\text{soda,f}}\right) - s_l\left(T_{\text{soda,i}}\right)\right], \tag{11}$$

where $s_{\text{CL}}(T, p) \approx s_l(T)$. Treating the soda as water and using Table A-2 in Moran et al., 8th ed.,

$s_l(T_{\text{soda},f} = 5\text{ °C}) = 0.0761$ kJ/(kg.K),
$s_l(T_{\text{soda},i} = 20\text{ °C}) = 0.2966$ kJ/(kg.K),
=> $\Delta S_{\text{soda}} = -0.0781$ kJ/K,

which is identical to the result found using the incompressible substance model.